

# Experimental verification of mathematical model for multiphase flow in air-agitated seed precipitation tank

CHEN Qiao-ping<sup>1,2</sup>, YAN Hong-jie<sup>1</sup>, GE Shi-heng<sup>1</sup>, ZHOU Jie-min<sup>1</sup>

1. School of Energy Science and Engineering, Central South University, Changsha 410083, China;

2. Henan Branch, Aluminum Corporation of China Limited, Zhengzhou 450041, China

Received 17 May 2010; accepted 15 August 2010

**Abstract:** In order to check the validity of the mathematical model for analyzing the flow field in the air-agitated seed precipitation tank, a scaled down experimental apparatus was designed and the colored tracer and KCl tracer were added in the apparatus to follow the real flow line. Virtue tracers were considered in the mathematical model and the algorithm of tracers was built. The comparison of the results between the experiment and numerical calculation shows that the time of the tracer flows out of stirring tube are 40 s in the experiment and 42 s in numerical calculated result. The transient diffusion process and the solution residence time of the numerical calculation are in good agreement with the experimental results, which indicates that the mathematical model is reliable and can be used to predict the flow field of the air-agitated seed precipitation tank.

**Key words:** model experiment; numerical simulation; multiphase flow; seed precipitation

## 1 Introduction

Gas-liquid reactors are widely used in process industries to carry out a variety of operations, like homogenization, gas dispersion, heat transfer and chemical reaction. To obtain high quality products and high efficiency processes, mixing must satisfy not only the needs of mass and heat transport but also the required homogeneity in the vessel in the shortest time. Over the years many experimental and numerical simulation studies have been made to investigate the characteristics of gas-liquid reactors [1–3].

Seed precipitation is one of the key processes in the industrial production of alumina using Bayer process, which has a significant effect on the output and quality of alumina and the technical economic index. Air-agitated tanks, which are the main facilities used for seed precipitation, are widely used in Chinese alumina industries. Many researches had been done on numerical simulation of flow field in seed precipitation tank in last years [4–5].

Generally, multi-fluid model was adopted to calculate the flow field in the air-agitated tank. In the multi-fluid model, there is separate solution field [6–7].

The interaction between different phases is described via inter-phase transfer terms. The multi-fluid model was solved using the inter-phase slip algorithm (IPSA) of SPALDING and LAUNDER [8–9].

The continuity equation of two-phase flow is

$$\nabla \cdot (r_\alpha \rho_\alpha U_\alpha) = 0 \quad (1)$$

where subscript  $\alpha$  refers to phase  $\alpha$ ;  $\gamma$  is the volume fraction of each phase (%);  $\rho$  is the fluid density ( $\text{kg/m}^3$ ); and  $U$  is the velocity ( $\text{m/s}$ ).

The momentum equation is

$$\nabla \cdot (r_\alpha \rho_\alpha U_\alpha U_\alpha) - \mu_\alpha (\nabla U_\alpha + (\nabla U_\alpha)^T) = r_\alpha (B - \nabla p_\alpha) + c_{\alpha\beta}^d (U_\beta - U_\alpha) \quad (2)$$

where subscript  $\beta$  refers to phase  $\beta$ ;  $B$  is the body force (N);  $p$  is the pressure (N); and  $U_\alpha U_\alpha$  is a dyad, which can be expressed as:

$$\nabla \cdot (U_\alpha U_\alpha) = (U_\alpha \cdot \nabla) U_\alpha + U_\alpha \nabla \cdot (U_\alpha) \quad (3)$$

$c_{\alpha\beta}^d$  is the drag force for single bubble, which can be expressed as

$$c_{\alpha\beta}^d = \frac{3}{4} \frac{C_d}{d} r_\beta \rho_\alpha |U_\beta - U_\alpha| \quad (4)$$

where  $C_d$  is a drag coefficient, which can be decided by

experimental drag curve [10–12]; and  $d$  is the diameter of the bubble (m).

Algebraic equation of the phase volume fraction is

$$r_{\alpha} + r_{\beta} = 1 \quad (5)$$

The pressure is the same on both phases, i.e. the interfacial pressure drop is neglected.

$$P_{\alpha} = P_{\beta} = P \quad (6)$$

The virtual mass force was neglected in the present simulations due to its small magnitude compared with the drag force [13–15].

In order to predict the physical phenomena accurately, the numerical solution of computational model should be compared with the exact solution or the experimental results. The multi-fluid model was verified in some specific cases [11–12]. However, the verification on mathematical model for air-agitated tank has not been reported yet. In this work, an experiment is carried out to check the validity of the mathematical model.

## 2 Apparatus design

The shear stress of the sodium aluminate solution was detected under different velocity gradients, and are results are shown in Fig. 1. It shows that the shear stress variation follows Newton's law of viscosity and the sodium aluminate solution is Newtonian fluid. It is accepted to simulate the fluid flow in the seed precipitate tank using the multi-fluid model.

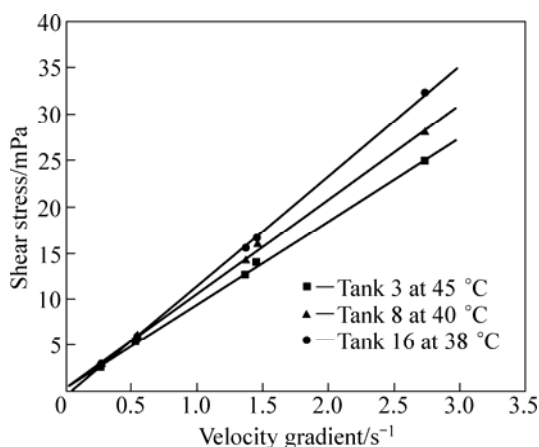


Fig. 1 Shear stress variation of solution under different velocity gradients

An experimental apparatus with a small size (see Fig. 2) was designed based on analog principle because of the complexity and hugeness of seed precipitation tank, and the scale was 1:27. Water was added in the model tank through inlet and compressed air was blown through windpipe, then air and water were mixed at the

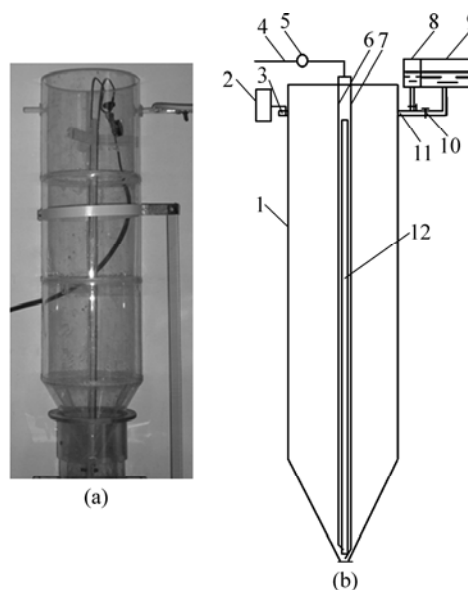


Fig. 2 Experimental apparatus (a) and its section plane (b) of air-agitated seed precipitation tank: 1—Body of tank; 2—Conductivity meter; 3—Outlet; 4—Air inlet; 5—Flow meter; 6—Main windpipe; 7—Secondary windpipe; 8—KCl solution tank; 9—Water tank; 10—Valve; 11—Water inlet; 12—Stirring tube

bottom of the stirring tube, and the water was lifted because of the density gradient between inside and outside of the tube. Reynolds number and Froude number were considered to ensure the dynamic similarity in stirring tube since the distribution of flow field plays a crucial role in the process of seed precipitation [16–18].

The colored tracer was used to follow the flow lines of the liquid in the tank and to observe the diffusive contour. When the flow field was steady, the colored tracer was added through the water inlet and its diffusion process was recorded with a camera. The results of experiment and numerical calculation were compared to verify the reliability of the mathematical model.

The solution residence time in tank could be calculated through the concentration of KCl tracer at the outlet and the concentration of KCl tracer was obtained by measuring the conductivity of liquid at the outlet. KCl tracer was widely used in model test of metallurgy facilities and was proved to be more accurate than NaCl tracer by ZHOU and DONG [19]. KCl saturated solution was added to the seed tank from KCl solution tank and the change of electric conductivity of water could be measured at outlet by using conductivity measuring apparatus. In order to reduce the influence of tracers, the solution level in the KCl solution tank should be coincident to the water level of the water tank. The conversion between KCl solution concentration and conductivity could be found in Fig. 3, which was obtained from experimental data.

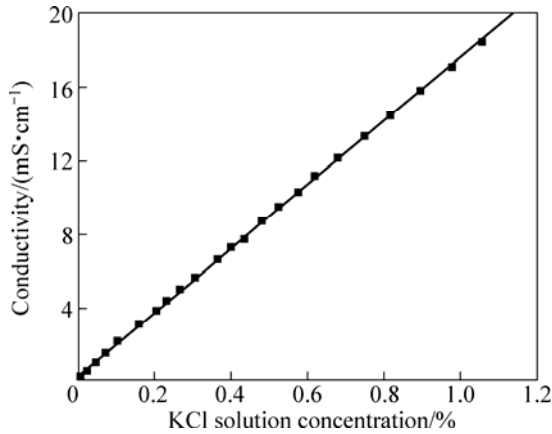


Fig. 3 Relationship between conductivity and KCl solution concentration

### 3 Numerical calculation of solution residence time

Virtual tracers were considered in flow field simulation on model tank to compare with experiment results. The diffusing process of the virtual tracer is a transient problem and can be given by

$$\frac{\partial}{\partial t}(r_a \rho_a \varphi_a) + \nabla \cdot (r_a \rho_a U_a \varphi_a) = \nabla \cdot (r_a \Gamma_a (\nabla \varphi_a)) \quad (7)$$

where  $\varphi$  is the scalar quantity and  $\Gamma$  is the diffusion coefficient.

The impulse concentration of the tracer at the inlet is

$$C_\varphi = \begin{cases} 1, & t \in (0, 1] \\ 0, & t \in (1, \infty) \end{cases} \quad (8)$$

where  $C_\varphi$  is the concentration of the tracer.

The algorithm pattern of solution residence time in seed precipitation tank is shown in Fig. 4. The solution residence time is described using the distribution density function and the mean residence time because the time of tracers staying in the tank is different. Tracers are added instantaneously at the inlet of the tank and then will flow out at different time. The proportion of tracers effusing at the time interval  $[t, dt]$  is  $P(t)dt$ , where  $P(t)$  is the distribution density function of solution residence time. Then  $P(t)$  can be described as:

$$P(t) = \frac{C(t)}{\int_0^\infty C(t)dt} \quad (9)$$

where  $C(t)$  is the rate of outflow of tracers.

The mean residence time of tracers,  $\bar{t}$ , is

$$\bar{t} = \frac{\int_0^\infty tP(t)dt}{\int_0^\infty P(t)dt} = \int_0^\infty tP(t)dt \quad (10)$$

If the record for tracers is discontinuous, then  $\bar{t}$  is

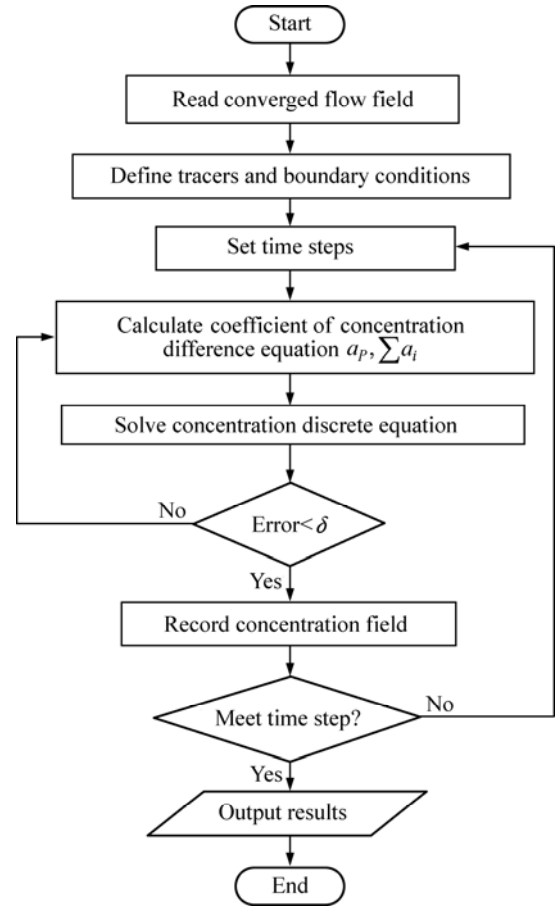


Fig. 4 Algorithm pattern of solution residence time in seed precipitation tank

$$\bar{t} = \frac{\sum_{i=1}^N t_i P(t_i) \Delta t_i}{\sum_{i=1}^N P(t_i) \Delta t_i} = \frac{\sum_{i=1}^N t_i C(t_i) \Delta t_i}{\sum_{i=1}^N C(t_i) \Delta t_i} \quad (11)$$

where  $\Delta t$  is the interval of record time.

In order to compare the calculated results,  $P(t)$  and  $t$  need to be dimensionless.

$$\theta = \frac{t}{t_{\text{avg}}} \quad (12)$$

$$P(\theta) = t_{\text{avg}} P(t) \quad (13)$$

where  $\theta$  is the dimensionless time;  $t_{\text{avg}}$  is the theoretical mean residence time of the solution in the tank:

$$t_{\text{avg}} = \frac{V}{Q} \quad (14)$$

where  $V$  is the volume of the tank and  $Q$  is the flux of the solution.

### 4 Results and discussion

The diffusing processes of the colored tracer from experiment and calculation at different time are shown in

Fig. 5. The tracer diffuses down along the right of the tank and mixes adequately with the liquid in the stirring tube. The colored tracer flows out from the stirring tube in 40 s during the experiment, and the numerical calculation result is 42 s. It shows that the experiment result agrees well with the simulation situation, which proves that the mathematical model is reliable.

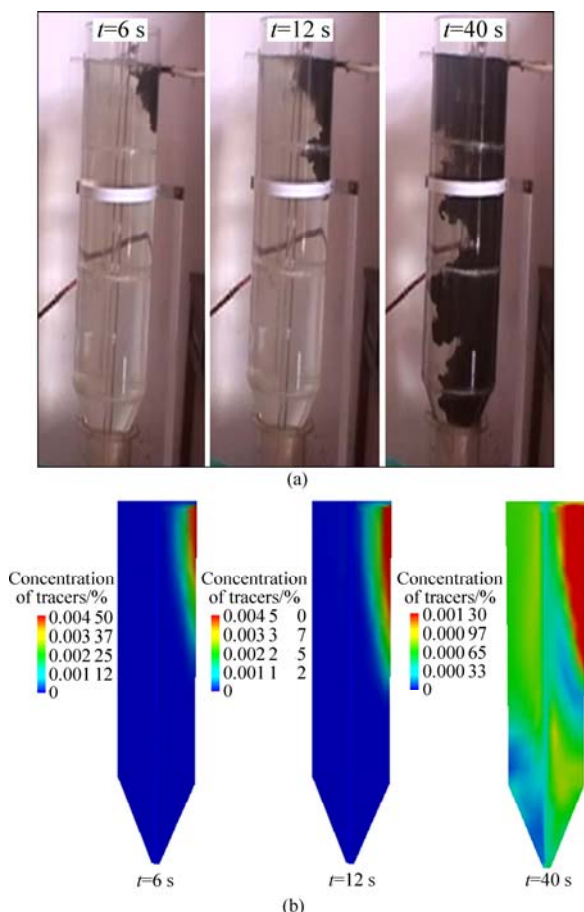


Fig. 5 Diffusion results of experiment (a) and calculation (b) for tracers

The colored tracer diffuses horizontally when moving down, which causes a part of tracer flow out of the tank without stirring. Therefore, the phenomena of liquid short-circuiting can be observed both in the experiment and calculation results.

Model experiment of solution residence time was carried out through detecting the conductivity of solution, which can be used to calculate the concentration of KCL tracers at outlet of the model tank. Figure 6 illustrates the distribution density function of solution residence time ( $P(\theta)-\theta$ ). There are two peaks on both of the curves. The first one indicates that KCL tracers flow out directly without stirring and the second one is the concentration of tracers which flow out through stirring tube. Figure 6 shows that the experiment value ( $P(\theta)$ ) is higher than the calculation value. The main reason lies in two factors: 1) The liquid (water) in model tank is an electric

conductor, which results in the conductivity of detected solution being higher than the real value, then the calculated concentration of KCL tracers is higher than the real value. 2) Numerical calculation brings little error because of hypothesis in the mathematical model. For example, the level fluctuation of liquid surface was not considered. However, the trend of curves can prove that the mathematical model is acceptable.

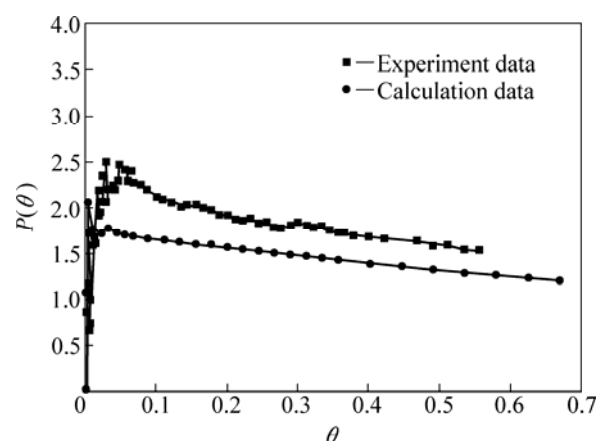


Fig. 6 Comparison of distribution density function of residence time

## 5 Conclusions

1) A model experiment is proceeded to certify the validity of the mathematical model of the flow field in air-agitated seed precipitation tank.

2) The numerical simulation result agrees well with the experimental result, which proves that the mathematical model is reliable and can be used to the numerical research on seed precipitation tank in industrial production of alumina.

## References

- [1] ZHANG Qing-hua, YONG Yu-mei, MAO Zai-sha, YANG Chao, ZHAO Cheng-jun. Experimental determination and numerical simulation of mixing time in a gas-liquid stirred tank [J]. Chemical Engineering Science, 2009, 64: 2926–2933.
- [2] KERDOUSS F, BANNARI A, PROULX P. CFD modeling of gas dispersion and bubble size in a double turbine stirred tank [J]. Chemical Engineering Science, 2006, 61: 3313–3322.
- [3] BUSCIGLIO A, GRISAFI F, SCARGIALI F, BRUCATO A. On the measurement of local gas hold-up and interfacial area in gas-liquid contactors via light sheet and image analysis [J]. Chemical Engineering Science, 2010, 65: 3699–3708.
- [4] YAN Hong-jie, ZHOU Jie-min, LÜ Zi-jian, ZHOU Huai-min. Numerical simulation of flow field and optimization of structure parameters of seed precipitation tank for production of alumina [J]. Journal of Central South University of Technology, 2005, 12(2): 229–232.
- [5] LÜ Zi-jian, YAN Hong-jie, ZHOU Ping, ZHOU Jie-min, WU Hong-ying, ZHOU Huai-min, WANG Shao-dong. Fluid field numerical simulation of air-agitated seed precipitation tank [J].

- Nonferrous Metals, 2003, 55(1): 63–65. (in Chinese)
- [6] HOSOKAWA S, TOMIYAMA A. Multi-fluid simulation of turbulent bubbly pipe flows [J]. Chemical Engineering Science, 2009, 64(24): 5308–5318.
- [7] SPICKA P, DIAS M M, LOPES J C B. Gas-liquid flow in a 2D column: Comparison between experimental data and CFD modeling [J]. Chemical Engineering Science, 2001, 56(21): 6367–6383.
- [8] SPALDING D B. The calculation of free-convection phenomena in gas-liquid mixtures [C]//ICHMT Seminar. Dubrovnik, 1976.
- [9] LAUNDER B E, SPALDING D B. The numerical computation of turbulent flows [J]. Comput Meth in Appl Mech Eng, 1974, 3(3): 269–289.
- [10] ZHANG L, CAI K, SHEN Y. Effect of interphase lift force on fluid flow in air stirred cylindrical vessel [J]. Ironmaking and Steelmaking, 2000, 27(2): 138–143.
- [11] JIN H, GLIMM J. Verification and validation for turbulent mixing [J]. Nonlinear Analysis, 2008, 69: 874–879.
- [12] GRACE J R, TAGHIPPOUR F. Verification and validation of CFD models and dynamic similarity for fluidized beds [J]. Powder Technology, 2004, 139: 99–110.
- [13] KENDOUSH A A, ABBAS H, SULAYMON, SAWSAN A M. Experimental evaluation of the virtual mass of two solid spheres accelerating in fluids [J]. Experimental Thermal and Fluid Science, 2007, 31(7): 813–823.
- [14] KENDOUSH A A. The virtual mass of an oblate-ellipsoidal bubble [J]. Physics Letters A, 2007, 366(3): 253–255.
- [15] TABIB M V, ROY S A, JOSHI J B. CFD simulation of bubble column—An analysis of interphase forces and turbulence models [J]. Chemical Engineering Journal, 2008, 139(3): 589–614.
- [16] CAMARASA E, MELEIRO L A C, CARVALHO E, DOMINGUES A, MACIEL FILHO R, WILD G, PONCIN S, MIDOUX N, BOUILLARD J. A complete model for oxidation air-lift reactors [J]. Computers and Chemical Engineering, 2001, 25: 577–584.
- [17] VIAL C, PONCIN S, WILD G, MIDOUX N. A simple method for regime identification and flow characterisation in bubble columns and airlift reactors [J]. Chemical Engineering and Processing, 2001, 40: 135–151.
- [18] BENDJABALLAH N, DHAOUADI H, PONCIN S, MIDOUX N, HORNUIT J M, WILD G. Hydrodynamics and flow regimes in external loop airlift reactors [J]. Chemical Engineering Science, 1999, 54: 5211–5221.
- [19] ZHOU Zhi-qing. DONG Lu-ren. Influence of tracer on measured residence time in water modeling [J]. Research on Iron and Steel, 1999(2): 14–16. (in Chinese)

## 空气搅拌式种分槽内多相流数学模型的实验验证

陈乔平<sup>1,2</sup>, 闫红杰<sup>1</sup>, 葛世恒<sup>1</sup>, 周子民<sup>1</sup>

1. 中南大学 能源科学与工程学院, 长沙 410083;
2. 中国铝业公司 河南分公司, 郑州 450041

**摘 要:** 为了验证空气搅拌式种分槽流场数学模型的可靠性, 设计了空气搅拌式种分槽的模型实验台架, 实验台架按照原始比例进行缩小。采用有色示踪剂与 KCl 示踪剂来研究种分槽内的流体流动过程。在种分槽流场的数学模型中增加虚拟示踪剂模块, 建立虚拟示踪剂跟踪计算的算法。对实验结果与数值计算结果进行了比较。结果表明: 示踪剂进入种分槽后从翻料管流出的时间实验结果为 40 s, 数值计算结果为 42 s; 数值计算所得有色示踪剂在种分槽内的扩散过程与停留时间分布与实验结果吻合良好。实验结果表明: 空气搅拌式种分槽的数学模型是可靠的, 可以用于预测分析种分槽内流体流动状况。

**关键词:** 模型试验; 数值计算; 多相流; 种分

(Edited by YANG Hua)