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# Flow stress equation in range of intermediate strain rates and high temperatures to predict roll force in four-pass continuous rod rolling

Sang-min BYON[1], Doo-hyun NA[2], Young-seog LEE[2]

1. Department of Mechanical Engineering, Dong-A University, Busan 604-714, Korea;
2. Department of Mechanical Engineering, Chung-Ang University, Seoul 156-756, Korea

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**Abstract:** A flow stress equation was proposed to compute the roll force in the finishing stands of an actual rod mill where the strain rate and the temperature of the material range from 100 to 400 $s^{-1}$ and from 900 to 1050 °C, respectively. The underlying idea is to modify the Shida model and Misaka model, which provide flow stress equations (constitutive equations) frequently used to depict deformation behavior of high temperature material at different strain rates. The modified model was coupled with finite element method to compute the roll force during four-pass continuous rod rolling, where strain rates are in the range of 100−400 $s^{-1}$ at high temperatures (900−1050 °C). The roll forces and the surface temperatures of the material at each stand were measured, and the measured data were compared with the computed values. Results reveal that the Misaka model is better than the Shida model for high temperatures and intermediate strain rates. The roll force error was −5.7 % when the Misaka model was used at 900 °C. However, the error increased by −15.2% at 1050 °C. When the modified Misaka model was used, the error was reduced to 1.8% on average. It can consequently be deduced that the modified Misaka model can be used to depict the deformation resistance behavior in intermediate ranges of strain rate and high temperature ranges in continuous rod rolling process.

**Key words:** roll force; strain rate; deformation resistance; rod rolling; Shida model; Misaka model

## 1 Introduction

In hot rod (or bar) rolling process, billets (or materials) heated above their recrystallization temperature are transformed into final products with acceptable thickness and shape tolerance as they pass through a series of stands positioned in tandem [1−3]. Here, the word stand denotes a unit machine with a pair of rolls, a screw-down device, a housing to contain these parts, and a drive motor. Sometimes a stand is also called a pass. The shape tolerance for the material at each stand is influenced by the chemical compositions of the material, the rolling temperature, the rolling speed, and the stiffness of the stand structure. Once the stiffness of the stand structure has been established, the ability to compute the roll force as a function of rolling temperature, rolling speed, and material grade becomes vital for designing various aspects of the rod (or bar) rolling process, such as roll groove shape, distance between stands, temperature of material, roll diameter, and roll speed at each stand.

The prediction of roll force depends largely on how accurately a change in material behavior (response) under an external loading at elevated temperatures (800−1200 °C) can be described and formulated into a mathematical equation, which is generally called the constitutive equation, deformation resistance equation or flow stress equation [4−7]. The material behavior, which is usually represented as stress, significantly depends on the temperature and the rate of deformation, i.e., the strain rate.

The roughing train of a rod mill is part of a slow forming process with a strain rate range of $10^{-3}$−$10^{2}$ $s^{-1}$ [8]. In this strain range, the Shida equation and the Misaka equation have both been widely used to calculate roll forces [9,10]. Both the Shida equation (model) and the Misaka equation (model) have strong point that they do not require any material constants to be determined from additional experiments. However, their application range for strain and for strain rate is limited (<0.7: Shida model and <0.5: Misaka model; <100 $s^{-1}$: Shida model

**Corresponding author:** Young-Seog LEE; Tel: +82-2-824-5256; E-mail: ysl@cau.ac.kr
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and <200 $s^{-1}$: Misaka model).

Therefore, the Shida and Misaka equations cannot be fully applied to the finishing block mill of an actual rod mill, where strain rates reach more than 200 $s^{-1}$. LEE et al [11] reported that the strain rates at the four stands (passes) of the finishing block mill are in the range of 100−400 $s^{-1}$.

JOHNSON and COOK [12] suggested a constitutive equation which can be used for a wide range of strain rates (0.002−650 $s^{-1}$) after performing torsion tests, Hopkinson bar tensile tests, and static tensile tests. They assumed that the dependence of stress on strain, strain rate, and temperature can be multiplicatively decomposed into three separate functions. To build up a constitutive equation for a specific material grade completely using Johnson and Cook's constitutive equation, it is necessary to determine the material constants after performing the tests. Note that the success of a constitutive model is dependent on the accuracy of the material constants. For this reason, Johnson and Cook's model is inappropriate for use in hot rolling processes which produces various grades of steel products.

Note that both the Shida model and the Misaka model, which are representative constitutive equations frequently used for describing the high temperature deformation behavior of material, have been known to be applicable to compute the deformation resistance of the material when strain rates are less than 100 $s^{-1}$ and 200 $s^{-1}$, respectively.

In this study, the possibility of applying the Shida model and the Misaka model to predicting the deformation resistance of material subjected to intermediate strain rates (100−400 $s^{-1}$) at high temperatures (900−1050 °C) was first examined. Secondly, the Misaka model was modified so that it can predict the deformation resistance of material subjected to strain rates and temperature ranges mentioned above. For this purpose, a hot rod rolling test was carried out using the four-pass high speed continuous hot rod rolling mill installed at Freiberg University, Germany. The roll forces and the surface temperatures of the material at each stand were measured. The roll forces were computed using the finite element method coupled with the Shida model and Misaka model. The measured roll forces and surface temperatures of the material were then compared with the computed values.

## 2 Four-pass high-speed continuous hot rod rolling test

Figure 1 shows a complete view of the pilot high-speed continuous rod rolling mill at the University of Freiberg, Germany. This university is the only institute in the world with its own high-speed continuous rolling mill. The general appearance of the four-pass continuous rod rolling mill and the distance between stands are shown in Fig. 2. Figure 3 shows the dimensions of the roll grooves (round and oval shapes) and the roll diameters of the four stands (passes). The rolling type of No. 1 and No. 3 stands is horizontal, and that of No. 2 and No. 4 stands is vertical. Roll speed at each stand (pass) is listed in Table 1. The material used in the rolling test is RSt 36 steel (C: 0.14, Si: 0.3, Mn: 0.25, P: 0.05, S: 0.05 in mass fraction). The initial diameter of the material is 12 mm, and the final diameter of the rolled material is 8.1 mm.


**Fig. 1** Complete view of pilot high-speed continuous rod rolling mill at University of Freiberg in Germany

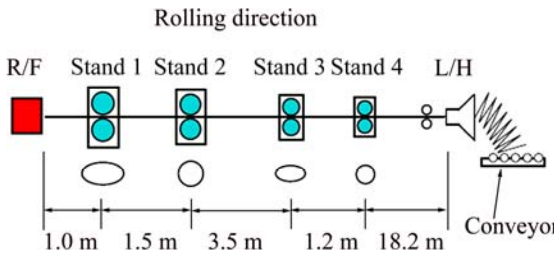


**Fig. 2** Schematic diagram for four-pass high-speed continuous rod rolling test and distance between stands (R/F: Reheating furnace; L/H: Laying head)

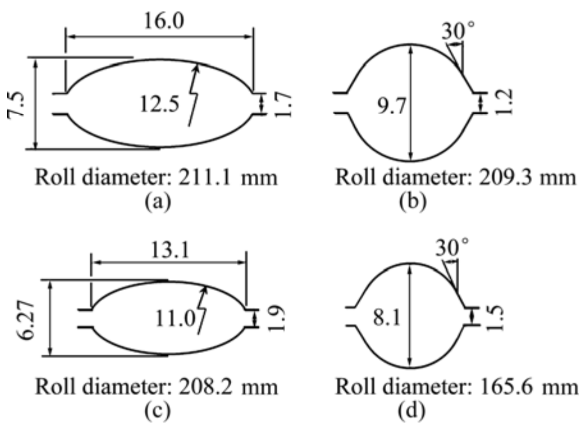


**Fig. 3** Dimensions of roll grooves and roll diameter at each stand (Unit: mm): (a) 1st pass; (b) 2nd pass; (c) 3rd pass; (d) 4th pass

**Table 1** Roll speed at each stand

| Stand 1 | Stand 2 | Stand 3 | Stand 4 |
|---|---|---|---|
| 9 m/s | 10 m/s | 12 m/s | 15 m/s |

The material (RSt 36 steel) was heated in the reheating furnace for homogenization for 30 min at 900 °C and 1050 °C, respectively. The heated material was rolled as soon as it came out of the reheating furnace. The roll forces at each stand were measured during the rolling test. The surface temperatures of the material entering No. 1 and No. 4 stands were measured as well. The rolling test was conducted three times at each temperature to confirm the reproducibility of the test.

## 3 Finite element analysis

Three-dimensional FE (finite element) analysis was conducted using a commercial FE code, ABAQUS to compute the roll forces at each stand and the surface temperatures of the material during rolling. The element type used for the material was C3D8RT (An 8-node thermally coupled brick, tri-linear displacement and temperature, reduced integration, hourglass control). Coulomb friction coefficient of 0.3 was used [13−15]. Figure 4 shows the mesh and boundary conditions. The number of material elements used was 27000 and of roll elements used was 11000. The symbolic parameters used in the finite element analysis are listed in Table 2.

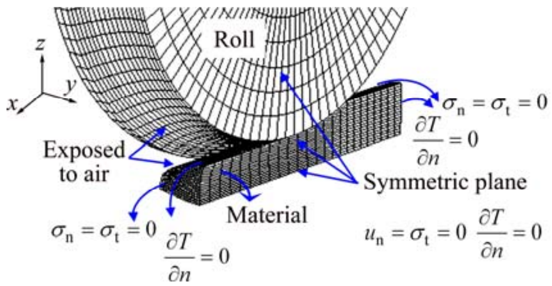


**Fig. 4** Mesh and boundary conditions used in finite element analysis

**Table 2** Description of parameters used in FE analysis

| Symbol | Meaning |
|---|---|
| $u_n$ | Normal displacement |
| $u_t$ | Tangential displacement |
| $\sigma_n$ | Normal surface traction |
| $\sigma_t$ | Tangential surface traction |
| $T$ | Temperature |
| $\partial T/\partial n$ | Temperature gradient |

To evaluate the interdependencies among the mechanical behaviors of the material and the thermal behavior of the material and the roll, a thermo-mechanically coupled analysis was performed. The material and the roll were taken as the analysis domain. The contact heat transfer coefficient between the material and the roll was set to be 2 kW/(m$^2$·°C) and the convection heat transfer coefficient of the material to be 10 W/(m$^2$·°C). The question of the constitutive equation used in FE analysis is treated in the next section.

## 4 Flow stress equation (constitutive equation)

To analyze hot deformation process, the change in mechanical response under external loading must be described by a constitutive equation which relates stress and strain to current conditions of temperature and strain rate because this change plays a crucial role in describing material response in terms of known material parameters. The constitutive equation is also called flow stress equation or deformation resistance equation.

SHIDA [9] proposed a constitutive equation (model) applicable under the following conditions: carbon content 0.07%−1.2%, temperature 700−1200 °C, and strain up to 0.7, but not applicable in a range of strain rates greater than 100 s$^{-1}$ [9]. The Shida model is expressed as

$$\bar{\sigma} = 0.28\exp\left(\frac{5.0}{T_n} - \frac{0.01}{C+0.05}\right)\cdot \left(1.3\left(\frac{\bar{\varepsilon}}{0.2}\right)^m - 0.3\left(\frac{\bar{\varepsilon}}{0.2}\right)\right)\left(\frac{\dot{\bar{\varepsilon}}}{10}\right)^n \tag{1}$$

where $C$, $\bar{\varepsilon}$, and $\dot{\bar{\varepsilon}}$ are respectively the equivalent carbon content, strain, and strain rate. The constants $m$ and $n$ are respectively the strain hardening coefficient and the strain rate sensitivity coefficient, respectively. In the Shida equation, $m$=0.41−0.07$C$, $n$=(−0.019$C$+0.126)$T_n$+(0.076$C$−0.05), and $T_n$=($T$+273)/1000, where $T$ is temperature (°C). The development of this constitutive equation is based on the experimental data obtained from compression-type high temperature−high strain rate testing machines specifically suited for flow stress measurement, i.e., the cam-plastometer and drop hammer types.

MISAKA and YOSHIMOTO [10] suggested another form of constitutive equation (model) that specifies the mean flow stress in terms of carbon content (up to 1.2 %), temperature (750−1200 °C), strain (up to 0.5) and strain rate (30−200 s$^{-1}$) after performing a series of drop hammer tests. The Misaka equation can be expressed as

$$\bar{\sigma} = \exp(0.126 - 1.75C + 0.594C^2 + \frac{2851+2968C-1120C^2}{T_a})\bar{\varepsilon}^m\,\dot{\bar{\varepsilon}}^n \tag{2}$$

where $T_a$ is the absolute temperature. Constants $m$ and $n$ are 0.21 and 0.13, respectively.

Figure 5 shows variations of stress in terms of strain at 900 °C and 1050 °C and strain rates varying from 100 $s^{-1}$ to 400 $s^{-1}$. Stresses obtained from the Misaka model increased monotonically after the material yielded. When the material temperature was 900 °C, the stresses computed by the Shida model were higher than those from the Misaka model over the whole range of strains. At temperature of 1050 °C, the stresses computed by the Shida model were higher than those from the Misaka model up to a strain of approximately 0.6. This indicates that the Misaka model is more sensitive to temperature variations than the Shida model.

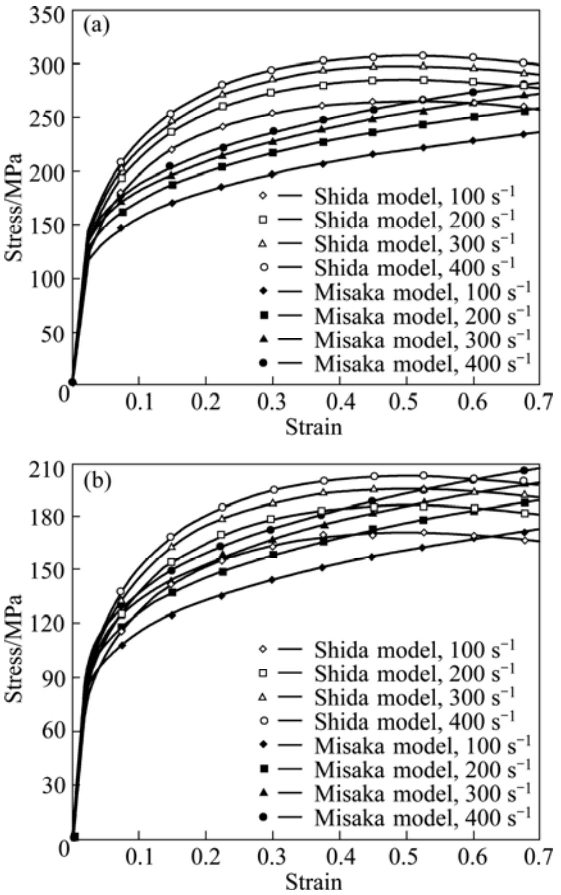


**Fig. 5** Comparison of Shida model and Misaka model at temperature of 900 °C (a) and 1050 °C (b)

## 5 Results and discussion

### 5.1 Roll forces from original Shida model and Misaka model

Table 3 and 4 show the roll forces measured at each stand and those calculated by the Shida model and Misaka model. The strain rates at each stand are as follows: 204.2 $s^{-1}$ at stand No. 1; 206.7 $s^{-1}$ at stand No. 2; 293.4 $s^{-1}$ at stand No. 3; and 378.5 $s^{-1}$ at stand No. 4. Note that stands No. 1 and No. 3 have rolls with an oval groove, while stands No. 2 and No. 4 have rolls with a round groove (see Fig. 3). Even though the Shida model and Misaka model can be used under rolling conditions in which the strain rates of the material during rolling are less than 100 $s^{-1}$ and 200 $s^{-1}$, for comparison, they were applied to rolling test conditions in which the strain rates of the material were in the range of 100−400 $s^{-1}$. The relative difference between the measured roll forces and the calculated values is error. The error is estimated on the basis of the measurements. For example, the error in the Shida (or Misaka) model denotes the difference between the measured roll forces and calculated ones in case that the Shida (or Misaka) model is used as a constitutive equation in FE analysis.

**Table 3** Roll forces at temperature of 900 °C

| Stand No. | Measured roll force/kN | Shida model | | Misaka model | |
|---|---|---|---|---|---|
| | | Calculated roll force/kN | Error/% | Calculated roll force/kN | Error/% |
| 1 | 63.0 | 74.3 | −17.9 | 61.3 | 2.7 |
| 2 | 36.7 | 40.1 | −9.3 | 38.8 | −5.7 |
| 3 | 71.3 | 71.1 | 0.3 | 71.7 | −0.6 |
| 4 | 37.3 | 35.2 | 5.6 | 38.4 | −2.9 |

**Table 4** Roll forces at temperature of 1050 °C

| Stand No. | Measured roll force/kN | Shida model | | Misaka model | |
|---|---|---|---|---|---|
| | | Calculated roll force/kN | Error/% | Calculated roll force/kN | Error/% |
| 1 | 44.7 | 49.8 | −11.4 | 45.3 | −1.3 |
| 2 | 25.0 | 28.2 | −12.8 | 28.8 | −15.2 |
| 3 | 52.3 | 51.5 | 1.5 | 54.0 | −3.3 |
| 4 | 28.5 | 26.9 | 5.6 | 29.2 | −2.5 |

The roll force error in the Shida model is larger than that in the Misaka model. The maximum error using the Shida model occurred when the material was rolled at 900 °C. It reaches −17.9 % at stand No. 1. Meanwhile, the Misaka model has a maximum roll force error (−15.2 %) at stand No. 2 when the temperature of the material is 1050 °C. This inconsistency in error between measured values and predicted values might be attributable to the variables considered in finite element analysis. There are many variables affecting the roll force in actual rolling process. However, the variables such as material shape change along the length direction during rolling, and variation in friction condition at the interface of roll and material was not considered in FE analysis. The measured roll force at stand No. 2 at 1050 °C seems to be misgauged from the viewpoint of the roll force difference between stands No. 2 and No. 4 at 900 °C. Note that the roll force difference between stands No. 2 and No. 4 at 900 °C is very small, but at

1050 °C, it is approximately 14.0 %. This means that the roll force at No. 2 stand at 1050 °C should be greater than 25.0 kN, but less than 28.5 kN. If the roll force was 26.0 kN or 27.0 kN at No. 2 stand at 1050 °C, the maximum roll force error (−15.2 %) will be reduced to −10.8 % or −6.7 %.

Except for the error at No. 2 stand at 1050 °C, the error of the Misaka model is less than ± 5.7 %. It can consequently be concluded that using the Misaka model to compute roll force is more reasonable than using the Shida model. Therefore, in this study, the Misaka model was modified to be applicable to rolling test conditions in which the strain rates of the material being deformed are in the range of 100−400 $s^{-1}$. The strain rate hardening coefficient $n$ in the original Misaka model was modified based on the least square method which minimizes the sum of squared residuals. The residual is the difference between observed values at each stand (pass) and the fitted values provided by modified Misaka model. As a result, the coefficient in original Misaka model was changed from 0.13 to 0.123.

### 5.2 Roll forces at each stand (pass) using modified Misaka model

Finite element analysis coupled with the modified Misaka model was performed to compute the roll forces at each stand, where the strain rates of the material during rolling are in the range of 100−400 $s^{-1}$. Figures 6 and 7 illustrate the flow stress—strain curve at 900 °C and 1050 °C, respectively, when the modified Misaka model is used. The degree of dependency of strain rate on flow stress is slightly reduced.

Table 5 and 6 show the measured roll forces and the computed values. In comparison with the error of the original Misaka model, the maximum error of the modified Misaka model is reduced from −15.2% to −9.6%. Except for the roll force at No. 2 stand at 1050 °C, the error is ±3.8%. Therefore, it can be concluded that the modified Misaka model (equation) can be applied to the finishing block mill of an actual rod mill in which the strain rates are in the range of 100−400 $s^{-1}$.

### 5.3 Surface temperature of material during rolling test

To confirm the usefulness of the modified Misaka model (equation), the surface temperatures of the material measured at the entry of No. 1 and No. 4 stands during the rolling test are compared with the computed values. Figure 8 shows the surface temperature of the

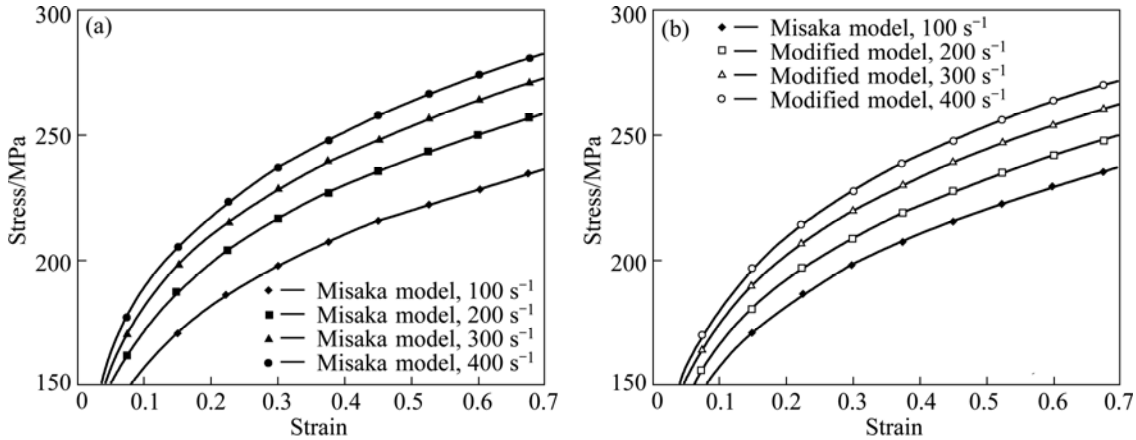


**Fig. 6** Comparison of original Misaka model (a) and modified Misaka model (b) at 900 °C

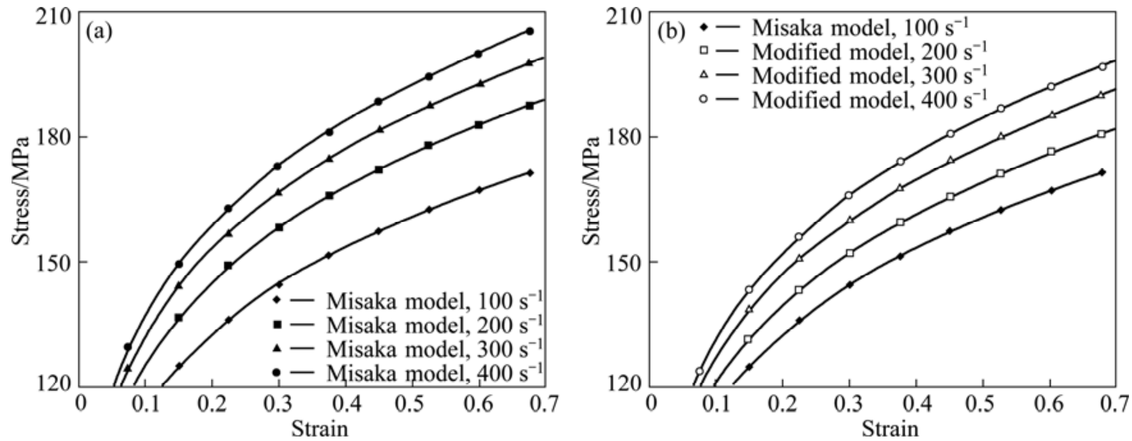


**Fig. 7** Comparison of original Misaka model (a) and modified Misaka model (b) at 1050 °C

**Table 5** Comparison of measured and calculated roll forces by modified Misaka model at 900 °C

| Stand No. | Measured roll force/kN | Calculated roll force/kN | Error/% |
|---|---|---|---|
| 1 | 63.0 | 60.6 | 3.8 |
| 2 | 36.7 | 38.1 | −3.8 |
| 3 | 71.3 | 70.9 | 0.6 |
| 4 | 37.3 | 37.2 | 0.3 |

**Table 6** Comparison of measured and calculated roll forces by modified Misaka model at temperature of 1050 °C

| Stand No. | Measured roll force/kN | Calculated roll force/kN | Error/% |
|---|---|---|---|
| 1 | 44.7 | 44.1 | 1.3 |
| 2 | 25.0 | 27.4 | −9.6 |
| 3 | 52.3 | 53.5 | −2.3 |
| 4 | 28.5 | 28.6 | −0.4 |

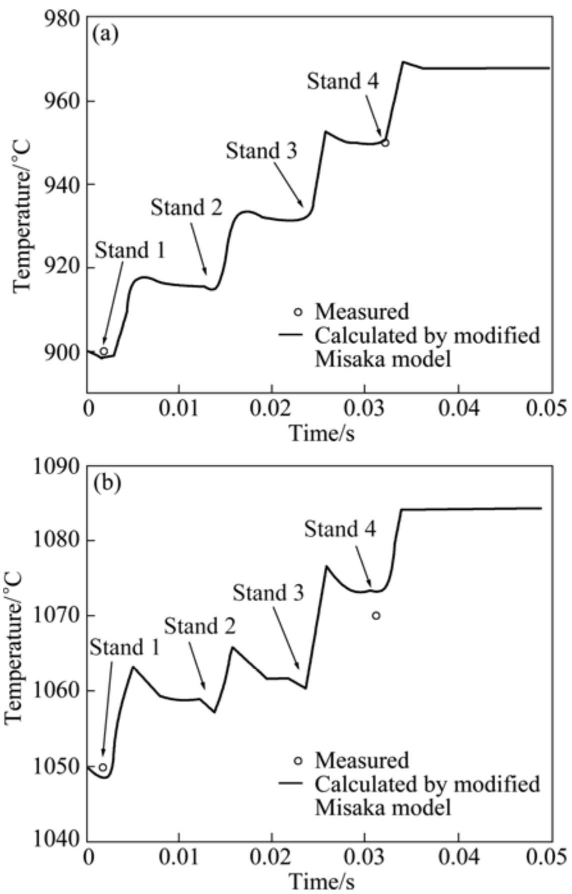


**Fig. 8** Surface temperature of material measured at two points and surface temperature profile of material as function of rolling time during rolling test at initial material temperature of 900 °C (a) and 1050 °C (b)

material measured at two stands and the surface temperature profile of the material as a function of rolling time. The surface temperature increases as the stand number, i.e., rolling time, increases. This is because heat inside the material is rapidly generated by high strain rate plastic deformation at each stand. Note that the rolling speed is high (9 m/s at No. 1 stand; 10 m/s at No. 2 stand; 12 m/s at No. 3 stand; 15 m/s at No. 4 stand), stepwise temperature increments at each stand (pass) are continuously observed. Overall, the measured surface temperatures are in good agreement with the computed values. Therefore, the modified Misaka model is good enough to be used as a constitutive equation which relates stress and strain under rolling conditions in which the strain rates of material deformed are in the range of 100−400 $s^{-1}$ at high temperatures (900−1050 °C).

## 6 Conclusions

This study examined whether the Shida model and the Misaka model can be used to compute the roll force at the front stands (passes) of the finishing block mill of an actual rod mill where the strain rate ranges from 100 to 400 $s^{-1}$. The Misaka model was modified to be applicable to the front stands of the finishing block mill. A four-pass high-speed continuous hot rod rolling experiment was also performed to compare measurements (the roll forces at each pass and the surface temperatures of the material) with predictions obtained from the modified Misaka model.

When the modified Misaka model was used at different temperatures (900 °C and 1050 °C), the difference between measurements and computed values was 2.8% on average. The measured surface temperatures of the material during rolling coincide with the computed values regardless of variations in the initial temperature of the material. Hence, it can be concluded that the modified Misaka model can be used to represent deformation resistance behavior for strain rates (100−400 $s^{-1}$) and temperatures (900−1050 °C).

## Acknowledgement

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# 高温、中应变速率下预测四道次连续轧制力的流变应力方程

Sang-min BYON[1], Doo-hyun NA[2], Young-seog LEE[2]

1. Department of Mechanical Engineering, Dong-A University, Busan 604-714, Korea;
2. Department of Mechanical Engineering, Chung-Ang University, Seoul 156-756, Korea

**摘　要**：为了计算在应变速率 100～400 $s^{-1}$、温度 900～1050 °C 条件下四道次连续线材轧制过程中的轧制力，提出了一个流变应力方程。基本概念是对 Shida 模型和 Misaka 模型进行改进。通常用这 2 种模型建立的流变应力本构方程来描述高温材料在不同应变率下的变形行为。将改进模型与有限元方法相结合来计算应变速率 100～400 $s^{-1}$、温度 900～1050 °C 条件下的四道次连续轧制过程中的轧制力。测量材料在每个道次的轧制力和表面温度，并与预测值进行比较。结果表明，在高温、中应变速率条件下，Misaka 模型比 Shida 模型更好。在 900 °C 时，采用 Misaka 模型的轧制力误差为−5.7 %。在 1050 °C 时，采用 Misaka 模型的轧制力误差为−15.2%，而采用改进的 Misaka 模型的轧制力误差降低到 1.8%。由此可以得出，对于高温、中应变速率的线材轧制过程，改进的 Misaka 模型能用来预测高温材料的变形行为。

**关键词**：轧制力；抗变形性；棒材压制；Shida 模型；Misaka 模型

**(Edited by Xiang-qun LI)**