Evaluation of interface fracture toughness in SiC fiber reinforced titanium matrix composite

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Abstract: The finite element method based on the equivalent domain integral technique was developed to simulate the push out test and evaluate the interfacial fracture toughness of SiC reinforced titanium matrix composites. A special subroutine was introduced while modeling the push-out test to control interfacial failure process. In addition, the residual stresses, Poisson ratio and friction stresses were all considered in the finite element analysis and the interface debonding was described as a continuous process. The results show that the interfacial fracture toughness of SiC/Timetal-834 is about 50 J/m². Moreover, the effects of various parameters on the interfacial fracture toughness and the variations of energy release rates at both ends of the specimen were analyzed in detail.

Key words: push out test; interface fracture toughness; finite element analysis (FEA); SiC/Ti composite

1 Introduction

The push out test has been developed for a few years as a means of quantifying interfacial properties of SiC fiber reinforced titanium matrix composites (TMCs)[1−4]. However, the values of interfacial shear strength and frictional shear stresses obtained from the push out test are only average assessments. Since the interface stress distribution is complex due to edge effect or crack growth during the push-out test[5], the average values do not represent the real behavior of the interfaces and may lead to misleading in some cases. The finite element method, which is used to model the push out test, just predicts the real stress distribution at the interface[5−7]. Moreover, the evaluation of interfacial fracture toughness using finite element method is independent of mesh density. Unfortunately, only very few finite element simulations based on the energy failure criterion have been done. MUKHERIEE et al[8−9] and ZENG et al[10] used the finite element method based on the energy failure criterion to evaluate interfacial fracture toughness of TMCs. However, in MUKHERIEE’s analysis, the effect of frictional stress on the interfacial fracture toughness was ignored. In ZENG’s analysis, the interface debonding process was not described as a continuous one.

In this work, the finite element method based on the equivalent domain integral technique is presented to evaluate the interfacial fracture toughness of TMCs. In the finite element analysis, the effects of the residual stresses, Poisson ratio and friction stresses are all considered, and the interface debonding process is described as a continuous one.

2 Finite element analysis

2.1 Finite element model and material properties

The push out test is analyzed using an axi-symmetric cylindrical model, as shown in Fig.1. The axi-symmetric cylindrical model consists of SiC fiber and titanium alloy matrix. The height of specimen is 500 µm, fiber radius is 70 µm, and matrix radius is 112 µm. The elements of SiC fiber and titanium alloy matrix are isoparametric 4-noded quadrilateral elements. The elements of interface are composed of contact-friction element and spring element. Fig.2 shows the details of the spring elements at the interface. In finite element
analysis, SiC fiber and titanium alloy matrix are treated as isotropic elastic materials. Moreover, the property dependency on temperature is considered. SiC/Timetal-834 is taken as the model material. Table 1 lists the thermo-mechanical properties of titanium alloy matrix and SiC fiber.

![Fig.1 Axi-symmetric finite element model](image1)

![Fig.2 Details of spring elements](image2)

**Table 1** Thermo-mechanical parameters of SiC fiber and Timetal-834 used in finite element analysis

<table>
<thead>
<tr>
<th>Material</th>
<th>E/GPa</th>
<th>ν</th>
<th>α/10⁻⁶ °C⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timetal-834(28 °C)</td>
<td>115.0</td>
<td>0.30</td>
<td>11.2</td>
</tr>
<tr>
<td>Timetal-834(300 °C)</td>
<td>96.4</td>
<td>0.30</td>
<td>11.2</td>
</tr>
<tr>
<td>Timetal-834(530 °C)</td>
<td>84.2</td>
<td>0.30</td>
<td>11.2</td>
</tr>
<tr>
<td>SiC</td>
<td>469.0</td>
<td>0.17</td>
<td>4.0</td>
</tr>
</tbody>
</table>

2.2 Numerical simulation procedure

Modeling of the push out test involves two steps.

1) Modeling the cooling process of composites. When TMCs are cooled from high temperature to room temperature, thermal residual stresses are induced due to the mismatch of the thermal expansion coefficients between fiber and titanium alloy matrix. In finite element analysis, the cooling process can be modeled by using temperature load, that is, the initial load and boundary load are added by using initial temperature and test temperature respectively. The free end surface of the model can be simulated via removing the existing tying constraint and spring elements[11–16].

2) Modeling the push out test. The push out test involves pushing a fiber out of composites through a flat indenter. In finite element analysis, the prescribing displacement is added to a rigid punch until the fiber is pushed out completely. Moreover, a special subroutine is used to control the interface debonding and frictional sliding process. The interface failure process is based on the energy failure criterion given by

$$G \geq G_{lc} \quad (1)$$

where $G$ is the energy release rate of the interface crack and $G_{lc}$ is the critical energy release rate. When the strain energy release rate of the interface crack ($G$) exceeds the critical value, the spring stiffness is reduced to zero and the debonding initiates.

3 Results and discussion

3.1 Evaluation of interface fracture toughness of TMCs

The finite element method based on the equivalent domain integral technique was used to simulate the push out test. When the simulated load—displacement curve has a good agreement with the experiment curve, the interfacial fracture toughness of TMCs can be obtained. In this work, SiC/Timetal-834 is selected as the model material. The load—displacement curve of the push out test for SiC/Timetal-834 is shown in Fig.3[10]. The specimen of the push-out test is the same as the finite element model. More information about the push out equipment and test procedure is given in Ref.[4].

The finite element method based on the equivalent domain integral technique is used to simulate the push out test of SiC/Timetal-834. First, the push out test is simulated with different interfacial frictional coefficients.

![Fig.3 Load—displacement curves of push out test[10]](image3)
Fig. 4 presents the simulated load—displacement curves with different frictional coefficients (0.3, 0.4 and 0.5). From Fig. 4(a), it can be obtained that, when the interfacial frictional coefficient is assumed to be 0.4, the frictional force evaluated is equal to the experimental one. So the interfacial frictional coefficient is equal to 0.4. Then, the push out test is simulated with different critical energy release rates and a constant value of interfacial frictional coefficient ($\mu = 0.4$). Fig. 4(b) shows the simulated load—displacement curves with different critical energy release rates. From Fig. 4(b), it can be obtained that when the critical energy release rate is assumed to be 50 J/m$^2$, good agreement is found between the simulated load—displacement curve and the experimental curve. So, the interfacial fracture toughness (the critical energy release rate) of SiC/Timetal-834 is about 50 J/m$^2$. It is noted that the effects of interface thickness and loading rate on the peak load are all neglected in the finite element analysis. Therefore, the interfacial fracture toughness evaluated by the finite element analyses has some error.

From the concept of fracture mechanics, we deduced the equation of interfacial fracture toughness:

$$G_{\text{f}} = \frac{r_f}{4E_f} \left[ \left( \sigma_{p} + \frac{2a_0}{r_f} + \sigma_{f,r,s} - \sigma_{f,t,s} \right) \frac{2a_0}{r_f} + \left( 1 - v_{f} \right) \sigma_{f,r,s}^2 + \left( \sigma_{f,t,s}^2 \right) \right]$$ (2)

where $r_f$ is the radius of the fiber, $E_f$ is elastic modulus of the fiber, $\sigma_p$ is the applied stress, $\tau$ is the interfacial frictional stress, $\sigma_{f,r}$ is radial thermal residual stress, $\sigma_{f,t}$ is tangential thermal residual stress, $v_f$ is the Poisson ratio of the fiber, $a$ is the complete length, $a_0$ is the initial debonding length[10]. These parameters are summarized and listed in Table 2. Using these parameters, the interfacial fracture toughness of SiC fiber reinforced Timetal-834 is calculated to be about 46.21 J/m$^2$. This further demonstrates the interfacial fracture toughness evaluated by the finite element analysis is reasonable.

### Table 2 Parameters used to examine predictions of analytical model

<table>
<thead>
<tr>
<th>$E_f$</th>
<th>$r_f$</th>
<th>$\sigma_{f,r}$</th>
<th>$\sigma_{f,t}$</th>
<th>$v$</th>
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<td>914</td>
<td>330</td>
<td>132</td>
<td>2014</td>
<td>12.5</td>
<td>150</td>
</tr>
</tbody>
</table>

3.2 Effect of peak load on interfacial fracture toughness of TMCs

Fig. 4(b) presents the simulated load—displacement curves with different critical energy release rates. From Fig. 4(b), it can be obtained that the critical energy release rate of TMCs increases with increasing the peak load. The reason is that the strain energy in composites increases with increasing the peak load. Therefore, the interfacial fracture toughness of TMCs increases with increasing the peak load. CHANDRA and ANANTH[5] also obtained the similar conclusion that the interfacial bond strength increases with increasing the peak load.

3.3 Effect of frictional stresses on interfacial fracture toughness of TMCs

Fig. 5 presents the simulated load—displacement curves of the push out test for SiC/Timetal-834 composite. The load—displacement curves in Fig. 5 have different critical energy release rates and the same peak load. In general, the critical energy release rate varies with the variation of the peak load, because the strain energy in composites varies with the peak load. However, in Fig. 5, the critical energy release rates of SiC/ Timetal-834 are different under the same peak load. That results from different interfacial frictional forces. With the increase of frictional force, the more strain energy in composites is consumed. So, the strain energy provided for creation of a new crack surface will decrease. That is to say that the interfacial fracture toughness of SiC/ Timetal-834 is decreased. LIU and KAGAWA[17], KIM
and MAI[18] also demonstrated that the interfacial fracture toughness of composites is inversely proportional to the interfacial friction stress.

### 3.4 Variations of energy release rate at both ends of specimen

A finite element method based on the equivalent domain integral technique is used to analyze the variations of the energy release rate at both ends of the specimen. Fig.6 shows the variations of energy release rates at both ends of the specimen for SiC/Timetal-834, evaluated by the finite element method. In Fig.6, the applied displacement \( u \) is normalized with respect to \( u_{\text{max}} \), at which the interface debonding initiates. It can be seen from Fig.6 that, the energy release rate at the supported end increases from 37.6 J/m\(^2\) to 45 J/m\(^2\); in contrast, the energy release rate at the loading end decreases from 37.6 J/m\(^2\) to 34 J/m\(^2\).

![Fig.6 Variations of energy release rate at two ends of specimen](image)

Fig.6 Variations of energy release rate at two ends of specimen

From positive at the loading end to negative at the supported end. When a compressive load is applied to the loading end, additional negative shear stresses, which are superimposed on shear residual stresses, are introduced at interface. The shear residual stresses at the loading end tend to decrease, due to the fact that the direction of shear stresses introduced by compressive load are opposite to that of shear residual stresses at the loading end. However, the shear stresses at the supported end increase, due to the fact that the direction of shear stresses introduced by compressive load are the same as that of shear residual stresses at the supported end. Therefore, the energy release rate at the supported end increases. This also explains the reason why the initial debonding of TMCs occurs at the supported end during the push out test.

![Fig.7 Interfacial shear stress distribution of titanium matrix composites](image)

Fig.7 Interfacial shear stress distribution of titanium matrix composites

### 4 Conclusions

1) The interfacial fracture toughness of SiC/Timetal-834 composites evaluated is about 50 J/m\(^2\).

2) The interfacial fracture toughness of TMCs increases with increasing the peak load and decreases with increasing the frictional force.

3) The distribution of shear residual stresses results in the increase of the energy release rate of SiC/Timetal-834 composites at the supported end and the decrease at the loading end.

### References


[3] KÖCK T, BRENDEL A, BOLT H. Interfacial reaction between


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