

# 弹性筛面动力学及难筛分物料的透筛机理<sup>①</sup>

赵跃民 刘初升

(中国矿业大学能源利用和化学工程系, 徐州 221008)

**摘要** 针对难筛分物料的弹性筛面筛分机, 建立了筛分机的弹性筛面动力学模型, 通过对动力学模型的理论分析, 推导出弹性筛面在筛分机振动过程的弹性变形规律, 并分析了各种粒度的颗粒在弹性筛面上透筛、堵孔的受力情况, 给出了透筛力计算公式, 指出弹性筛面透筛力大于刚性筛面透筛力, 揭示了弹性筛面克服堵孔主要是筛面弹性变形的作用机理。提出在设计弹性筛面筛分机时, 应将筛分机工作转速设计为筛面一阶固有频率。最后, 进行了金属筛面和弹性筛面筛分机对各种含外在水分煤炭筛分的对比试验, 结果表明, 弹性筛面筛分机的筛分效率优于金属筛面筛分机。

**关键词** 弹性筛面 弹性筛面动力学 难筛分物料

**中图法分类号** TD452

潮湿细粒物料的干法深度筛分是当前国际上筛分技术研究的重点。由于细颗粒粒度小、比表面积大以及一些粘土成分在外在水分作用下很容易造成堵孔, 使得筛分过程难以进行<sup>[1, 2]</sup>。近十年来筛分领域的一个重要进展是在传统的筛分机上将金属筛面改为聚氨酯、橡胶等弹性筛面<sup>[3-8]</sup>, 国外已普遍采用, 国内处于起步阶段。本文对弹性筛面的动力学模型、参数设计及其克服堵孔的机理进行研究, 为弹性筛面的设计、应用提供理论依据。

## 1 弹性筛面动力学

弹性筛面主要是指以聚氨酯、橡胶、琴弦等具有较大弹性的材料制成的筛面。尽管传统的金属筛面在振动筛的振动过程中也能呈现一定的弹性, 但由于金属材料的弹性模量较大, 一般都在 200 GPa 左右, 所以其弹性振动特性几乎微不足道。而现在的弹性筛面, 如聚氨酯筛面, 其弹性模量只有 34~ 50 MPa<sup>[3]</sup>, 所以用弹性材料制成的筛面在振动筛振动过程中其弹

性振动特性就必须考虑。因此需研究弹性筛面的动态特性, 并阐述弹性筛面克服堵孔的机理, 建立筛面动力学模型。

### 1.1 弹性筛面的动力学模型

基于筛分机刚性运动和筛面弹性运动互不耦合的假设, 分别讨论筛分机刚性运动和筛面弹性运动。

筛面刚性运动的简图如图 1 所示。 $K$  为弹簧刚度,  $C$  为粘性阻尼系数,  $m$  为偏心块质量,  $e$  为偏心距, 刚性运动方程为

$$(M + 2m)\ddot{y} + C\dot{y} + Ky = 2me\omega^2 \sin \omega t \quad (1)$$

式中  $M = M_j + K_\omega \times M_\omega$ ,  $M_j$  为筛箱的设计质量,  $M_\omega$  为筛上物料,  $K_\omega$  为物料结合系数,  $K_\omega = 0.15 \sim 0.3$ ,  $\omega$  为激振力频率。方程(1)的强迫振动解为

$$\bar{y} = A \sin(\omega t - \phi) \quad (2)$$

式中  $A = 2me\lambda^2 / \{(M + 2m)[(1 - \lambda^2)^2 + (2\xi\lambda)^2]\}^{1/2}$ , 且  $\tan \phi = 2\xi\lambda(1 - \lambda^2)$ ,  $\xi = C/C_c$ ,  $C_c$  为临界阻尼,  $\lambda = \frac{\omega}{\omega_n}$ ,  $\omega_n$  为系统的

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固有频率。式(2)即为筛面的刚性运动表达式。

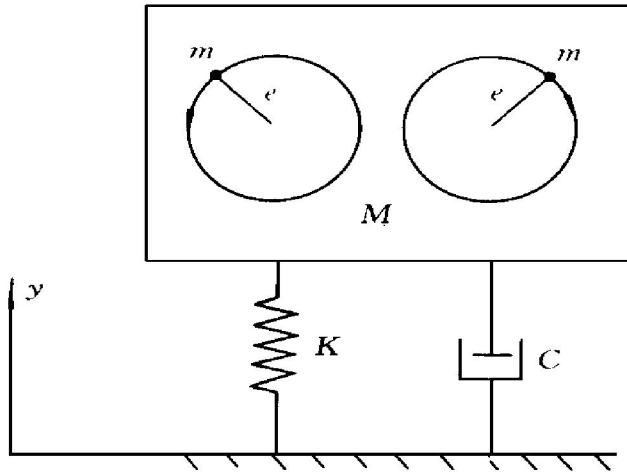


图1 筛分机振动模型

Fig.1 Vibration model of screening machine

基于前述基本假定，采用瞬时凝固法，将筛面刚性运动引起的惯性力和物料对筛面的非线性作用力作用到筛面上，从而引起筛面的弹性变形运动，其运动简图如图2所示。筛面弹性运动微分方程为

$$D \nabla^4 Z = q - (\rho_A + \rho_m) \ddot{Z} \quad (3)$$

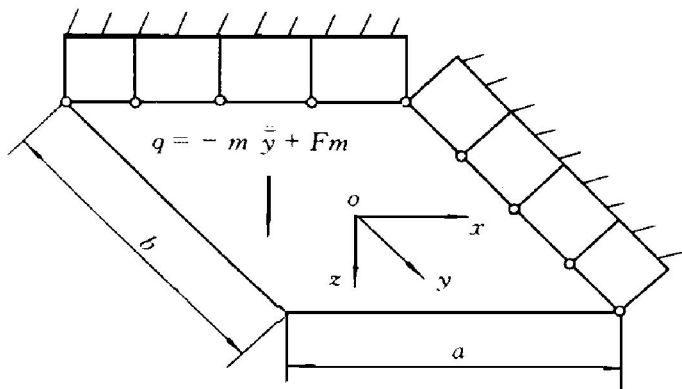


图2 筛面弹性振动力学模型

Fig.2 Dynamic model of elastic vibration of screen surface

式中  $D = Eh^3/(12 - 12\mu^2)$ ,  $q$  为外加激振力,  $Z$  为筛面在  $Z$  轴方向上的弹性位移,  $\rho_A$  为筛面单位面积的质量,  $\rho_m$  为粘附到筛面上的物料质量,  $E$  为筛面弹性模量,  $h$  为筛面厚度,  $\mu$  为泊松比,  $\nabla$  为汉密尔顿算子;  $q = -\rho_A \ddot{y} +$

$F_m$ ,  $F_m$  为非线性冲击力, 为分析方便起见, 只考虑物料附着在筛面上与筛面一起运动的一种状态, 则  $q$  的表达式为

$$q = \rho_A A \omega^2 \sin(\omega t - \phi) + \rho_m A \omega^2 \sin(\omega t - \phi) \quad (4)$$

将式(4)代入式(3)可得

$$D \nabla^4 Z + (\rho_A + \rho_m) \ddot{Z} = (\rho_A + \rho_m) A \omega^2 \sin(\omega t - \phi) \quad (5)$$

将弹性筛面视为四边简支薄板, 所以筛面弹性运动振动解为四边简支薄板振动解。在方程(5)中, 自由振动将随时间的延长由于阻尼的作用而消失, 剩下的只是强迫振动响应, 设式(5)解为

$$Z(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} A_{m,n} \sin(\omega t - \phi)$$

利用线性微分方程的迭加性将上式第  $m, n$  项代入式(5)得

$$D \cdot A_{m,n} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]^2 \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \sin(\omega t - \phi) - (\rho_A + \rho_m) \cdot A_{m,n} \omega^2 \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \sin(\omega t - \phi) = (\rho_A + \rho_m) A \omega^2 \sin(\omega t - \phi)$$

将上式两边同乘以  $\sin \frac{r\pi x}{a} \sin \frac{s\pi y}{b}$ , 并沿整个筛面积分得

$$DA_{m,n} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]^2 \int_0^a \int_0^b \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \sin \frac{r\pi x}{a} \sin \frac{s\pi y}{b} \sin(\omega t - \phi) dx dy - (\rho_A + \rho_m) \omega^2 A_{m,n} \int_0^a \int_0^b \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \cdot \sin \frac{r\pi x}{a} \sin \frac{s\pi y}{b} \sin(\omega t - \phi) dx dy = (\rho_A + \rho_m) A \omega^2 \int_0^a \int_0^b \sin \frac{r\pi x}{a} \sin \frac{s\pi y}{b} \cdot \sin(\omega t - \phi) dx dy$$

利用三角函数的正交性, 经过简单运算可得

$$A_{m,n} = ab\omega^2 / (\omega_{m,n}^2 - \omega^2) \quad (m, n \text{ 为奇数})$$

故方程(5)的解为

$$Z = \sum_m \sum_n A_{m,n} \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \sin(\omega t - \psi) \quad (6)$$

式中  $\omega_{m,n} = \pi^2 \sqrt{\frac{D}{\rho_A + \rho_m}} \times [(\frac{m}{a})^2 + (\frac{n}{b})^2]$  为筛板第  $m, n$  阶固有频率。

## 1.2 弹性筛面克服堵孔的机理

筛面静止时筛孔形状如图 3(a) 所示。筛面作弹性振动时, 由式(6)可知筛孔形状呈近似锥形孔。筛孔在筛面振动的前半个周期内形状如图 3(b) 所示, 在后半个周期内筛孔形状如图 3(c) 所示。所以, 筛孔在一个周期性内进行一张一缩的变化。

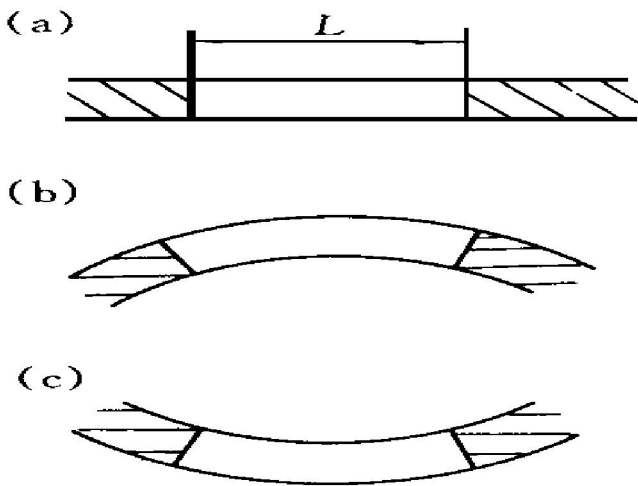


图3 振动过程中筛孔形状的变化

Fig. 3 Shape vibration of screen aperture during process of vibration

(a) —Static aperture shape;

(b) —Aperture shape in former semiperiod;

(c) —Aperture shape in later semiperiod

堵孔有两种情况, 一是难筛颗粒( $0.75d_p \leq d \leq d_p$ ,  $d_p$  为分离粒度,  $d$  为颗粒粒度)落入筛孔时, 由于颗粒形状不规则, 容易卡死在筛孔中造成堵孔; 另一种堵孔问题是, 细颗粒在外在水分表面张力和粘性物料如粘土等作用下, 能在筛孔上形成覆盖膜造成堵孔。对难筛颗粒堵孔问题来说, 筛孔不变形时, 颗粒落入筛孔后受力简图如图 4(a) 所示。当筛孔变形处于后半周期时颗粒落入筛孔后受力简图如图

4(b) 所示。图 4 中  $f$  为粘结力,  $F_k$  为筛孔对颗粒的挤压力,  $N$  为筛孔对颗粒的弹性压力,  $N_\mu$  和  $F_{\mu,k}$  为摩擦力,  $F_m$  为颗粒的惯性力,  $\alpha$  为筛孔变形时弹性转角, 其值为  $\arctan(\frac{\pi}{l} \cdot \cos \frac{\pi x}{l})$ 。

筛孔不变形时颗粒的透筛力为

$$F_T^1 = F_m - 2F_{\mu,k} - f$$

筛孔变形时颗粒的透筛力为

$$F_T^2 = F_m + 2N \sin \alpha - 2f - 2N_\mu \cos \alpha$$

式中  $F_k = E \cdot \frac{L}{A} \cdot \Delta L = E \cdot \frac{L}{A} \cdot (d - L)$ ,  $A$  为筛孔横截面积,  $N$  可近似表达为  $N = E \cdot \frac{L}{A} \cdot (d - L)$  即  $N = F_k$ 。

从两者的透筛力来看  $F_T^2 > F_T^1$ , 使得堵孔可能性减小。此外, 在筛孔变形过程中, 筛面对颗粒有搓揉运动, 能将粘附在大颗粒上的细小颗粒搓揉下来而成为筛下物。粘附成团的细小颗粒落入筛孔后, 筛孔边缘对颗粒产生的正压力将这种颗粒压碎而成为筛下物, 从而有效地提高筛分效率。

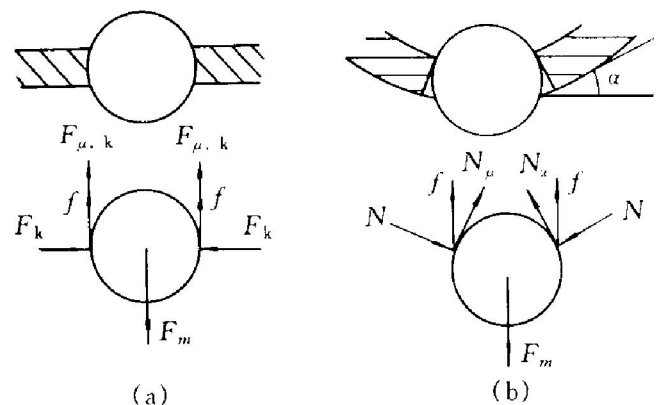


图4 颗粒在筛孔中受力情况

Fig. 4 Penetrating force of particle in screen aperture

(a) —Penetrating force of rigid screen surface;

(b) —Penetrating force of elastic screen surface

## 1.3 筛分机参数设计

增加筛面的弹性变形对克服堵孔是有益的, 因此我们在设计振动筛时应将筛分机的工作频率设计为筛面弹性振动的基频, 即

$$\omega = \omega_{1,1} = \pi^2 \sqrt{\frac{D}{\rho_A + \rho_m}} \cdot \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{1}{b} \right)^2 \right] \quad (7)$$

从而引起筛面的弹性共振以解决堵孔问题, 提高筛分效率。

## 2 实验研究

为了验证上述理论的合理性, 在实验室振动筛的模型机上将金属筛面改装成聚氨酯筛面。每块筛板的尺寸为 500 mm × 200 mm, 筛板厚度为  $h = 4$  mm; 聚氨脂板的弹性模量为 40 MPa, 泊松比  $\mu = 0.4$ ; 单位面积筛面的质量  $\rho_A = 1.2$ , 粘附于筛面上的物料质量为  $\rho_m = 1.5$ 。其一阶固有频率:

$$\begin{aligned} \omega_{1,1} &= \pi^2 \sqrt{\frac{D}{\rho_A + \rho_m}} \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{1}{b} \right)^2 \right] \\ &= 87.7 \text{ rad/s} \end{aligned}$$

将工作转速设计为 840 r/min, 进行金属筛面和弹性筛面的筛分效果的对比试验, 筛分物料为烟煤, 外在水分从 0% ~ 20% 变化, 分级粒度为 6 mm, 其筛分效率和筛上物限下率示于图 5, 由此可以看出弹性筛面的筛分效果较金属筛面要好。

## 3 结论

(1) 建立了弹性筛面的动力学模型, 给出了弹性筛面强迫振动解和固有频率表达式, 其解为四边简支薄板振动的强迫振动解和固有频率。指出弹性筛面由于弹性变形促使筛孔运动减轻了堵孔。

(2) 给出了振动筛工作转速的设计公式, 将工作转速设计在  $\omega_{1,1} = \pi^2 \sqrt{\frac{D}{\rho_A + \rho_m}} \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{1}{b} \right)^2 \right]$ , 能促使弹性筛面共振, 以提高筛分效率。

(3) 进行了金属筛面和弹性筛面的对比实

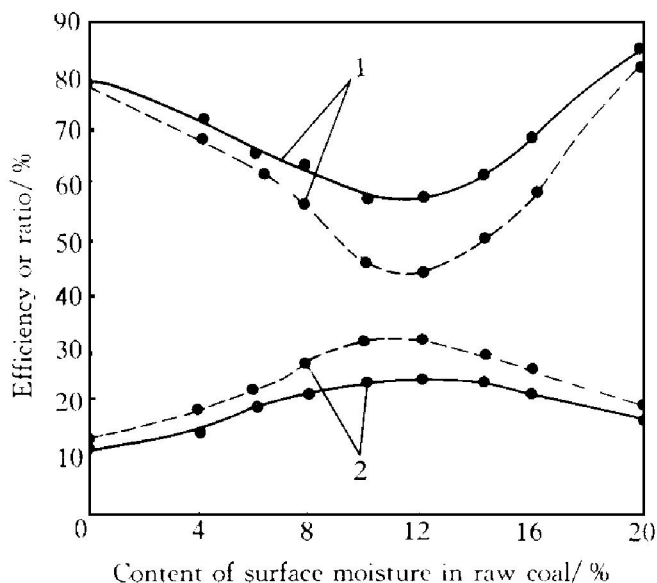


图 5 筛分指标与外在水分含量的关系

Fig. 5 Relationship between screen index and contents of surface moisture

1—Screen efficiency;

2—Ratio of particles to be holding on screen surface;

Real line—Elastic screen surface;

Dotted line—Rigid screen surface

验, 实验表明弹性筛面筛分效率优于金属筛面。

## REFERENCES

- 1 Blurstein E. Iron and Coal, 1959, (3): 791–798.
- 2 Liu Chusheng (刘初升) and Zhao Yaomin (赵跃民). Coal Processing Technology (选煤技术), 1997, 3: 23–26.
- 3 Ma Huaning (马华宁). J. of Foreign Metallic Mineral Processing (国外金属选矿), 1991, (1): 27–32.
- 4 Ma Lianfu (马连福). Mining Equipment (采矿设备), 1991, (9): 27–31.
- 5 Ye Hengdi (叶恒棣). Mining Equipment (采矿设备), 1992, (10): 15–17.
- 6 Dietz G. Aufbereitungs-Technik, 1994, 35(8): 404–412.
- 7 Grescher J. Aufbereitungs-Technik, 1995, 36(8): 372–378.
- 8 Hirsch W. Aufbereitungs-Technik, 1992, 33(12): 686–690.

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# RELATIVE MOTION BETWEEN SOLID PARTICLES AND LIQUID MEDIUM IN HYDROCYCLONES

## PART III: EFFECT OF TURBULENCE FREQUENCY

Xu Jirun and Luo Qian<sup>†</sup>

*Department of Chemical Engineering,*

*Dalian University, Dalian 116622, P. R. China*

<sup>†</sup> *Department of Resources and Environment Engineering,*

*Northeastern University, Shenyang 110006, P. R. China*

**ABSTRACT** The effect of fluid turbulence frequency on the relative motion between solid particles and liquid medium in hydrocyclones was examined with follow-up parameter  $\eta$  and turbulence frequency  $\omega$ . It is shown that the parameters in tangential and vertical directions have simpler relation with turbulence frequency, i. e. the relative motion, as expected, gets more obvious with the increase of turbulence frequency; but in radial direction the lags between particles and fluid behavior have much more complicated than in other two directions due to the existence of centrifugal force. Three basic behavior forms are found for particles with different diameters and the mechanism are explained.

**Key words** hydrocyclones particle motion turbulence frequency follow-up situation

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# DYNAMICS OF ELASTIC SCREEN SURFACE AND PENETRATING MECHANISM OF DIFFICULT SCREENING MATERIAL

Zhao Yaomin and Liu Chusheng

*Department of Energy Utilization and Chemical Engineering,*

*China University of Mining Technology, Xuzhou 221008, P. R. China*

**ABSTRACT** The dynamic model of elastic screen surface of screening machine which screens difficult screening material has been set up. By theoretically analyzing the dynamic model, the deformation formula of elastic screen surface in the vibration of screening machine was derived. The penetrating and blocking forces of all kinds sizes of particles were analyzed, and the formula of penetrating force was given, then it was pointed out that the penetrating force of elastic screen surface is bigger than that of rigid screen surface. The mechanism overcoming the blocking of elastic screen surface was explained. The design formula of working revolution of elastic screening machine, which is equal to the fundamental frequency of elastic vibration of screen surface, was given. The comparison experiments between on metal screen surface and on elastic screen surface were made, the results of experiments showed that the screening technical index of the screening machine with elastic screen surface is better than that of the screening machine with metal screen surface.

**Key words** elastic screen surface dynamics of elastic screen surface difficult screening material

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