Rock matrix-fractured media model for heterogeneous and fractured coal bed

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Abstract: Coal was considered rock matrix-fractured media composed of rock matrix and fractures, and the rock matrix-fractured media model for heterogeneous and fractured coal bed was presented. In this model the rock matrix is heterogeneous, and the mechanical parameters such as elastic modulus and strength follow Weibull distribution. Fractures in coal bed were generated with the discrete fracture network method, and the properties of fractures were simulated with Desai element. Then the virtual generating system (VGS) of natural heterogeneous and fractured coal bed was developed in Matlab 6.0. The coupled model of gas flow and deformation process based on the rock matrix-fractured media model method and VGS for heterogeneous and fractured coal bed was presented, and the numerical code was developed in Matlab 6.0. The gas flow process in the heterogeneous and fractured coal bed was simulated in a numerical case. The main conclusions are: 1) The natural heterogeneous and fractured coal bed could be simulated by the rock matrix-fractured media model and VGS; 2) The fractures connected with the well have much more effects on gas flow than those non-connected.

Key words: heterogeneity; fractures; rock matrix-fractured media

1 Introduction

Methane in coal bed is an important natural energy resource, although ignition and the resulting explosion hazard remain a major problem during coal mining. Degassing bed is an important method to mitigate this hazard and results in beneficial recovery of a clean-burning and low-carbon fuel resource [1–9].

Coal is a heterogeneous and fractured geological material. On one hand, coal is a heterogeneous geological material which contains natural weakness at various scales. When methane recovery begins, these pre-existing weaknesses can close, open, extend or induce new fractures due to the changes of the effective stress, which can in turn change the structure of the coal and alter its flow properties. On the other hand, coal is a fractured geological material which contains natural faults and fractures at various scales. Mechanical response and flow properties of coal are greatly influenced, even controlled by the faults and fractures in coal bed. So how to simulate the heterogeneous and fractured coal bed is a key issue for the analysis of methane recovery.

The failure processes of heterogeneous rock have been modeled in some references [10–12]. In early 1980s, the discrete fractures network (DFN) method was presented to consider fluid flow and transport processes in fractured rock masses. The method enjoys wide applications for problems of fractured rocks, mainly due to its conceptual attractiveness. The stochastic simulation of fracture systems is the geometric basis of the DFN approach [13–14]. However, the model for the heterogeneous and fractured coal bed has never been reported yet.

In this work, the model for the heterogeneous and fractured coal bed based on Monte Carlo technology was presented, and the virtual generating system (VGS) of natural heterogeneous and fractured coal bed was developed. Furthermore, the model of coupled gas flow and deformation process for heterogeneous and fractured coal bed was demonstrated.

2 Model description

2.1 Simulation of heterogeneity for coal bed

Mechanical properties are assumed locally heterogeneous (including elastic modulus and strength)
and are represented by the Weibull distribution [10],

\[
\varphi = \frac{m}{s_0} \left( \frac{s}{s_0} \right)^{m-1} \exp \left[ -\left( \frac{s}{s_0} \right)^m \right]
\]  

(1)

where \( s \) represents the variables of elastic modulus and strange \( s_0 \) represents the corresponding mean value; \( m \) is a homogeneity index, which may be derived from the statistical distribution of rock parameters.

2.2 Simulation of fractures in coal bed

The fracture system geometry in coal bed is generated based on stochastic Monte Carlo simulations of fracture systems, using the PDFs (probability density functions) of fracture parameters formulated according to the field mapping results, in addition to the assumption about elliptical fracture shape (plane problem).

The fractures geometrical shape parameters include: density, orientation (dip), centre, trace length and width. The density of the fractures follows Poisson distribution, and the centre of a fracture follows uniform distribution. Orientation (strike) follows normal or uniform distribution. Both the trace length and the width follow lognormal distribution [13].

To transform the virtual coal into finite element model, the fractures and the matrix are viewed as different materials. During generating virtual coal bed, the fractures and rock matrix are automatically divided.

The finite element meshes for the virtual fractured coal are demonstrated in Fig. 1.

2.3 Virtual system of heterogeneous and fractured coal bed

Natural coal is viewed as rock matrix-fractured media, and virtual coal bed is composed of fractures element and coal matrix. Based on the above analysis, virtual heterogeneous and fractured coal is generated as follows.

Firstly, with experiments, the statistical distribution and corresponding parameters of mechanical properties of coal matrix are analysed and obtained. Then, the PDFs of fracture parameters are formulated according to field mapping results, and DFN model for coal is set up. Thirdly, the fracture system is generated with Monte Carlo technology, and the finite element meshes are generated. Lastly, based on the mechanical parameters statistical analysis for the heterogeneous matrix, the heterogeneous mechanical properties of coal are simulated.

Based on the above process, the virtual generating system of natural heterogeneous and fractured coal bed (VGS) is developed in Matlab 6.0.

Fig. 1 Virtual fractures and corresponding FEM grids: (a) FEM grid of single set of non-filled discontinuous fracture; (b) FEM grid of double sets of filled and non-filled discontinuous fractures; (c) FEM grid random filled and non-filled fractures

3 Model of coupled gas flow and deformation process of rock matrix-fractured media

3.1 Matrix

Suppose that the matrix is Mohr–Coulomb material and composed of a solid matrix that contains interstitial pore space filling with freely diffusing pore gas. The ideal gas law is used to describe the relation between gas density and pressure. Gas flow follows Darcy’s law, and
the gravitational term is relatively small and ignored. Mechanical equilibrium equation can be expressed as:

$$\sigma_{ij} + f_i = (ap \delta)_{ij} = 0$$  \hspace{1cm} (2)

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})$$  \hspace{1cm} (3)

$$d\sigma_{ij} = D_{ijkl} d\varepsilon_{kl}$$  \hspace{1cm} (4)

$$\sigma_{ij} = \sigma_{ij} + ap \delta_{ij}$$  \hspace{1cm} (5)

where $\sigma_{ij}$, $\varepsilon_{ij}$ and $a$ are total stress, effective stress and Biot coefficient, respectively; $D_{ijkl}$ is elasto-plastic matrix; and $\varepsilon_{ij}$ is strain; $f_i$ and $u_{ij}$ ($i= x, y, z$) are the components of the net body force and displacement in the i-direction; $p$ is gas pressure, MPa.

Free-phase and physical absorbed methane exist in coal bed. Methane content in coal bed can be similarly expressed as [5]:

$$W = A \sqrt{P}$$  \hspace{1cm} (6)

where $W$ is the methane content, m$^3$/t; $A$ is the coefficient of the methane content (ranging from 1 to 4), m$^2$/(t·MPa$^{1/2}$).

Under isothermal conditions, the gas flow in porous media is governed by a mass balance equation,

$$\nabla^2 K \cdot P = S(P) \frac{\partial P}{\partial t} + 2\sqrt{P} \frac{\partial \varepsilon_{ij}}{\partial t}$$  \hspace{1cm} (7)

where $S(P) = 0.25AP^{-0.75}$; $\varepsilon_{ij}$ is volumetric strain; $K$ is gas permeability coefficient; $P$ is gas square pressure, $P=\sqrt{P}$.

Permeability coefficient varies with element stress state [9]:

$$K = \begin{cases} K_0 \exp(-\beta \sigma_{ij}'), \text{ In elastic state} \\ \xi K_0 \exp(-\beta \sigma_{ij}'), \text{ In shear failure state} \\ \xi' K_0 \exp(-\beta' \sigma_{ij}'), \text{ In tension failure state} \end{cases}$$  \hspace{1cm} (8)

where $K_0$ is gas permeability coefficient without stress; $\xi$ and $\xi'$ are the modified indexes of shear and tension failure, respectively, which can be derived from experiments; $\beta$ is the experimental parameter; and $\sigma_{ij}'$ is the minimal effective stress.

### 3.2 Fractures

Fractures are simulated with Desai thin layer element [15]. The thin layer element is linear and elastic and the constitutive model can be expressed as:

$$\begin{bmatrix} d\sigma'_{ij} \\ d\sigma_n' \\ d\sigma_n'' \end{bmatrix} = \begin{bmatrix} D_{ss} & D_{sn} & 0 \\ D_{sn} & D_{nn} & 0 \\ 0 & 0 & D_{nn} \end{bmatrix} \begin{bmatrix} d\varepsilon_{ij} \\ d\varepsilon_n \\ d\varepsilon_n' \end{bmatrix}$$  \hspace{1cm} (9)

where $d\sigma'_{ij}$, $d\sigma_n'$, and $d\sigma_n''$ are the vectors of increments of the effective shear and normal stresses, respectively; $D_{ss}$, $D_{nn}$ are shear component and normal component in constitutive matrix, respectively; $d\varepsilon_{ij}$, $d\varepsilon_n$ are the vectors of increments of shear and normal strains, respectively.

The gas flow in fractures is expressed as:

$$q_i = K_{\ell,i} \frac{\partial P}{\partial s_i}$$  \hspace{1cm} (10)

where $K_{\ell,i}$ is the permeability coefficient in the direction of the fracture, which is decided by normal effective stress:

$$K_{\ell,i} = K_{\ell,0} \exp(-\beta_i \sigma_n')$$  \hspace{1cm} (11)

where $\beta_i$ is the experimental parameter; $\sigma_n'$ is the effective normal stress.

The gas mass in fractures is expressed as:

$$W_{\ell} = A_\ell \sqrt{P}$$  \hspace{1cm} (12)

where $W_{\ell}$ is the gas mass in fractures, m$^3$/t; $A_\ell$ is the coefficient of gas content of fracture, 0.1–0.3 m$^3$/ (t·MPa$^{1/2}$).

The gas flow equation in fractures is expressed by:

$$\frac{\partial}{\partial x} \left( K_{\ell,i} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{\ell,i} \frac{\partial P}{\partial y} \right) = S_{\ell}(P) \frac{\partial P}{\partial t} - 2\sqrt{P} \frac{\partial \varepsilon_{ij}}{\partial t}$$  \hspace{1cm} (13)

where $S_{\ell}(P) = 0.25 A_\ell P^{-0.75}$.

The model of coupled gas flow and deformation process for the heterogeneous fractured coal bed is represented by Eq. (1)–Eq. (13) with the boundary and initial conditions. The code for the model is developed in Matlab 6.0.

### 4 Case study

A methane production well in Liaoning Province, China, is studied. The well depth is 173 m. The unit mass of overburden is 2.3 kg/m$^3$. The self-weight stress is 4 MPa. The gas pressure is 1.1 MPa. The radius of gas production well is 0.18 m. The study region is 20 m x 20 m with the fixed boundary and Dirichlet boundary $p=1.1$ MPa. The dirichlet boundary of the well is $p=0.1$ MPa. And the other parameters are listed in Table 1.

Based on the parameters in Table 1, VGS is adopted to generate the virtual heterogeneous and fractured coal bed, and the FE meshes are also generated. The FE meshes are inputted into VGS, and gas pressure and the permeability coefficient in 2880 d can be seen in Fig. 2 and Fig. 3. From Fig. 2 it can be found that gas flow path is shortened due to the fractures in coal bed, and methane production is improved. Besides, the fractures connected with the well have much more effects on gas flow than those non-connected.

The permeability coefficient can be seen from Fig. 3.
It is found that some failure regions occur due to the mechanical heterogeneity of coal and gas flow. In failure regions the permeability enhances, and gas pressure is not perfectly symmetrical.

Table 1 Parameters of case study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Remark</th>
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<tbody>
<tr>
<td>Elastic modulus/MPa</td>
<td>2 100</td>
<td>Weibull distribution (Wd), ( m = 4 )</td>
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<td>Poisson ratio</td>
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<tr>
<td>Internal friction angle(°)</td>
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<td>Shear failure modified Index, ( \xi )</td>
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</tr>
<tr>
<td>Tension failure modified index, ( \xi )</td>
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<td>Permeability coefficient, ( K_f ) ( (m^2 MPa^{-2} d^{-1}) )</td>
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<td></td>
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<tr>
<td>Content coefficient of methane, ( A_f ) ( (m^2 MPa^{-2} d^{-1}) )</td>
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<tr>
<td>Density of fractures/m(^{-2} )</td>
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<td>Permeability coefficient, ( K_{fg} ) ( (m^2 MPa^{-2} d^{-1}) )</td>
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</tbody>
</table>

5 Conclusions

1) The naturally heterogeneous and fractured coal bed could be simulated by virtual generating system (VGS).
2) The rock matrix-fractured media model can simulate the coupled gas flow and mechanical progress in heterogeneous and fractured coal bed.
3) The fractures connected with the well have much more effects on gas flow than those non-connected.

References


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