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Predicting pillar stability for underground mine using Fisher discriminant analysis and SVM methods

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Abstract: The purpose of this study is to apply some statistical and soft computing methods such as Fisher discriminant analysis (FDA) and support vector machines (SVMs) methodology to the determination of pillar stability for underground mines selected from various coal and stone mines by using some index and mechanical properties, including the width, the height, the ratio of the pillar width to its height, the uniaxial compressive strength of the rock and pillar stress. The study includes four main stages: sampling, testing, modeling and assessment of the model performances. During the modeling stage, two pillar stability prediction models were investigated with FDA and SVMs methodology based on the statistical learning theory. After using 40 sets of measured data in various mines in the world for training and testing, the model was applied to other 6 data for validating the trained proposed models. The prediction results of SVMs were compared with those of FDA as well as the measured field values. The general performance of models developed in this study is close; however, the SVMs exhibit the best performance considering the performance index with the correct classification rate P_{rs} by re-substitution method and P_{cv} by cross validation method. The results show that the SVMs approach has the potential to be a reliable and practical tool for determination of pillar stability for underground mines.

Key words: underground mine; pillar stability; Fisher discriminant analysis (FDA); support vector machines (SVMs); prediction

1 Introduction

Pillars are key structural columns that are commonly applied in underground mining. They are usually made of in situ intact ores and do not have additional reinforcements. Their main function is to provide temporary or permanently support for the weight of overburden material between adjacent underground openings and ore ceiling of drilling rooms during excavation and mining [1−6]. As mining goes deeper and deeper, pillar failure becomes more and more frequent and critical due to the remarkable increase in ambient stresses. Because of their significance in safe and economical extraction of underground ores, mine pillars and their design have been investigated by a number of researchers and engineers [1, 6].

Over the past decades, deterministic (empirical, statistical or analytical) methods for the estimation of mine pillar stability have been developed [4−5, 7]. Pillar design is typically carried out by estimating the strength and the stress of the pillars, and then sizing the pillars so that an adequate margin exists between the expected pillar strength and stress. Because the uniaxial compressive strength of the rock plays an important role in pillar instability, the stability of a pillar can be evaluated by calculating a factor of safety (FoS), which is the ratio of the average strength (S) to the average stress (σ_p) in the pillar (FoS= S/σ_p) [6]. Theoretically, the FoS value greater than 1 means that the pillar is stable, while the FoS value lower than 1 means unstable [7]. Sometimes, these methods, however, are questionable because failures in pillars did occur even though the failed pillars had been considered stable, i.e., FoS >1 [6]. Besides this, due to the nonlinear behaviour of rock pillars at the high stress levels associated with deep mining conditions, the failure mechanism is not considered explicitly in these methods. And empirical methods are based on interpretation of available databases collected from ongoing or completed projects,

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it is therefore difficult to generalize the obtained results beyond the scope of the original site characteristics [8]. Moreover, under actual field conditions, several important factors affect the pillar stability of underground excavations including in situ stress, rock mass properties, site geology, mining method, opening spans, and time, since excavation has no complete theoretical solution for its prediction [8−11]. In such situations, a wide range of statistical and numerical solutions, software computing methods and machine learning models like currently artificial intelligence (AI), have been developed and applied in this field to estimate pillar stability for underground mines, and the researchers have made admirable efforts over the pillar design and layout applied in rocks. GRIFFITHS et al [12] presented a probabilistic analysis of underground pillar stability. HUTCHINSON et al [13] recommended the use of simulation methods for considerations of crown pillar stability risk assessment in mine planning. DENG et al [6] proposed a pillar design based on Monte Carlo simulation by combining finite element methods, neural networks and reliability analysis. CAUVIN et al [7] used probabilistic method to assess uncertainties in mining pillar stability. GRIFFITHS et al [14] combined random field theory with an elasto-plastic finite element algorithm in a Monte Carlo framework to estimate the stability of pillars. PALEI and DAS [15] presented a logistic classification model for prediction of roof fall risks in bord and pillar workings in coal mines. JAISWAL and SHRIVASTVA [16] and MOHAN et al [10] established a three-dimensional finite element model for estimation of coal-mass pillar strength through calibration of a numerical model. ELMO and STEAD [9] investigated the use of the hybrid FEM/DEM code ELFEN in studying the failure modes of jointed pillars. MONJEZI et al [17] developed a multilayer perceptron neural network model methodology to predict the pillar stress concentration in the bord and pillar method and compared the results with BEM numerical solution.

Fisher discriminant analysis (FDA) is a well-known classical linear method for classifying two or multiple classes and it has been shown to be the optimal linear techniques for fault diagnosis [18−19], rockburst prediction [20] and reliability assessment for mine ventilation system safety [21]. Thus, FDA will be used as a representative linear technique for comparison with nonlinear technique. Among AI tools, support vector machine (SVM) is an efficient ML technique derived from statistical learning theory by VAPNIK [22]. As a representative nonlinear technique, SVM will be used since it has been shown to be an effective technique for classifying nonlinear dataset [18−19, 22−25]. It is therefore motivating to investigate the capability of FDA, and SVM in pillar stability for underground mines prediction.

2 Calculation theory of Fisher discriminant analysis

The basic idea of FDA [18−21] is an optimal dimensionality reduction technique in terms of maximizing the separability of these classes. It determines a set of projection vectors that maximizes the scatter between the classes while minimizes the scatter within each class. A short mathematical description is introduced as follows.

Define n as the number of observations, m as the number of measurement variables, p as the number of classes, and n_j as the number of observations in the jth class. Represent the vector of measurement variables for the ith observation as $\boldsymbol{x}_i$. If the training data for all classes have already been stacked into the matrix $\boldsymbol{X}\in R^{n\times m}$, then the transposition of the ith row of $\boldsymbol{X}$ is the column vector $\boldsymbol{x}_i$, the total-scatter matrix is

$$\boldsymbol{V}_{\mathrm{t}}=\sum_{i=1}^{n}(\boldsymbol{x}_i-\bar{\boldsymbol{x}})(\boldsymbol{x}_i-\bar{\boldsymbol{x}})^{\mathrm{T}} \tag{1}$$

where $\bar{\boldsymbol{x}}$ is the total mean vector whose elements correspond to the means of the columns of $\boldsymbol{X}$.

Let the matrix $\boldsymbol{X}_j$ be defined as the set of vectors $\boldsymbol{X}_j$ that belong to class j, and then, let $\boldsymbol{V}_{\mathrm{w}}$ and $\boldsymbol{V}_{\mathrm{b}}$ be the within-class scatter matrix and the between-class scatter matrix, which are respectively defined as:

$$\boldsymbol{V}_{\mathrm{w}}=\sum_{j=1}^{p}\sum_{x_i\in X_{jj}}(\boldsymbol{x}_i-\bar{\boldsymbol{x}}_j)(\boldsymbol{x}_i-\bar{\boldsymbol{x}}_j)^{\mathrm{T}} \tag{2}$$

$$\boldsymbol{V}_{\mathrm{b}}=\sum_{j=1}^{p}m_j(\bar{\boldsymbol{x}}_j-\bar{\boldsymbol{x}})(\bar{\boldsymbol{x}}_j-\bar{\boldsymbol{x}})^{\mathrm{T}} \tag{3}$$

where $\bar{\boldsymbol{x}}_j=\dfrac{1}{nj}\sum_{\boldsymbol{x}_i\in x_j}\boldsymbol{x}_i$ is the mean vector for class j; $\bar{\boldsymbol{x}}=\dfrac{1}{j}\sum_{i=1}^{j}\boldsymbol{x}_i$ is the total mean vector; m_j is the number of observations in class j.

The total-scatter matrix is equal to the sum of the between-scatter matrix and the within-scatter matrix:

$$\boldsymbol{V}_{\mathrm{t}}=\boldsymbol{V}_{\mathrm{b}}+\boldsymbol{V}_{\mathrm{w}} \tag{4}$$

FDA looks for a projection matrix, $\boldsymbol{\beta}$, which maximizes the Fisher's criterion.

$$J_{\mathrm{Fisher}}(\boldsymbol{\beta})=\arg\max_{\boldsymbol{\beta}}\left|\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{V}_{\mathrm{b}}\boldsymbol{\beta}\right|/\left|\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{V}_{\mathrm{w}}\boldsymbol{\beta}\right| \tag{5}$$

Thus, the maximization problem reduces to solve

$$\boldsymbol{V}_{\mathrm{b}}\boldsymbol{\beta} = \eta \boldsymbol{V}_{\mathrm{w}}\boldsymbol{\beta} \tag{6}$$

where the eigenvalues η indicate the degree of overall separability among the classes. If $\boldsymbol{V}_{\mathrm{w}}$ is nonsingular, we can obtain a conventional eigenvalue problem by the following expression:

$$\boldsymbol{V}_{\mathrm{w}}^{-1}\boldsymbol{V}_{\mathrm{b}}\boldsymbol{\beta} = \eta\boldsymbol{\beta} \tag{7}$$

Notice that there will be t=min(K−1, d), eigenvalues $\eta_1 \geq \eta_2 \geq \ldots \geq \eta_t$ and t corresponds to eigenvectors $\boldsymbol{\beta}=(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_t)$. So, the discriminant function can be given by

$$\boldsymbol{y} = \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x} \tag{8}$$

With the discriminant function, the Mahalanobis distance (M_{D}) can be used to identify which class the new measured data G_i belong to. If a new sample is denoted as $x = (x^1, x^2, \cdots, x^m)^{\mathrm{T}}$, then

$$M_{Di} = (\boldsymbol{x}-\boldsymbol{\mu}_i)^{\mathrm{T}}\sum{}^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_i) \tag{9}$$

where $\boldsymbol{\mu}_i = (\boldsymbol{\mu}_{i1}, \boldsymbol{\mu}_{i2}, \cdots, \boldsymbol{\mu}_{im})^{\mathrm{T}}$ is the mean vector of $\boldsymbol{G}^i$.

Therefore, the fault diagnosis problem is to compare the M_{Ds} of all the candidates and select the minimum.

$$\min_{i=1,k}\{M_{Di}\} \Rightarrow i \tag{10}$$

If i = 1, 2, ⋯, k, which means that the new data belong to G^1, G^2 , ⋯, G^k, indicating various stability conditions. The FDA model of discriminant procedure is shown in Fig. 1.

3 SVM classifier

3.1 Brief theory of SVM

SVM methodology [18−19, 22−25] is a kind of machine learning technique based on statistical learning theory. The basic idea of applying SVM to patterning classification can be stated as follows.

Give the training set (x_i, y_i), where i=1, 2, ⋯, m, $\boldsymbol{x} \in \Omega^d$, $\boldsymbol{y} \in \{-1, +1\}$ can be separated by the hyperplane $\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x} + b = 0$, where $\boldsymbol{w}$ is the weight vector and b is the bias. If this hyperplane maximizes the margin, then the following inequality is valid for all input data:

$$(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}_i + b)\boldsymbol{y}_i \geq 1, \text{ for all } \boldsymbol{x}_i \text{ where } i=1, 2, \cdots, r \tag{11}$$

The margin of the hyperplane is equal to 2/||$\boldsymbol{w}$||. Thus, the problem is maximizing the margin by minimizing ||$\boldsymbol{w}$||/2 subject to Eq. (11). This is a convex quadratic programming problem. Lagrange multipliers (LM) (α_i>0, i=1, ⋯, r) are used to solve

$$L_{\mathrm{P}} = -\sum_{i=1}^{m}\alpha_i[(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}_i + b)\boldsymbol{y}_i - 1] + \| w \|^2 / 2 .$$

After minimizing L_{p} with respect to both $\boldsymbol{w}$ and b, the optimal weights are given by: $\boldsymbol{w}^* = \sum_{i=1}^{m}\alpha_i^*\boldsymbol{y}_i\boldsymbol{x}_i$.

Only if α_i>0, $\boldsymbol{x}_i$ are called support vectors. The dual of the problem is given by:

$$L_{\mathrm{d}}(\alpha) = -\frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\boldsymbol{a}_i\boldsymbol{a}_j\boldsymbol{y}_i\boldsymbol{y}_j\boldsymbol{x}_i\boldsymbol{x}_j + \sum_{i=1}^{r}\boldsymbol{a}_i \tag{12}$$

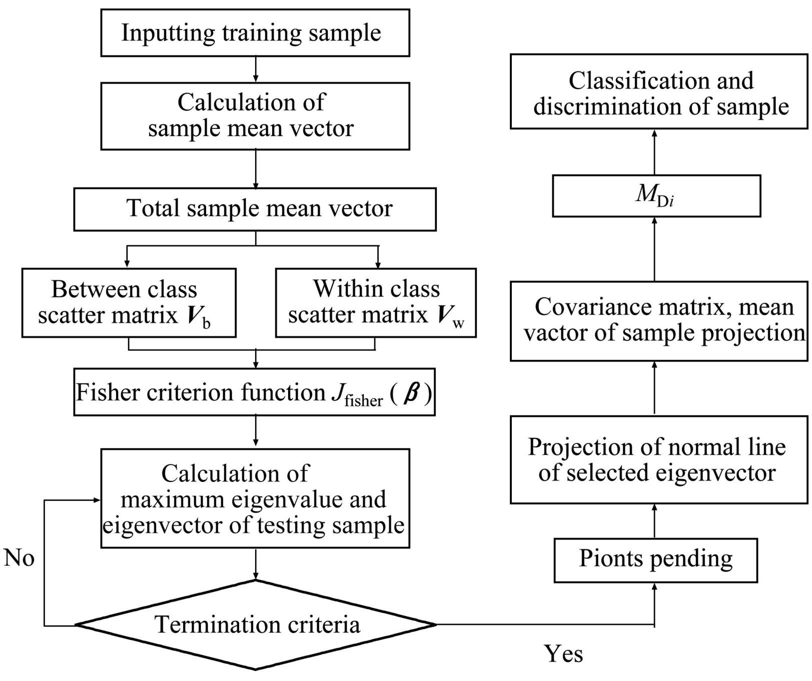

Fig. 1 Flow chart for evaluating implication of FDA

The LM is only non-zero, when $(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}_i+b)\boldsymbol{y}_i=1$. The optimal bias for any support vector $\boldsymbol{x}_i$ is given by: $b^*=\boldsymbol{y}_i-\boldsymbol{w}^{*\mathrm{T}}\boldsymbol{x}_i$. When SVMs are trained, the decision function takes the form of:

$$F(\boldsymbol{x})=\mathrm{sign}(\sum_{i=1}^{r}a_i^*\boldsymbol{y}_i\boldsymbol{x}_i\boldsymbol{x}^{\mathrm{T}}+b^*) \tag{13}$$

where a_i^* are optimal LM; sign(·) is the signum function. It gives +1 (stable pillar) if the element is greater than or equal to zero and −1 (failed pillar) if it is less than zero.

For input data with a high noise level, SVM uses soft margins that can be expressed as follows with the introduction to the non-negative slack variables ξ_i, $i=1, \cdots, r$:

$$(b+\boldsymbol{w}^{*\mathrm{T}}\boldsymbol{x}_i)\boldsymbol{y}_i\geq 1-\xi_i \text{ for } i=1, 2, \cdots, r \tag{14}$$

To obtain the optimal separating hyperplane, it should be minimized by $\psi=C\sum_{i=1}^{r}\xi_i^k+\frac{1}{2}\|\boldsymbol{w}\|^2$ subject to Eq. (14), where C is the penalty parameter controlling the tradeoff between the complexity of the decision function and the number of training misclassified examples.

For a linear non-separable case, SVM performs a nonlinear mapping of the input vector $\boldsymbol{x}$ from the input space $\boldsymbol{R}^{\mathrm{N}}$ into a higher dimensional Hilbert space, where the mapping is determined by kernel function $K(\boldsymbol{x}, \boldsymbol{y})$. Radial basis function (RBF) is one of the kernel functions that are given by $K(\boldsymbol{x}, \boldsymbol{y})=\exp(-\|\boldsymbol{x}-\boldsymbol{y}\|^2/2g^2)$, where g is the width of the RBF kernel [23]. After a kernel function is selected, the decision function will become:

$$F(x)=\mathrm{sign}\left[\sum_{i=1}^{r}a_i^*\boldsymbol{y}_iK(\boldsymbol{x}_i,\boldsymbol{x})+b^*\right] \tag{15}$$

where C and g are user-determined parameters by an iterative process selecting an optimum value based on the full training data set. The election of the parameters plays an important role in the performance of SVMs. Further detailed mathematical description over SVMs can be referred from Refs. [18−19, 22−25].

3.2 SVMs model of discriminant procedure

The approach for the development of the SVMs-based correlation can be divided into five stages: ① Collection of database in which the data have been divided into two sub-sets, a training dataset and a testing dataset; ② Linear scaling of the train data set from 0 to 1 and calculation of the various parameters (model parameters) for establishing the classification function; ③ Estimation of the optimal model parameters (C, g) using the combined approach of K-fold cross validation and grid search method (GSM) [25]; ④ Establishment of the final SVMs model for pillar stability with the help of the best parameters and ⑤ Evaluation and validation of the SVMs model by evaluation with testing data and comparing it with literature correlations. For better understanding of a flow diagram, the establishment of the SVM-based model for prediction of pillar stability is described as shown in Fig. 2.

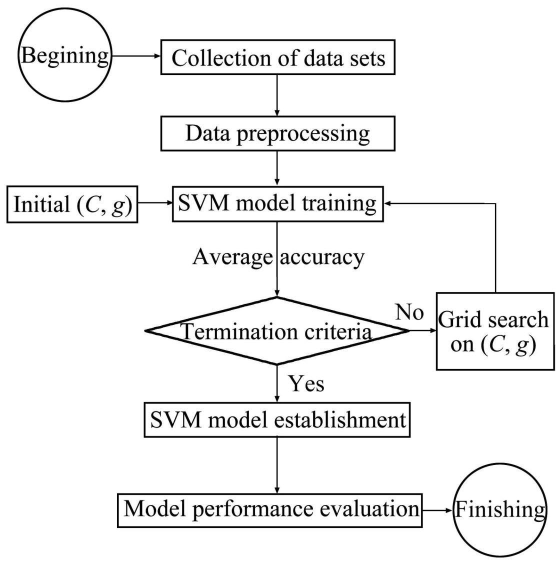

Fig. 2 Flowchart of key steps for establishment of SVM model

4 Hazard of pillar prediction of FDA model and SVM model

4.1 Data collection

Underground mine excavation is usually achieved by the room and pillar method, bord and pillar method, longwall mining method from the aspects of work safety and cost issue [1, 2, 14, 17, 26−29]. The present study aims to establish predictive models for pillar stability for underground mines purposes using FDA and SVMs technique, and also aims to attract the mining engineering attention to predicting pillar stability. For this purpose, a total of 46 case histories were collected from JAISWAL and SHRIVASTVA [16], MOHAN et al [10] and ESTERHUIZEN et al [11]. Tables 1 lists the details of the failed and stable cases of coal pillars of Indian coal mines and the USA underground stone mines, respectively. The scatter plot matrix of the original data set is given in Fig. 3. For implementing the FDA and SVMs models, the data have been divided into two sub-sets:

1) A training dataset. This is required to train the model. In this study, 40 out of a total of 46 data sets are considered for training.

2) A testing dataset. This is required to estimate the model performance. In this study, the remaining 6 data sets are used for testing.

Table 1 Summary of characteristics of stable and failed pillars

No.	Mine	W/m	H/m	W/H	σ_{UCS}/MPa	σ_p/MPa	Pillar stability
1	Amritnagar (Nega Jamehari)	3.60	4.50	0.80	45	5.01	F
2[a]	Amritnagar (Nega Jamehari)	3.60	6.00	0.60	45	4.68	F
3	Begonia (Begonia)	3.90	3.00	1.30	26	5.80	F
4	Amlai (Burhar)	4.50	5.40	0.83	25	3.61	F
5	Sendra Bansjora (X)	4.65	8.10	0.57	24	2.77	F
6	W. Chirimiri (Main)	5.40	3.75	1.44	45	8.12	F
7	Birsingpur (Johilla top)	7.50	3.60	2.08	38	10.45	F
8	Pure Kajora (Lower Kajora)	5.40	3.60	1.50	33	6.02	F
9[a)]	Pure Kajora (Lower Kajora)	4.95	3.60	1.38	33	7.43	F
10	Shankarpur (Jambad bottom)	4.50	4.80	0.94	47	4.20	F
11	Ramnagar (Begunia)	2.85	1.80	1.58	26	7.76	F
12	Ramnagar (Begunia)	3.00	1.80	1.67	26	6.17	F
13	Kankanee (XIII)	19.80	6.60	3.00	27	5.88	F
14	Kankanee (XIV)	18.60	8.40	2.20	27	5.83	F
15	Bellampalli (Ross)	5.40	3.00	1.80	48	4.01	S
16	Nimcha (Nega)	9.90	6.00	1.70	50	3.09	S
17	Morganpit (Salarjung)	8.10	3.00	2.70	46	14.08	S
18	Ramnagar (Ramnagar)	9.90	2.70	3.70	28	5.20	S
19[a)]	Lachhipur (Lower Kajora)	7.20	5.10	1.40	33	2.25	S
20	N. Salanpur (X)	9.00	5.10	1.80	21	2.08	S
21[a)]	Bankola (Jambad top)	10.10	4.80	2.10	35	3.09	S
22	Bankola (Jambad top)	6.30	3.00	2.10	35	5.20	S
23	Suraka cchar (G-I)	160.00	3.50	4.60	29	4.14	S
24	Lachhipur (Lower Kajora)	18.30	5.10	3.60	33	1.40	S
25	E. Angarapatra (XII)	6.00	2.10	2.90	19	3.00	S
26	Kargali Incline (Kathara)	9.30	3.60	2.60	40	2.34	S
27	Jamadoba 6 and 7 Pits (XVI)	5.80	2.00	2.90	29	7.59	S
28	Topsi (Singharan)	7.00	1.80	3.90	41	5.15	S
29	Stone mines (USA)	10.70	18.30	0.58	215	9.00	F
30	Stone mines (USA)	10.70	18.30	0.58	215	9.40	F
31	Stone mines (USA)	10.70	18.30	0.58	215	10.30	F
32	Stone mines (USA)	15.20	27.40	0.56	153	12.60	F
33[a)]	Stone mines (USA)	10.70	18.30	0.58	215	12.80	F
34	Stone mines (USA)	12.20	27.40	0.44	150	17.20	F
35	Stone mines (USA)	8.50	15.80	0.54	150	17.20	F
36	Stone mines (USA)	12.20	27.40	0.44	150	17.30	F
37	Stone mines (USA)	7.90	9.80	0.81	160	19.00	F
38	Stone mines (USA)	12.80	7.30	1.73	160	17.40	F
39	Stone mines (USA)	12.50	15.20	0.82	160	17.80	F
40	Stone mines (USA)	6.10	12.20	0.49	160	19.00	F
41[a)]	Stone mines (USA)	6.70	12.20	0.54	160	20.00	F
42	Stone mines (USA)	3.70	8.50	0.43	215	24.10	F
43	Stone mines (USA)	8.20	9.10	0.90	160	25.00	F
44	Stone mines (USA)	5.50	7.30	0.75	160	27.00	F
45	Stone mines (USA)	12.20	15.80	0.77	165	8.40	F
46	Stone mines (USA)	12.20	15.80	0.77	165	7.60	F

Test data from 1 to 28 are referred to Refs. [10, 16]; Test data from 29 to 46 are referred to Ref. [11]; a) denotes testing sample by selecting stochasticly; The total number of data is 46.

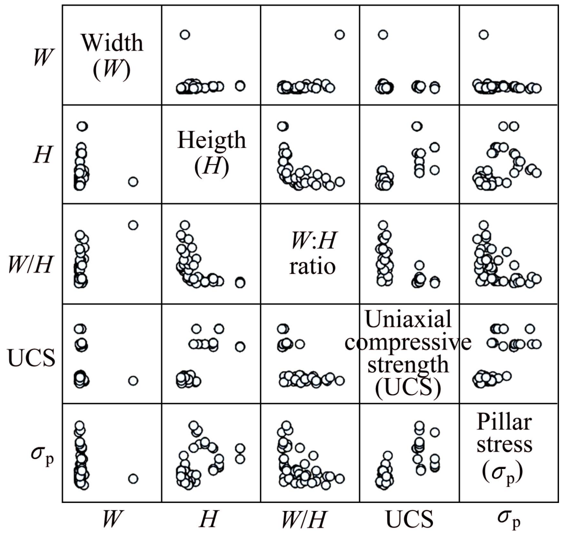

Fig. 3 Scatter plot matrix of original data set

4.2 Model development and validation

In order to establish predictive models, five main pillar parameters are used in the developed SVMs model on the basis of previous researching indices of pillar stability [10−17]. They are the width (W), height (H), the ratio of the pillar width to its height (W/H), the uniaxial compressive strength of the rock (σ_{UCS}), pillar stress(σ_{p}). All these parameters are controllable. Thus 5 parameters are used to establish pillar stability prediction models based on FDA and SVMs incorporating the pillar parameters. Table 2 indicates the relevant input parameters used to develop pillar stability prediction models with their maximum, minimum, mean and standard deviation, respectively. Moreover, the outputs of FDA and SVMs models for the classification prediction of pillar stability are as follows: S is defined as stable pillar and F is defined as failed pillar according to characteristics of pillar stability for underground mines. Figure 4 illustrates typical failure mechanism of a naturally fractured pillar [9], including 1) failure by lateral kinematic release of preformed blocks due to the increasing vertical load, 2) failure as a result of the formation of inclined shear fractures transecting the pillar, typically in relative low width-to-height (W/H) ratio pillars and 3) failure along a set of transgressive fractures when the angle of inclination of the fractures to the pillar principal axis of loading exceeds the angle of friction. Overall, the mechanical response of a pillar is directly linked to the presence of geological structures and it can be safely assumed that these effects would be more noticeable for slender pillars.

Table 2 Descriptive statistics of input parameters for SVMs model

Item	W/m	H/m	W/H	σ_{UCS}/MPa	σ_{p}/MPa
Maximum	19.8	27.4	4.6	215.0	27.0
Minimum	2.85	1.80	0.43	19.00	1.40
Mean	8.679	8.699	1.522	88.739	9.510
Standard deviation	4.284	7.166	1.072	71.203	6.832

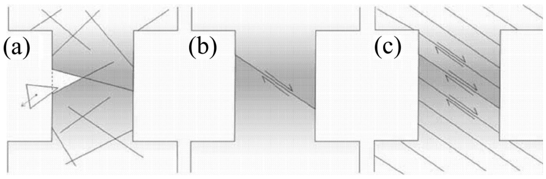

Fig. 4 Typical failure mechanism of a naturally fractured pillar [9]: (a) Occurrence of preformed blocks; (b) Formation of inclined shear fractures transecting pillar; (c) Transgressive fractures

The boxplot of the original data set is given in Fig. 5. For most of the data groups, the median is not in the centre of the box, which indicates that the distribution is not symmetric. In addition, all dependent variables do not have any outlier except W, H and W/H.

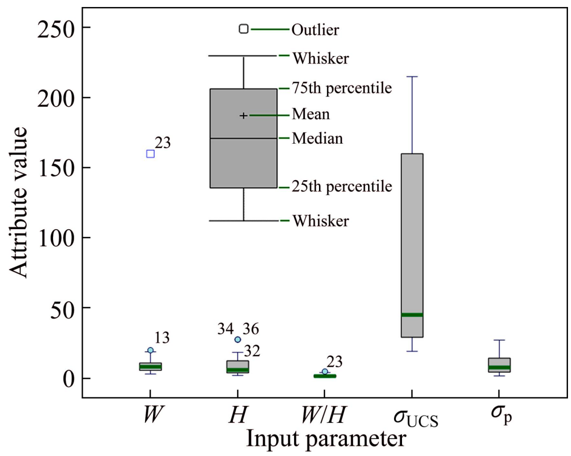

Fig. 5 Boxplot of original data set

4.2 1 Testing and validation of FDA model

In the present approach, the engineering data [10−11, 16] are introduced to show how the FDA method is applied in practice, and the 40 sets of samples are selected as the training samples of FDA model (Table 1). FDA model for the prediction of pillar stability is established after developing the model using the FDA theory discussed above. The Fisher discriminant function generated by the FDA has the following form:

$$y=0.082W+1.975H+0.006W/H-0.073\sigma_{\text{UCS}}-2.046\sigma_{\text{p}}-0.187 \tag{14}$$

Equation (14), canonical discriminant function, was used in the analysis. Table 3 and Table 4 show that the discrimination capability of Eq. (14) is significant. The corresponding feature value of discriminant function 1 is 1.856, with variance ratio (discriminant efficiency) 100%>85%, which can explain all of the sample information; the correlation coefficient is as high as 0.806; η value and the P value are very small, as 0<<0.05. So it is concluded that the discriminant function can well distinguish the various categories through significance test. In order to investigate the validity and accuracy of the FDA model evaluation of pillar stability, the proposed model with 40 groups of training data is tested and the results are shown in Table 4. On each back to the sub, and its actual situation, the test results included are listed in Table 5. It can be seen that 95.0% of original grouped cases are correctly classified by FDA method from Table 5. Cross validation is done only for those cases in the analysis [30]. In cross validation, each case is classified by the functions derived from all cases other than one case. Therefore, 90.0% of cross-validated grouped cases is correctly classified by FDA method. The results show that this model has high prediction accuracy and can be used in practical engineering.

Table 3 Eigenvalue of Fisher discriminant function

Function	Eigenvalue	Variance/ %	Cumulative value/%	Canonical correlation
1	1.856	100.0	100.0	0.806

Table 4 Wilks' Lambda of Fisher discriminant function

Test of function	Wilks' Lambda	Chi-square	f	Significant
1	0.350	37.252	5	0

Table 5 Prediction results obtained for pillar stability by FDA

Type	Predicted value(FDA)	F	S	Total
Train	F	28	0	28
	S	2	10	12
Test	F	4	0	4
	S	1	1	2
Total		35	11	46

The rows indicate the number of points that the FDA predicted corrcectly and the number of errors committed compared with the actual condiction, as also the pillar stability assessed by FDA in each case

The validity of the proposed method is shown in two aspects [19−21]: 1) high correct rate for training data and 2) high accuracy for testing data based on the predicted and measured (real) values, about 6 testing samples (listed in Table 1) by the FDA model. The results are identical with actual pillar conditions expect one sample.

4.2 2 Testing and validation of SVMs model

CHANG and LIN [24] developed a LIBSVM toolbox (software available at http://www.csie.ntu.edu.tw/~cjlin/libsvm) for SVMs modeling in Matlab application. This toolbox is used here for the application of SVMs in predicting pillar stability for underground mines. In SVMs, each of the input variables (W, H, W/H, σ_{UCS}, σ_p) is first normalized to their respective maximum values by using Eq. (15) [18−19, 24−25]. In this equation, x_{norm} is the normalized value, x is the actual value, x_{max} is the maximum value, and x_{min} is the minimum value. The output variable pillar stability is also normalized with respect to the actural value.

$$x_{norm}=(x-x_{min})/(x_{max}-x_{min}) \tag{15}$$

The engineering data in Table 1 [10−11, 16] are introduced to show how the SVMs method was applied in practice. In the present study, the above SVMs model has been used for the prediction of pillar stability. In SVMs, each of the input variables (W, H, W/H, σ_{UCS}, σ_p) is first normalized to their respective maximum values [18−19, 23−25]. The output variable pillar stability is also normalized with respect to the actual value. 40 sets of samples are selected as the training samples of SVMs model (listed in Table 1). SVMs model is established using the SVMs theory discussed above. When applying SVMs, the goodness of fit is determined by the penalty factor C and insensitive parameter g. LIBSVM provides a parameter selection tool using the RBF kernel: cross validation via parallel GSM [24−25]. For the grid search, currently we support only C-SVM with two parameters C and g. They can be easily modified for other kernels such as linear and polynomial, or for SVMs. In the current study, the free parameters of SVMs are selected, following a K-fold cross-validation (K=5) experiment to control generalization capability of SVMs, and the RBF kernel is used as the kernel function for training the samples, obtaining the best parameters of SVMs because it tends to give better performance. Figure 6 shows an example of the GSM result, where the x-axis and the y-axis are $\log_2 C$ and $\log_2 g$, respectively. The z-axis is the 5-fold average performance. The findings of this experiment are that SVMs are quite robust against parameter selections.

Then we perform the 5-fold cross-validation on the training set to choose the proper parameters of $C = \{2^{-8}, 2^{-7}, \cdots, 2^{8}\}$ and $g=\{2^{-8}, 2^{-7}, \cdots, 2^{8}\}$, respectively. 8×8=64 parameter combinations of (C, g) are tried and the one with the best cross-validation accuracy is chosen as the parameter values of the RBF kernel. Then the best parameter pair (C, g) is used to create the model for training. After obtaining the predictor model, we conduct the prediction on each testing set accordingly. The result of the SVMs parameter selection by GSM is shown in

Fig. 6. The optimum estimated values of the model parameters obtained using cross-validation are as follows: capacity constant C is 32, Gaussian kernel parameter g is 2 and the average value of MSE is CVmse, which is equal to 97.5%. 40 sets of training sample data were back evaluated one by one using the SVMs model of pillar stability and compared with the actual situation. The compared pillar stability test results of training data are shown in Fig. 7 and Table 6. SVMs have good performance for classification forecast from Fig. 7 and Table 6, which prove that the model has stable and reliable prediction ability. Therefore, the SVMs model is feasible and effective for pillar stability forecast and can be put into use.

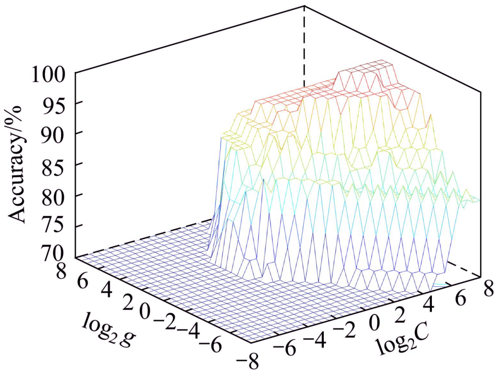

Fig. 6 MSE values for different combinations of $\log_2 C$ and $\log_2 g$

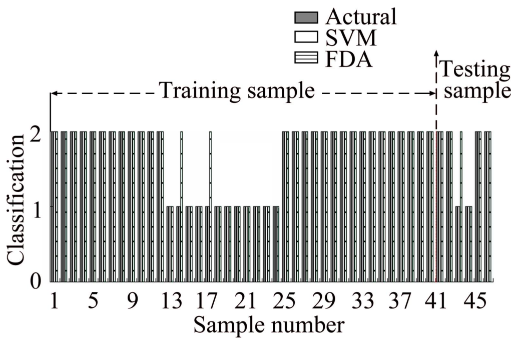

Fig. 7 Comparison results of pillar stability prediction using FDA and SVMs method ("1" denotes "Stable pillar"; "2"denotes "Failure pillar")

Table 6 Prediction results obtained for pillar stability by SVMs

Type	Predicted value	F	S	Total
Train	F	28	0	28
	S	0	12	12
Test	F	4	0	4
	S	0	2	2
Total		32	14	46

The rows indicate the number of points that the SVMs predicted corrcectly and the number of errors committed compared to the actual condition, as also the pillar stability assessed by SVMs in each case

To validate the predictive models based on the predicted and measured (real) values, 6 testing samples (listed in Table 1) were validated by the SVMs model. The results are shown in Fig. 7 and Table 6. The results are identical with actual pillar conditions and the accuracy of this SVMs classification model is well.

4.2 3 Sensitivity analysis

To reflect the discriminant criterion of pillar stability for determining the ability of size discrimination, using a useful concept of sensitivity analysis (F statistics) to identify it [21]. F statistics is the mean-square deviation's ratio between group variation and the standard deviation, reflecting the different ability of indicators' discrimination. The stronger the F is, the greater the recognition is. After being calculated, the involved identification of the W, H, W/H, σ_{UCS} and σ_{p} was normalized, and the values of F were 0.1494, 0.2102, 0.0050, 0.4654, 0.1700, as shown in Fig. 8. Thus, σ_{UCS} and H determine the strongest, followed by σ_{p}, W, W/H.

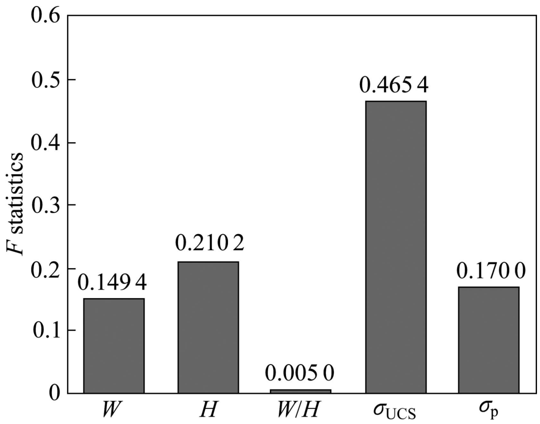

Fig. 8 Sensitivity analysis between pillar stability and each input parameter

4.3 Comparison of FDA and SVMs models

In estimating the SVMs model prediction performance, the results of SVMs models are compared with those of FDA method. Index such as the correct classification rate P_{rs} by re-substitution method [21], P_{cv} by cross validation method [30] was defined as the ratio of number of correct classification in test samples to total number of samples in test set, which can be used to evaluate the prediction accuracy of the proposed models. Comparison of the results of pillar stability evaluation obtained by the FDA and SVMs methods for the testing dataset are presented in Table 7. Original grouped cases and cross-validated grouped cases correctly classified by FDA method are 95.0%, 90.0%, respectively, whereas the corresponding values for SVMs method are 100%, 97.5%. Table 7 show that the SVMs method performs better than the FDA method. It can be concluded that the SVMs model can be applied to forecasting the pillar stability for underground mines classification with a high accuracy.

Table 7 Index values showing performance calculated by models

Model	Dataset	P_{rs}/%	P_{cv}/%
FDA	Train	97.5	90.0
	Test	83.3	
SVMs	Train	100	97.5
	Test	100	

The bar graph in Fig. 7 of the measured and predicted pillar stability by FDA and SVMs models shows that the prediction by SVMs is very close to the measured pillar stability, whereas the prediction by FDA has some error and is not able to predict the pillar stability in the best versatile way.

The above-mentioned comparisons indicate that both the two models are competitive with each other in pillar stability prediction for classification of underground mines, as shown in Fig. 7 and Table 7, but the performance of the SVMs is relatively superior to the FDA model. Moreover, the SVMs have some added advantages, which come from the specific formulation of an objective function with constraints. This function is solved using LM and has some inherent advantages and characteristics: 1) a global optimal solution will be found; 2) the result is a general solution avoiding overtraining; 3) the solution is sparse and only a limited set of training points contribute to this solution; and 4) nonlinear solutions can be calculated efficiently due to the use of inner products.

5 Conclusions

1) FDA and SVMs methodologies are proposed to forecast the pillar stability for underground mines by using the statistical learning algorithm. The factors influencing the pillar W, H, W/H, σ_{UCS} and σ_p are taken into consideration to build models on the determination of pillar stability from various coal and stone mines.

2) The FDA and SVMs model are obtained through training 40 sets of practical measuring samples and another 6 sets. The correct classification rate P_{rs} by re-substitution method and P_{cv} by cross validation method are introduced to verify the stability of FDA model and SVMs model. The P_{rs} and P_{cv} between the observed and predicted values by SVMs model are found to be 100% and 97.5%, respectively; the values by FDA are found to be 95% and 90% respectively. Compared with the FDA method, the results show that SVMs model has high prediction accuracy and can be used in practical engineering.

3) Sensitivity analysis shows the most important parameters on the induced stresses: σ_{UCS} and H determine the strongest, followed by σ_p, W, W/H. It is probably due to the fact that the variation of these parameters is not very much.

References

[1] BRADY B, BROWN E T. Rock mechanics for underground mining [M]. 2nd edn. London: Chapman and Hall, 2003.
[2] TESARIK D R, SEYMOUR J B, YANSKE T R. Long-term stability of a backfilled room-and-pillar test section at the Buick Mine, Missouri, USA [J]. International Journal of Rock Mechanics and Mining Sciences, 2009, 46(7): 1182−1196.
[3] NAJAFI M, JALALI S E, YARAHMADI BAFGHI A R, SERESHKI F. Prediction of the confidence interval for stability analysis of chain pillars in coal mines [J]. Safety Science, 2011, 49(5): 651−657.
[4] SALAMON, M D G. Stability, instability and design of coal pillar workings [J]. International Journal of Rock Mechanics and Mining Science & Geomechanics Abstracts, 1970, 7(6): 613−631.
[5] KOŠŤÁK B. Pillar strength prediction from representative sample of hard rock [J]. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 1971, 8(5): 523−526.
[6] DENG J, YUE Z Q, THAM L G, ZHU H H. Pillar design by combining finite element methods, neural networks and reliability: A case study of the Fenghuangshan Copper Mine, China [J]. International Journal of Rock Mechanics and Mining Sciences, 2003, 40(4): 585−599.
[7] CAUVIN M, VERDE T, SALMON R. Modeling uncertainties in mining pillar stability analysis [J]. Risk Analysis, 2009, 29(10): 1371−1380.
[8] WANG S Y, SLOAN S W, HUANG M L, TANG C A. Numerical study of failure mechanism of serial and parallel rock pillars [J]. Rock Mechanics Rock Engineering, 2011, 44(2): 179−198.
[9] ELMO D, STEAD D. An integrated numerical modeling—Discrete fracture network approach applied to the characterisation of rock mass strength of naturally fractured pillars [J]. Rock Mechanics Rock Engineering, 2010, 43(1): 3−19.
[10] MOHAN G M, SHEOREY P R, KUSHWATA A. Numerical estimation of pillar strength in coal mines [J]. International Journal of Rock Mechanics and Mining Sciences, 2001, 38: 1185−1192.
[11] ESTERHUIZEN G S, DOLINAR D R, ELLENBERGER J L. Pillar strength in underground stone mines in the united states [J]. International Journal of Rock Mechanics and Mining Sciences, 2011, 48(1): 42−50.
[12] GRIFFITHS D V, FENTON G A, LEMONS C B. Probabilistic analysis of underground pillar stability [J]. International Journal for Numerical and Analytical Methods in Geomechanics, 2002, 26(8): 775−791.
[13] HUTCHINSON D J, PHILLIPS C, CASCANTE G. Risk considerations for crown pillar stability assessment for mine closure planning [J]. Geotechnical and Geological Engineering, 2002, 20(1): 41−63.
[14] GRIFFITHS D V, FENTON G A, LEMONS C B. The random finite element method (RFEM) in mine pillar stability analysis [J]. Probabilistic Methods in Geotechnical Engineering, 2007, 491: 271−294.
[15] PALEI S K, DAS S K. Logistic classification model for prediction of roof fall risks in bord and pillar workings in coal mines: An approach [J]. Safety Science, 2009, 47(1): 88−96.
[16] JAISWAL A, SHRIVASTVA B K. Numerical simulation of coal pillar strength [J]. International Journal of Rock Mechanics and Mining Sciences, 2009, 46(4): 779−788.

[17] MONJEZI M, HESAMI S M, KHANDELWAL M. Superiority of neural networks for pillar stress prediction in bord and pillar method [J]. Arabian Journal of Geosciences, 2011, 4(5−6): 845−853.

[18] CHIANG L H, KOTANCHEK M E, KORDON A K. Fault diagnosis based on fisher discriminant analysis and support vector machines [J]. Computers and Chemical Engineering, 2004, 28(8): 1389−1401.

[19] CHEN Hui-ling, LIU Da-you, YANG Bo, LIU Jie, WANG Gang. A new hybrid method based on local fisher discriminant analysis and support vector machines for hepatitis disease diagnosis [J]. Expert Systems with Applications, 2011, 38(9): 11796−11803.

[20] ZHOU Jian, SHI Xiu-zhi, DONG Lei, HU Hai-yan, WANG Huai-yong. Fisher discriminant analysis model and its application for prediction of classification of rockburst in deep-buried long tunnel [J]. Journal of Coal Science & Engineering (China), 2010, 16(2): 144−149.

[21] SHI X Z, ZHOU J. Reliability assessment for mine ventilation system safety using fisher discriminant analysis [J]. Journal of Mining and Safety Engineering, 2010, 27(4): 562−567.

[22] VAPNIK V. The nature of statistical learning theory [M]. New York: Springer-Verlag, 1995.

[23] GUNN, S R. Support vector machines for classification and classification [R]. Technical Report. University of Southampton, 1998.

[24] CHANG C C, LIN C J. LIBSVM: A library for support vector machines [EB/0L]. 2010-05. http://www.csie.ntu.edu.tw/~cjlin/libsvm

[25] ZHOU Jian, LI Xi-bing, SHI Xiu-zhi. Long-term prediction model of rockburst in underground openings using heuristic algorithms and support vector machines [J]. Safety Scinece, doi: 10.1016/j.ssci.2011.08.065.

[26] VOZNESENSKII A S, ZAKHARCHENKO G V. Substantiation of roof and pillar stability monitoring in mines with the help of information criterion [J]. Journal of Mining Science, 2001, 37(2): 140−148.

[27] KUSHWAHA A, BANERJEE G. Exploitation of developed coal mine pillars by shortwall mining—A case example [J]. International Journal of Rock Mechanics and Mining Sciences, 2005, 42(1): 127−136.

[28] LOUI J P, SHEOREY P R. Estimation of non-effective width for different panel shapes in room and pillar extraction [J]. International Journal of Rock Mechanics and Mining Sciences, 2002, 39(1): 95−99.

[29] MARTIN C D, MAYBEE W G. The strength of hard-rock pillars [J]. International Journal of Rock Mechanics and Mining Sciences, 2000, 37: 1239−1246.

[30] LI Qiang, DOI K. Comparison of typical evaluation methods for computer-aided diagnostic schemes: Monte Carlo simulation study [J]. Medical Physics, 2007, 34(3): 871−876.

用 Fisher 判别法和支持向量机预测地下矿山矿柱稳定性

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摘　要：利用 Fisher 判别分析(FDA)和支持向量机(SVMs)等来识别地下矿山矿柱稳定性，从多种煤矿和石材矿山中提取一些指标和力学参数作为识别因子，包括矿柱宽度、高度、矿柱的高宽比、岩石单轴抗压强度和矿柱应力。包括取样、训练、建模和评估 4 个主要步骤。在建模阶段，基于统计学习理论，建立两类矿柱稳定性预测的 FDA 和 SVMs 模型，以 40 组世界不同矿山的实测数据进行模型的训练和测试，并将其模型应用于其他 6 组待测样本来验证建立模型的有效性，将 SVMs 模型预测结果与 FDA 模型及实际情况进行对比，采用指标回代估计法和交叉验证法来考察模型的识别能力。研究表明，SVMs 和 FDA 模型都能较好地预测矿柱的稳定性，但 SVMs 的优势更明显，有望成为一种可靠、实用的地下矿山矿柱稳定性的评价工具。

关键词：地下矿山；矿柱稳定性；Fisher 判别分析(FDA)；支持向量机(SVMs)；预测

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