



A model of continuum damage mechanics for high cycle fatigue of metallic materials

ZHANG Liang¹, LIU Xue-song¹, WANG Lin-sen^{1,2}, WU Shuang-hui¹, FANG Hong-yuan¹

1. State Key Laboratory of Advanced Welding and Joining, Harbin Institute of Technology, Harbin 150001, China;

2. Welding Process Section, Dongfang Boiler Group Co., Ltd., Zigong 643001, China

Received 8 March 2012; accepted 16 July 2012

Abstract: A non-linear continuum damage model was presented based on the irreversible thermodynamics framework developed by LEMAITRE and CHABOCHE. The proposed model was formulated by taking into account the influence of loading frequency on fatigue life. The parameters H and c are constants for frequency-independent materials, but functions of cyclic frequency for frequency-dependent materials. In addition, the expression of the model was discussed in detail at different stress ratios (R). Fatigue test data of AlZnMgCu1.5 aluminium alloy and AMg6N alloy were used to verify the proposed model. The results showed that the model possesses a good ability of predicting fatigue life at different loading frequencies and stress ratios.

Key words: AlZnMgCu1.5 alloy; AMg6N alloy; continuum damage model; cyclic frequency; high cycle fatigue; loading frequency; fatigue life; fatigue crack

1 Introduction

Most engineering components are subjected to cyclic loading, hence, fatigue damage is one of the main forms of failure in engineering structures. Therefore, it is important to formulate a method for the assessment of reliability and prediction of engineering components. Conventionally, people paid more attention to the stage of fatigue crack growth [1,2]. However, the phase of fatigue crack initiation occupies even 80% or more of the total fatigue life for many components in service [3]. Scores of models based on fracture mechanics have been developed to model the fatigue crack growth rate successfully [4,5]. But the fatigue crack initiation life cannot be predicted using the fracture mechanics based on the existing models. Therefore, it is urgent to develop a new approach for predicting the total fatigue life. Fortunately, continuum damage mechanics (CDM) solves the problem successfully. Several models for high cycle fatigue have been proposed based on the continuum damage mechanics [6–8]. LEMAITRE [6] stated that the micro-plastic strain was responsible for the high cycle fatigue damage. However, XIAO et al [7] developed a high cycle fatigue damage model from a brittle damage mechanism. In order to describe the high

degree of localization for high cycle fatigue damage, DESMORAT et al [8] proposed a three-dimensional two scale damage model. The modeling consists in the micromechanics analysis of a weak micro-inclusion subjected to plasticity and damage embedded in an elastic meso-element. But the influence of loading frequency on fatigue life is not taken into account in the existing models. Many studies showed that fatigue performances of some materials are insensitive to loading frequency, such as AlZnMgCu1.5 aluminium alloy [9], ULTIMET alloy [10] and Ti–6Al–4V [11], whereas, other materials, such as AMg6N [12], commercially pure tantalum [13], 1Kh2M steel [14], and low carbon steel [15], exhibit significant frequency-dependent fatigue behaviors. For the latter materials, ignoring the influence of loading frequency on fatigue life can lead to much waste of the materials or potential dangerous over-prediction of fatigue life.

In this work, a modified high cycle fatigue model based on continuum damage mechanics is proposed, which takes into account the influence of loading frequency on fatigue property. The fatigue test data of frequency-independent material and frequency-dependent material are used to verify the proposed model, respectively.

2 Thermodynamics

2.1 One-dimensional damage

Damage is a progressive degeneration of material property caused by initiation and growth of microcracks and cavities. For one-dimensional case, the damage can be defined as a dimensionless quantity [16]:

$$D = \frac{S_D}{S} \quad (1)$$

where S is the overall cross-sectional area and S_D is the total area of all microcracks and cavities, as shown in Fig. 1.

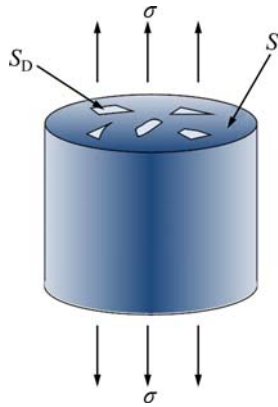


Fig. 1 Definition of one-dimensional damage

For the propose of describing three-dimensional damage phenomena, CHABOCHE [16] and LEMAITRE [17] introduced state potential and dissipation potential based on continuum mechanics and irreversible thermodynamics.

2.2 State potential

The state potential, which is a function of state variables, is defined as the power involved in each physical process. Taking Helmholtz free energy as the state potential, which is a continuum scalar function, concave with temperature, convex with other variables [17]:

$$\varphi = \varphi(\varepsilon, T, \varepsilon^e, \varepsilon^p, r, \alpha, D) \quad (2)$$

where ε is the total strain tensor, T the temperature, ε^e the elastic strain tensor, ε^p the plastic strain tensor, r the damage accumulated plastic strain, α the back strain tensor, and D the damage variable.

For high cycle fatigue, the total strain tensor $\varepsilon = \varepsilon^e$, and

$$\varphi = \varphi(T, \varepsilon^e, D) \quad (3)$$

The analytical expression for φ is chosen through

the “State Kinetic Coupling Theory” [18]. For an isothermal process,

$$\varphi = \frac{1}{\rho} \left[\frac{1}{2} a_{ijkl} \varepsilon_{ij}^e \varepsilon_{kl}^e (1-D) \right] \quad (4)$$

where a_{ijkl} is the fourth order elastic stiffness tensor; ρ is the density.

The law of elasticity coupled with damage is

$$\sigma_{ij} = \rho \frac{\partial \varphi}{\partial \varepsilon_{ij}} = a_{ijkl} \varepsilon_{kl}^e (1-D) \quad (5)$$

by inversion for the isotropic material:

$$\varepsilon_{ij}^e = \frac{1+\nu}{E} \frac{\sigma_{ij}}{1-D} - \frac{\nu}{E} \frac{\sigma_{kk}}{1-D} \delta_{ij} \quad (6)$$

where E is the elastic modulus, ν poisson ratio and δ_{ij} the Kronecker delta.

The variable Y , called “the strain energy density release rate” and associated with D , is defined as [17]

$$Y = -\rho \frac{\partial \varphi}{\partial D} = \frac{\sigma_{eq}^2 R_v}{2E(1-D)^2} \quad (7)$$

where σ_{eq} is the Von Mises equivalent stress,

$$\sigma_{eq} = \left(\frac{3}{2} \sigma_{ij}^D \sigma_{ij}^D \right)^{1/2} \quad (8)$$

R_v is the triaxiality function,

$$R_v = \frac{2}{3} (1+\nu) + 3(1-2\nu) \left(\frac{\sigma_H}{\sigma_{eq}} \right)^2 \quad (9)$$

where σ_{ij}^D and σ_H are stress deviator and hydrostatic stress, respectively.

$$\sigma_{ij}^D = \sigma_{ij} - \delta_{ij} \sigma_H \quad (10)$$

$$\sigma_H = \frac{1}{3} \sigma_{kk} \quad (11)$$

2.3 Dissipation potential

Dissipation potential, associated with the state potential φ , describes the kinetic laws of damage evolution.

Starting from the Clausius-Duhem inequality:

$$\sigma_{ij} \dot{\varepsilon}_{ij} - \rho(\dot{\varphi} + s\dot{T}) - q_i \frac{T_{,i}}{T} \geq 0 \quad (12)$$

With the definition of associated variables, for an isothermal case, it is required:

$$Y\dot{D} \geq 0 \quad (13)$$

According to the principle of thermodynamics, damage evolution is derived from a dissipation potential, which is a scalar continuum convex function of dual variables, and the state variables act as parameters.

For an isothermal case, using Legendre-Fenchel transformation, a dual dissipation potential can be obtained:

$$\psi = \psi(Y, \dot{\pi}; D, r) \quad (14)$$

where π is microplastic strain, r is dual variable for π . Then [6],

$$\dot{D} = \frac{\partial \psi}{\partial Y} \quad (15)$$

3 High cycle fatigue damage model

For high cycle fatigue, plastic strain is negligible compared with elastic strain, and microplastic is considered as the main causing of fatigue damage. Therefore, the law of damage evolution for high cycle fatigue can be represented as [17]

$$\dot{D} = \frac{\eta c E^{\eta/2} d^{\eta/2} e}{l^2 n^{\eta/2-1}} Y^{\eta/2-1} \dot{Y} \quad (16)$$

where η, c are material dependent coefficients; d, e, l and n are geometrical parameters of the microstructure; E is the elastic modulus. Considering no plastic strain occurring for high cycle fatigue, microplastic strain is introduced into the law of damage evolution. Here, Y is related formally to microplastic strain rate $\dot{\pi}$ through plastic constitutive equation:

$$\dot{D} \sim Y^p \dot{\pi} \quad (17)$$

So the proper expression is chosen as

$$\dot{D} = \frac{B}{p+1} Y^{p+1} \dot{\pi} \quad (18)$$

where B, p are material and temperature dependent parameters. In a similar way as for plastic strain, π is microplastic strain,

$$\pi = (\tilde{\sigma}_{eq} / K)^{1/M} \quad (19)$$

or, in reverse

$$\tilde{\sigma}_{eq} = K \pi^M \quad (20)$$

where $\tilde{\sigma}_{eq}$ is effective stress,

$$\tilde{\sigma}_{eq} = \sigma_{eq} / (1-D) \quad (21)$$

and M is strain hardening exponent. The studies by LIN and CHEN [19] and LUO et al [20] showed that, for some metallic materials, strain hardening exponent is dependent on strain rate. Hence, M is not a constant but a function of loading frequency f under different frequencies for cyclic load,

$$M = M(f) \quad (22)$$

From Eq. (15), we have

$$\dot{D} = \frac{\partial \psi}{\partial Y} = B Y^p \dot{\pi} = \frac{M(f) B \sigma_{eq}^{2p+M(f)-1} \dot{\sigma}_{eq} R_v^p}{(2E)^p (1-D)^{2p+M(f)} K^{M(f)}} \quad (23)$$

Defining fatigue damage criterion:

$$\begin{cases} \dot{D} = 0 & (\sigma^* < \sigma_l) \\ \dot{D} = \frac{M(f) B \sigma_{eq}^{2p+M(f)-1} \dot{\sigma}_{eq} R_v^p}{(2E)^p (1-D)^{2p+M(f)} K^{M(f)}} & (\sigma^* > \sigma_l) \end{cases} \quad (24)$$

where σ_l is the fatigue limit, and σ^* is the damage equivalent stress, $\sigma^* = R_v \tilde{\sigma}_{eq}$.

Let $Q(f) = BM(f) / [(2E)^p K^{M(f)}]$, $q(f) = 2p + M(f) - 1$, Eq. (23) reduces to

$$\dot{D} = \frac{Q(f) \sigma_{eq}^{q(f)} \dot{\sigma}_{eq} R_v^p}{(1-D)^{q(f)+1}} \quad (25)$$

For uniaxial cyclic loading, $\sigma_{eq} = |\sigma|$ and there are two cases for different stress ratios R ($R = \sigma_{min} / \sigma_{max}$), as shown in Fig. 2.

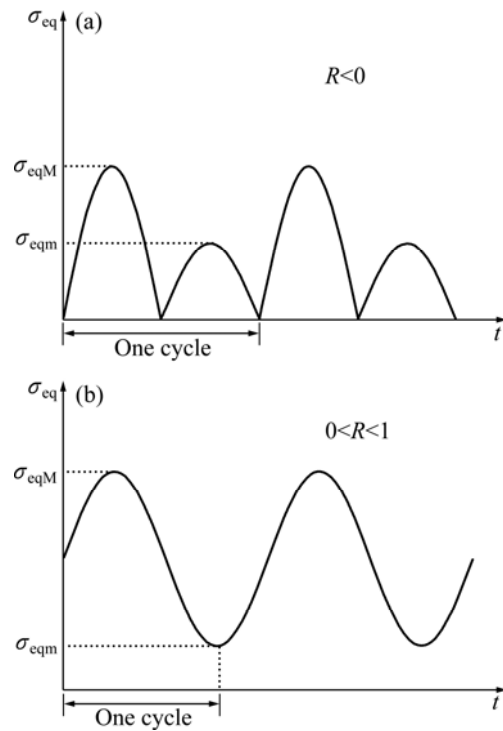


Fig. 2 Typical cases for uniaxial cyclic loading in form of sinusoidal waveform: (a) $R < 0$; (b) $0 < R < 1$

Case I: $R < 0$, for proportional loading, $R_v = 1$. Integrating Eq. (25) over one cycle gives

$$\begin{aligned} \int_D^{D+\frac{\partial D}{\partial N}} dD &= 2 \int_0^{\sigma_{eqM}} \frac{Q(f) \sigma_{eq}^{q(f)} R_v^p}{(1-D)^{q(f)+1}} d\sigma_{eq} + \\ &2 \int_0^{\sigma_{eqm}} \frac{Q(f) \sigma_{eq}^{q(f)} R_v^p}{(1-D)^{q(f)+1}} d\sigma_{eq} \end{aligned} \quad (26)$$

namely,

$$\frac{\partial D}{\partial N} = \frac{2Q(f)R_v^p}{[q(f)+1](1-D)^{q(f)+1}} \left[\sigma_{eqM}^{q(f)+1} + \sigma_{eqm}^{q(f)+1} \right] \quad (27)$$

Case II: $0 < R < 1$, integration of Eq. (25) over one cycle gives

$$\int_D^{D+\frac{\partial D}{\partial N}} dD = 2 \int_{\sigma_{eqm}}^{\sigma_{eqM}} \frac{Q(f)\sigma_{eq}^{q(f)} R_v^p}{(1-D)^{q(f)+1}} d\sigma_{eq} \quad (28)$$

namely,

$$\frac{\partial D}{\partial N} = \frac{2Q(f)R_v^p}{[q(f)+1](1-D)^{q(f)+1}} \left[\sigma_{eqM}^{q(f)+1} - \sigma_{eqm}^{q(f)+1} \right] \quad (29)$$

The two cases mentioned above are identical for $R=0$, namely,

$$\frac{\partial D}{\partial N} = \frac{2Q(f)R_v^p}{[q(f)+1](1-D)^{q(f)+1}} \sigma_{eqM}^{q(f)+1} \quad (30)$$

In this model, microcrack closure effect is ignored, meanwhile, D is considered a constant over one cycle.

For the overall periodic loading,

$$\begin{cases} \int_0^D (1-D)^{q+1} dD = \\ \int_0^N \frac{2Q(f)R_v^p}{[q(f)+1]} \left[\sigma_{eqM}^{q(f)+1} + \sigma_{eqm}^{q(f)+1} \right] dN \quad (R < 0) \\ \int_0^D (1-D)^{q+1} dD = \\ \int_0^N \frac{2Q(f)R_v^p}{[q(f)+1]} \left[\sigma_{eqM}^{q(f)+1} - \sigma_{eqm}^{q(f)+1} \right] dN \quad (0 < R < 1) \\ \int_0^D (1-D)^{q+1} dD = \\ \int_0^N \frac{2Q(f)R_v^p}{[q(f)+1]} \left[\sigma_{eqM}^{q(f)+1} \right] dN \quad (R = 0) \end{cases} \quad (31)$$

The number of cycle to failure N_R is obtained by Eq. (31) with the initiation conditions: $N=0$, $D=0$ and $N=N_R$, $D=1$. We have

$$\begin{cases} N_R = \frac{[q(f)+1] \left[\sigma_{eqM}^{q(f)+1} + \sigma_{eqm}^{q(f)+1} \right]^{-1}}{2[q(f)+2]Q(f)R_v^p} \quad (R < 0) \\ N_R = \frac{[q(f)+1] \left[\sigma_{eqM}^{q(f)+1} - \sigma_{eqm}^{q(f)+1} \right]^{-1}}{2[q(f)+2]Q(f)R_v^p} \quad (0 < R < 1) \\ N_R = \frac{[q(f)+1] \left[\sigma_{eqM}^{q(f)+1} \right]^{-1}}{2[q(f)+2]Q(f)R_v^p} \quad (R = 0) \end{cases} \quad (32)$$

when $R=-1$, $\sigma_{eqM}=\sigma_{eqm}$,

$$N_R = \frac{[q(f)+1] \left[\sigma_{eqM}^{q(f)+1} \right]^{-1}}{4[q(f)+2]Q(f)R_v^p} \quad (33)$$

4 Identification and validation of model

To simplify Eq. (33), let us define

$$H(f) = \frac{[q(f)+1]}{4[q(f)+2]Q(f)R_v^p} \quad (34)$$

$$c(f) = q(f)+1 \quad (35)$$

Equation (33) reduces to

$$N_R = H(f) \left[\sigma_{eqM}^{c(f)} \right]^{-1} \quad (36)$$

$H(f)$ and $c(f)$ may be identified from $S-N$ curve.

4.1 Fatigue behavior of frequency-independent material

Fatigue behaviors of some materials are insensitive to loading frequency. The result of fatigue tests performed on AlZnMgCu1.5 aluminium alloy with fully reversed tension—compression loading ($R=-1$) at 100 Hz and 20 kHz does not indicate a frequency influence on the fatigue properties, as shown in Fig. 3 cited from Ref. [9]. Hence, the fatigue lives of AlZnMgCu1.5 aluminium alloy with different frequencies may be predicted using the same model with constant H and c .

$$N_R = (1.83534 \times 10^{44}) \sigma_{eqM}^{-16.16} \quad (37)$$

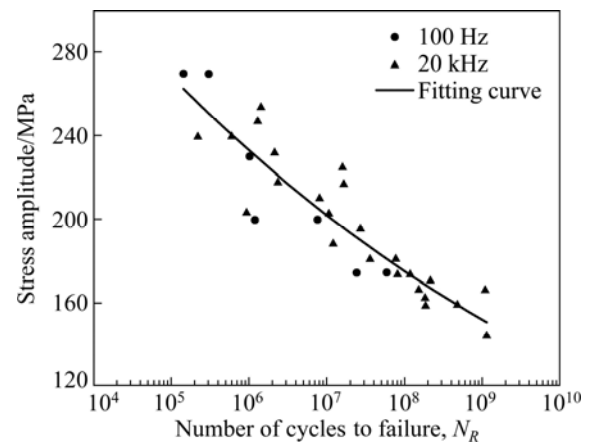


Fig. 3 Fatigue life of AlZnMgCu1.5 aluminium alloy with cyclic frequencies of 100 Hz and 20 kHz

4.2 Fatigue behavior of frequency-dependent material

The results of studies on the influence of cyclic frequency for AMg6N [12], commercially pure tantalum [13], 1Kh2M steel [14] and so on, reveal a pronounced frequency influence on fatigue behavior. Therefore, $H(f)$ and $c(f)$ are no more constants but functions of loading frequency. Figure 4 shows the results of fatigue tests of AMg6N at different frequencies with $R=-1$. The arrows indicate unbroken specimens. Increasing of loading frequency leads to increasing fatigue lives for AMg6N.

The data are from Ref. [12]. The values of H and c for each frequency rise with increasing loading frequency, as summarized in Table 1. The fitting curves of $c(f)$ vs $\lg f$ and $\lg[H(f)]$ vs $\lg f$ are plotted in Fig. 5 and Fig. 6, respectively. Thus, for AMg6N, the fatigue lives at different frequencies can be predicted using the following model, which exposes the influence of loading frequency on fatigue properties. Good agreement is obtained.

$$\begin{cases} N_R = H(f)\sigma_{\text{eqM}}^{-c(f)} \\ c(f) = 9.76 + 0.731e^{0.871\lg f} \\ \lg[H(f)] = 26.46 + 1.507e^{0.899\lg f} \end{cases} \quad (38)$$

Table 1 Values of H and c for each frequency

f/Hz	$c(f)$	$H(f)/\text{MPa}$	Fatigue damage model
35	12.32	9.57×10^{31}	$N_R = 9.574 \times 10^{31} \sigma_{\text{eqM}}^{-12.32}$
500	18.05	7.36×10^{44}	$N_R = 7.364 \times 10^{44} \sigma_{\text{eqM}}^{-18.05}$
3000	24.23	4.17×10^{59}	$N_R = 4.167 \times 10^{59} \sigma_{\text{eqM}}^{-24.23}$
10000	33.71	7.75×10^{81}	$N_R = 7.753 \times 10^{81} \sigma_{\text{eqM}}^{-33.71}$

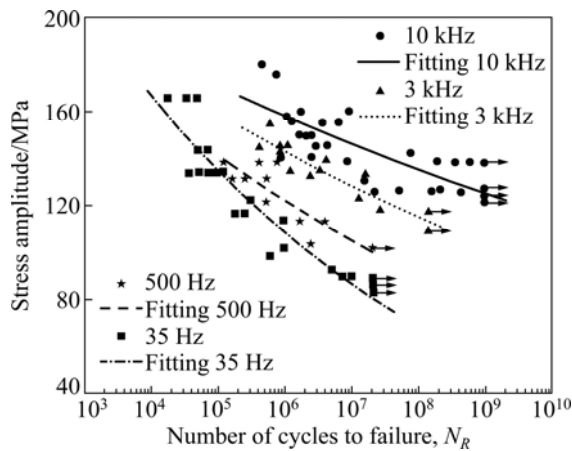


Fig. 4 Fatigue life of AMg6N alloy with loading frequency

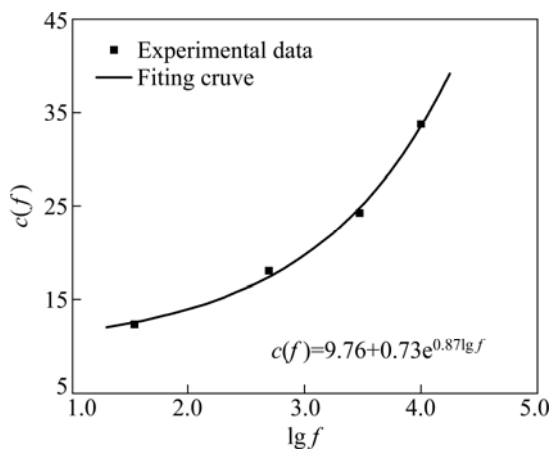


Fig. 5 Fitting curve of relationship between $c(f)$ and $\lg f$

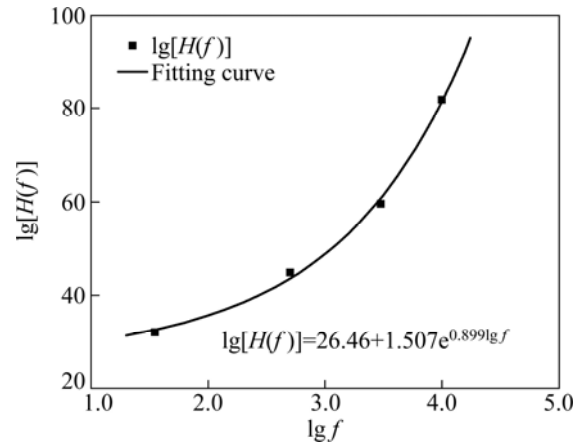


Fig. 6 Fitting curve of relationship between $\lg[H(f)]$ and $\lg f$

5 Conclusions

1) A new continuum damage model for high cycle fatigue was presented, which took into account the influence of cyclic frequency on fatigue life.

2) The parameters in the model H and c are constants for frequency-independent materials, but functions of loading frequencies for frequency-dependent materials.

3) The proposed model exhibits an excellent capacity of predicting the fatigue life both for frequency-independent and frequency-dependent materials.

References

- [1] MALARVIZHI S, BALASUBRAMANIAN V. Effect of welding processes on AA2219 aluminium alloy joint properties [J]. Transactions of Nonferrous Metals Society of China, 2011, 21(5): 962–973.
- [2] CHEN J Z, ZHEN L, YANG S J. Effects of precipitates on fatigue crack growth rate of AA 7055 aluminum alloy [J]. Transactions of Nonferrous Metals Society of China, 2010, 20(12): 2209–2214.
- [3] YANG X, LI N, JIN Z. A continuous low cycle fatigue damage model and its application in engineering materials [J]. International Journal of Fatigue, 1997, 19(10): 687–692.
- [4] MOHANTY J R, VERMA B B, RAY P K. Prediction of fatigue crack growth and residual life using an exponential model: Part I (constant amplitude loading) [J]. International Journal of Fatigue, 2009, 31(3): 418–424.
- [5] MOHANTY J R, VERMA B B, RAY P K. Prediction of fatigue crack growth and residual life using an exponential model: Part II (mode-I overload induced retardation) [J]. International Journal of Fatigue, 2009, 31(3): 425–432.
- [6] LEMAITRE J. How to use damage mechanics [J]. Nuclear Engineering and Design, 1984, 80(2): 233–245.
- [7] XIAO Y C, LI S, GAO Z. A continuum damage mechanics model for high cycle fatigue [J]. International Journal of Fatigue, 1998, 20(7): 503–508.
- [8] DESMORAT R, KANE A, SEYEDI M. Two scale damage model and related numerical issues for thermo-mechanical high cycle fatigue [J]. European Journal of Mechanics: A/Solids, 2007, 26(6): 1015–1030.

- 909–935.
- [9] MAYER H, PAPAKYRIACOU M, PIPPAN R. Influence of loading frequency on the high cycle fatigue properties of AlZnMgCu1.5 aluminium alloy [J]. Materials Science and Engineering A, 2001, 314(1–2): 48–54.
- [10] JIANG L, BROOKS C R, LIAW P K. High-frequency metal fatigue: The high-cycle fatigue behavior of ULTIMET® alloy [J]. Materials Science and Engineering A, 2001, 314(1–2): 162–175.
- [11] MORRISSEY R J, NICHOLAS T. Fatigue strength of Ti–6Al–4V at very long lives [J]. International Journal of Fatigue, 2005, 27(10–12): 1608–1612.
- [12] KOFTO D G. Effect of loading frequency and cycle asymmetry on the fatigue resistance of alloy AMg6N [J]. Strength of Materials, 1990, 22(2): 283–289.
- [13] PAPAKYRIACOU M, MAYER H, PYPEN C. Influence of loading frequency on high cycle fatigue properties of b.c.c. and h.c.p. metals [J]. Materials Science and Engineering A, 2001, 308(1–2): 143–152.
- [14] KUZ'MENKO V A, ISHCHENKO I I, TROYAN I A. Effect of loading frequency, temperature, and cycle asymmetry on the endurance of heat-resistant steels 1Kh2m and Kh18N9: Part 1 [J]. Strength of Materials, 1980, 12(4): 439–444.
- [15] TSUTSUMI N, MURAKAMI Y, DOQUET V. Effect of test frequency on fatigue strength of low carbon steel [J]. Fatigue & Fracture of Engineering Materials & Structures, 2009, 32(6): 473–483.
- [16] CHABOCHE J L. Continuum damage mechanics: Part I—General concepts [J]. Journal of Applied Mechanics, 1988, 55(1): 59–64.
- [17] LEMAITRE J. A course on damage mechanics [M]. Berlin: Springer-Verlag, 1996: 39–94.
- [18] LEMAITRE J, MARQUIS D. Modeling complex behavior of metals by the “State-Kinetic Coupling Theory” [J]. Journal of Engineering Materials and Technology, 1992, 114(3): 250–254.
- [19] LIN X Z, CHEN D L. Strain hardening and strain-rate sensitivity of an extruded magnesium alloy [J]. Journal of Materials Engineering and Performance, 2008, 17(6): 894–901.
- [20] LUO J, LI M, YU W. The variation of strain rate sensitivity exponent and strain hardening exponent in isothermal compression of Ti–6Al–4V alloy [J]. Materials and Design, 2010, 31(2): 741–748.

一个金属材料高周疲劳损伤力学模型

张 亮¹, 刘雪松¹, 王林森^{1,2}, 吴双辉¹, 方洪渊¹

1. 哈尔滨工业大学 先进焊接与连接国家重点实验室, 哈尔滨 150001;

2. 东方锅炉股份有限公司 工艺部焊接室, 自贡 643001

摘 要: 基于不可逆热力学提出一个新的高周疲劳损伤力学模型, 该模型考虑载荷频率对疲劳寿命的影响。模型中的参数 H 和 c 对于无频率效应的材料是常数, 而对于有频率效应的材料则是与频率有关的函数。同时, 讨论了不同应力比时模型的表达形式。利用 AlZnMgCu1.5 和 AMg6N 两种材料在不同频率下的疲劳实验数据验证提出的模型。结果表明, 该模型能够准确预测材料在不同加载频率和应力比条件下的疲劳寿命。

关键词: AlZnMgCu1.5 合金; AMg6N 合金; 连续介质损伤模型; 频率; 高周疲劳; 加载频率; 疲劳寿命; 疲劳裂纹

(Edited by YUAN Sai-qian)