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Elasto-plastic constitutive model of aluminum alloy foam subjected to impact loading

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Abstract: A multi-parameter nonlinear elasto-plastic constitutive model which can fully capture the three typical features of stress—strain response, linearity, plasticity-like stress plateau and densification phases was developed. The functional expression of each parameter was determined using uniaxial compression tests for aluminum alloy foams. The parameters of the model can be systematically varied to describe the effect of relative density which may be responsible for the changes in yield stress and hardening-like or softening-like behavior at various strain rates. A comparison between model predictions and experimental results of the aluminum alloy foams was provided to validate the model. It was proved to be useful in the selection of the optimal-density and energy absorption foam for a specific application at impact events.

Key words: elasto-plastic; constitutive model; metallic foam; strain rate effect; energy absorption

1 Introduction

Cellular materials, such as metallic foams, can dissipate considerable energy by large plastic deformation under quasi-static or dynamic loading[1−4]. Their cellular microstructures offer the ability to undergo large plastic deformation at nearly constant stress, and thus the materials can absorb a large amount of kinetic energy before collapsing to a more stable configuration or fracture. With this excellent feature and other advantages[1−4] including low density, high specific strength, high specific stiffness and high energy absorption ability, metallic foams are very suitable as inside structural material for protective structures and shock absorbers. Now they have been increasingly used in a wide range of protective applications[4−7]. A full mathematical description of the mechanical properties of metallic foams at various strain rates is significant for the corresponding impact events. However, the current metallic foam models[8−13] are mostly presented on the base of empirically obtained stress—strain curves under quasi-static test, which could only represent a narrow range of behaviors. The mechanical behavior of metallic foams can be modified in a wide range to meet specific requirements by choosing the cell wall materials, type, size and statistical distribution of the individual cells. However, this advantage combined effect of material properties and structure on the stress—strain behavior of foams complicates the description and prediction of their properties in terms of these parameters. Only a few models, such as the Gibson model[1], are based on the deformation mechanism and therefore could account for the effects of the characteristic parameters, such as density and cell size. These models are too complex to be applied in industry since the Gibson density dependency laws or the proposition of other laws[2−4] must be characterized by analyzing the foam structure.

A number of researchers[8−13] implemented simplified structural analysis by some nonlinear empirical constitutive models in finite element method (FEM) codes. Basically, these models can be expressed by the following equation,

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$$\sigma = H(T)G(\rho)M(\dot{\varepsilon})f(\varepsilon) \quad (1)$$

where H, G and M are the functions of temperature T, initial density ρ and strain rate $\dot{\varepsilon}$, respectively; $f(\varepsilon)$ is called the shape function and is used to describe the fundamental stress—strain response. Obviously, all the functions above play the role of scaling of the shape function.

It is important to obtain mechanical properties of metal foams from a single constitutive model which is capable of describing the stress—strain behavior at a wide range of specific foams densities and strain rates. Such data are essential in realistic numerical simulations for the safety design of structures.

In this investigation, a more comprehensive formulation of elasto-plastic constitutive model which can describe the entire dynamic compression behavior of aluminum alloy foams including three typical deformation stages was developed based on experimental results. Relative density and strain rate are two most important parameters determining the mechanical properties of aluminum alloy foams, and the dependency of the model parameters on them was analyzed. The dynamic compressive behavior of aluminum alloy foams can be characterized by using a single constitutive equation at various strain rates. It was noted that this model reduces the complexity related to cell morphologies. Moreover, more precise parameters of the foams in its optimal design, especially in evaluating the optimal density in the specific strain rate, can be obtained through this constitutive model.

2 Experimental

The open-cell aluminum alloy foams were made by infiltration casting process. The composition of the cell wall material is Al-3%Mg-8%Si-1.2%Fe (mass fraction). Specimens were cut into cylinders with sizes of d35 mm×30 mm for quasi-static tests and d35 mm×10 mm for dynamic tests, respectively, using an electrical discharge machine from blocks of the foam material. With this choice of dimensions, the specimens have at least 6−8 cells in all directions. Prior to tests, each specimen was weighted and measured in order to calculate its effective density.

Aluminum alloy foams with approximate relative density (defined as the density of the foam ρ^* divided by the density of the cell wall material ρ_s) ranging from 0.25 to 0.40 were investigated. The average cell sizes of these foams are approximately 0.9 and 1.6 mm, respectively. For comparison, a universal material testing system (MTS810.25) was used to perform the quasi-static compressive tests at a strain rate of 1×10^{-3} s^{-1}. For foam with each relative density, at least three repetitions of the compression test were performed. Figure 1 shows the typical SEM photographs of foam microstructures.

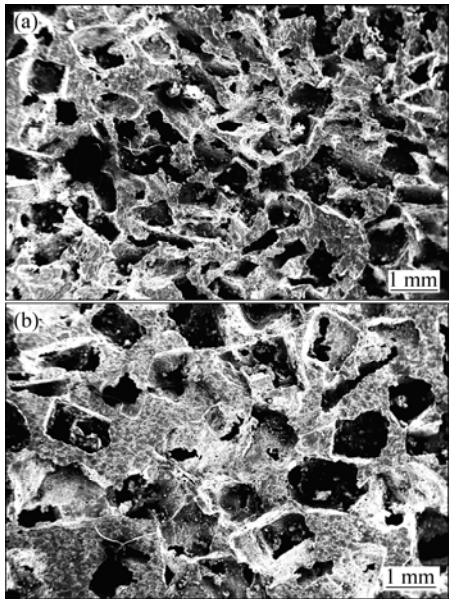

Fig.1 SEM images of aluminum foams with different open-cell sizes: (a) 0.9 mm; (b) 1.6 mm

3 Dynamic measuring techniques and optimized data processing

High strain-rate tests were conducted on the cylindrical samples described above using a split Hopkinson pressure bar apparatus[14−15], as shown in Fig.2. The striker, incident and transmitter bars were made of aluminum with yield stress equal to 200 MPa. They have an identical diameter of 37 mm and different lengths of 800, 2 000 and 2 000 mm, respectively. The end surfaces of the bars were lubricated to reduce the frictional restraint. The axial impact of striker bar and incident bar generates a compressive pulse, which is partially reflected when reaching the interface between the incident bar and specimen. The other portion of the wave is transmitted through the transmitter bar. The incident pulse and reflect wave in the incident bar were

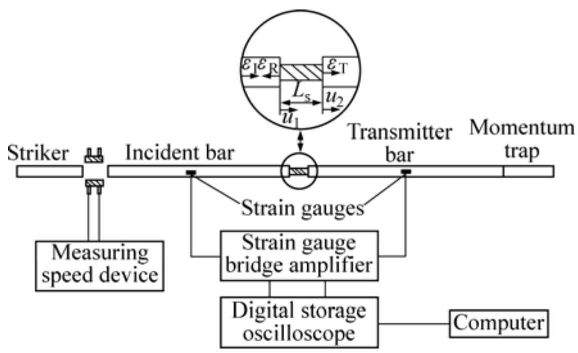

Fig.2 Schematic diagram of split Hopkinson pressure bar apparatus

recorded by the resistance strain gauge attached to the middle of the incident bar surface. The transmitted wave was recorded by the semiconductor strain gauge attached to the middle of the transmitter bar surface. Then the dynamic compressive stress—time and strain—time curves can be further obtained from the transmitted and reflected waves recorded.

From the experimental point of view, two major difficulties might be met in the impact tests of aluminum alloy foam using a split Hopkinson pressure bar (SHPB) technique: 1) weak signal due to the weak strength of cellular materials, which leads to a low signal to noise ratio; and 2) large scatter of signal due to the small size ratio of the specimen and its cells. To resolve the first issue, the transmitted wave was recorded by the semiconductor strain gauge. As for the second problem, a pressure bar with large diameter is necessary for a larger specimen. In this way, however, the wave dispersion effect increases greatly with increasing diameter of the bars. As signals are usually recorded in the middle of the surfaces of the bars to limit the signal overlap, their wave shapes at specimen-bar interfaces must be estimated using the shifting method which reflects the true responses of the foam specimens under impact[16]. Here the wave dispersion effect in shifting was corrected by the inverse analysis method in order to obtain an accurate material property of aluminum alloy foams. The inverse analysis procedure is briefly summarized as follows, with complete details given in Ref.[16].

Suppose a bar subjected to an impact load $f(t)$ at one end and consider the strain response $e(t)$ at the middle of the bar. If a linear system is assumed where input and output are $f(t)$ and $e(t)$, respectively, then the input—output relationship of the system is expressed by convolution integral as:

$$\begin{cases} e(t) = f(t) \times h(t) = \int_0^t h(t-\tau) f(\tau) \mathrm{d}\tau \\ f(t) = h(t) = e(t) = 0, \ t < 0 \end{cases} \tag{2}$$

where $h(t)$ is the impulse response function of the linear system. If the impulse response function for the chosen point on the bar is known and if the response $e(t)$ is measured there, the impact force $f(t)$ can be estimated by solving the integral Eq.(2). Thus, the problem of estimating the time history can be reduced to the process of deconvolution. A basic scheme for deconvolution is to transform the convolution in the time domain into a multiplication in the frequency domain using Fourier transforms:

$$E(\omega) = H(\omega)F(\omega) \tag{3}$$

where the characters in uppercase denote the Fourier transforms of the corresponding ones in lowercase. If the transfer function $H(\omega)$ is known in advance, the impact force can be estimated by evaluating the Fast Fourier Transforms (FFT) of the measured response, finding $F(\omega)$ from Eq.(3) and evaluating its inverse. It is well known that the use of FFT makes the computational task for deconvolution in the time domain. In fact, an FFT with an exponential window is equivalent to the Laplace transform and hence numerical inversion to obtain $f(t)$ from $F(\omega)$ by FFT can be regarded as a numerical inverse Laplace transformation. The typical incident, reflected and transmitted waves corrected are shown in Fig.3.

Fig.4 shows the experimental stress—strain curves

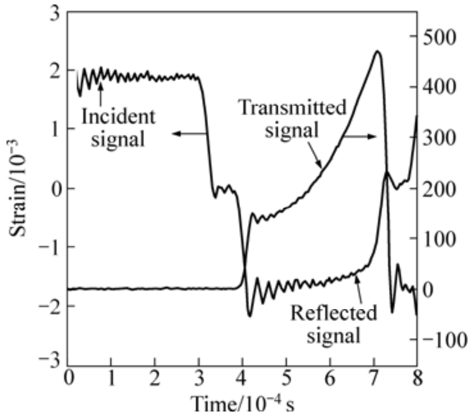

Fig.3 Typical waves signals corrected from incident and transmitter bars

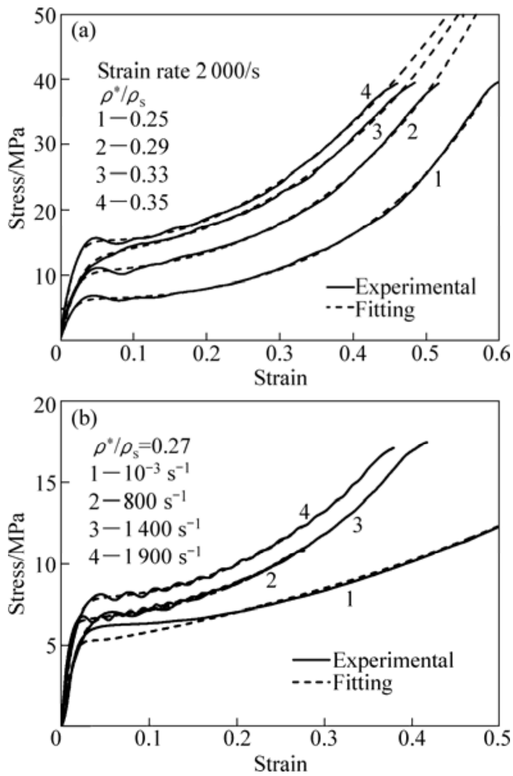

Fig.4 Experimental and fitting stress—strain curves of aluminum alloy foams: (a) With different relative densities; (b) With different strain rates

of the foams with different relative densities at various strain rates. It can be found that the compressive stress—strain curves of aluminum alloy foams exhibit universal three deformation characteristics: an initial linear-elastic region; an extended plateau region where the stress increases slowly as the cells deform plastically and a final densification region where collapsed cells are compacted together. Experimental results also show that the foams with high relative density have higher stiffness and yield stress, a small strain due to the crushing of cells and an earlier densification region than those of the low relative density foams.

4 Constitutive model

In this work, a multi-parameter constitutive model was proposed to depict the entire stress—strain response of aluminum alloy foams at different loading rates. The uniaxial compressive stress σ can be calculated:

$$\sigma = \left(A\bar{\rho}\frac{e^{\alpha\bar{\rho}\varepsilon}-1}{B+e^{\beta\bar{\rho}\varepsilon}} + C\bar{\rho}\left(\frac{\varepsilon}{1-\varepsilon}\right)^n \right)\left(1+D\lg\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right) \tag{4}$$

where ε is the compressive strain; $\bar{\rho}=\rho^*/\rho_s$ is the relative density of the foams; $\dot{\varepsilon}$ and $\dot{\varepsilon}_0$ are the average strain rate and reference strain rate (here, the reference strain rate was set as $1\times10^{-3}\ s^{-1}$), respectively. The first term of Eq.(4) is used to represent the universal three deformation characteristics of aluminum alloy foams[13], an initial linear-elastic region, an extended plateau region and a final densification region, while the second term is used to depict the strain rate effect. The parameters A, B, α, β, C and D can characterize the primary features of the stress—strain response under large deformation, as shown in Fig.5. For example, the parameter A can capture the elevated yield stress in compression with increasing relative density. The parameter B which captures the tensile yield strength is set equal to 1 for simplicity. Parameters α and β together capture the features of inelastic response with $\alpha>\beta$ depicting hardening-like response, $\alpha=\beta$ for the ideal plasticity-like response, and $\alpha<\beta$ for softening-like response. The parameters C and n can readily capture the initiation and the intensity of the densification region, respectively. The last parameter D can depict the dependence of strain rate. For simplicity, it was assumed that the parameters A, B, α, β and C are density dependent, while n and D are density independent. It is obvious to note that the formula have a vertical asymptote corresponding to the physical limit of compression (ε=1).

Figure 5 illustrates all these features for various values of parameters among which the parameter A is set as a constant of 15. From Fig.5, it can be found that the

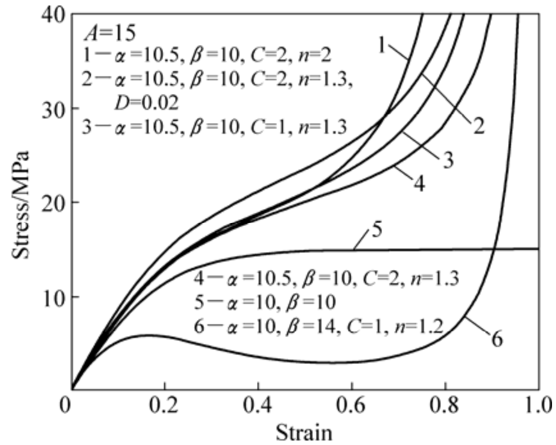

Fig.5 Illustration of model for various parameters

model fully capture the entirely dynamic compressive behavior for aluminum alloy foams at various strain rates. Note that the first term of the model provides an equivalent elastic modulus for each density of specific foam. Its derivative with respect to the strain is obtained:

$$\frac{\partial}{\partial\varepsilon}\left(A\bar{\rho}\frac{e^{\alpha\bar{\rho}\varepsilon}-1}{B+e^{\beta\bar{\rho}\varepsilon}} + C\bar{\rho}\left(\frac{\varepsilon}{1-\varepsilon}\right)^n \right) \tag{5}$$

when B is set to unity, the equivalent modulus can be written as:

$$E=\lim_{\varepsilon\to0}\frac{\partial\sigma}{\partial\varepsilon}=\lim_{\varepsilon\to0}\left(\frac{\partial}{\partial\varepsilon}\left(A\bar{\rho}\frac{e^{\alpha\bar{\rho}\varepsilon}-1}{B+e^{\beta\bar{\rho}\varepsilon}} + C\bar{\rho}\left(\frac{\varepsilon}{1-\varepsilon}\right)^n \right)\right)=\frac{A\alpha}{2} \tag{6}$$

It is noted that the tangent modulus near the origin of the stress—strain curve which can be considered the initial elastic modulus of the aluminum alloy foam is equal to the value of $A\alpha/2$. Clearly, the modulus varies with parameter A and α.

5 Parameters identification

It should be emphasized that the work of determining the parameters that describe the entire experimental compression stress—strain curves is not trivial. In this study, the nonlinear optimization procedure was developed to fit the experimental data. This function employs a Gauss-Newton iterative algorithm to minimize the mean squared error between the experimental data and the function prediction. On the base of each experimental curve, only the density dependent parameters are identified, while the density independent parameters are identified on the whole set of curves together. Utilizing the portion modeling the elastic-plateau region of Eq.(4), only a number of the data points before rapid densification phase are used to acquire a convergent parameter set (A, α and β). And then, the first term of Eq.(4), in which the values of the parameter A, α and β are known and C and n are unknown, is used to fit all the experimental data to obtain the initial parameter values of C and n. Finally, all the above predetermined parameters are used to acquire a final set of convergent parameters for the function defined in Eq.(4).

According to the parameter values obtained above, the functional forms of the model parameters were determined in terms of initial relative density, as shown in Fig.6. An attempt is then made to characterize the crushability behavior of the aluminum alloy foams at various strain rates. Based on the observed trends in Fig.6, the exponential form functions are chosen to depict the relationship between the parameters A, α, β, C and relative density $\bar{\rho}$.

$$A = 44.5\,\bar{\rho}^{1.42} \tag{7}$$

$$\alpha = 138.4/(1-0.076\bar{\rho}^{-1}) \tag{8}$$

$$\beta = 137.7\big/(1-0.0675\bar{\rho}^{-1.086}) \tag{9}$$

$$C = 0.00711\bar{\rho}^{1.816} \tag{10}$$

It is noted that n and D are two density independent empirical constants and are equal to 1.515 and 0.064 74, respectively.

A comparison between the model predictions and the experimental results of the aluminum alloy foams with different relative densities and strain rates under the dynamic compression is shown in Fig.4. It can be seen that a good agreement is obtained between the experimental results and the analytical prediction, and the quality of the fit is remarkable in the elastic-plateau region and densification region.

6 Conclusions

1) A multi-parameter rate-dependent phenomenological constitutive model of aluminum alloy foams was developed, which can describe the entire dynamic deformation behavior of aluminum alloy foams. The experimental results obtained from aluminum alloy foams with different relative density at various loading rates demonstrate the effectiveness of the model.

2) The functional forms of the model parameters in a term of relative density that may be responsible for the changes in yield stress, the hardening-like and densification behavior are also determined so that the dynamic compressive behavior can be characterized by

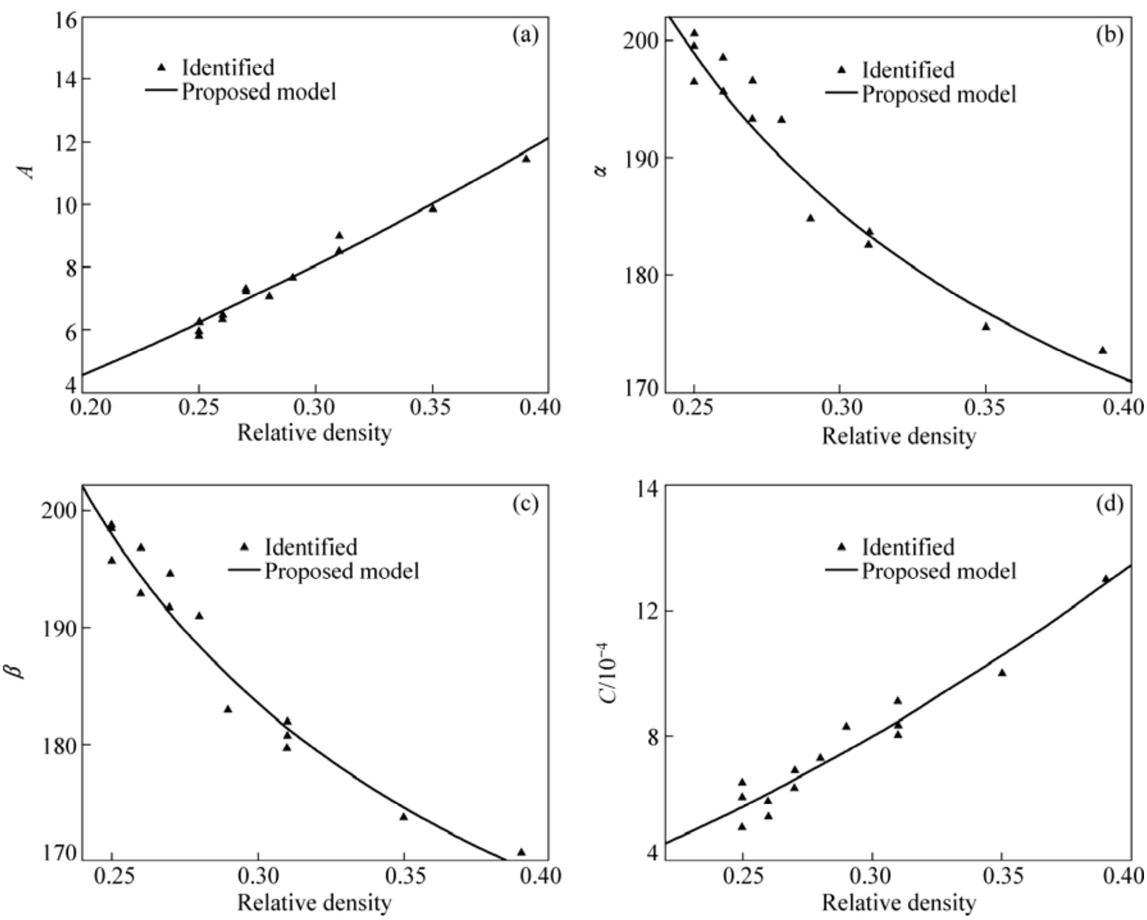

Fig.6 Functional relationship of model parameters A (a), α (b), β (c) and C (d) determined in terms of initial relative density

using a single constitutive equation under various loading conditions.

3) It is more convenient for specific applications in impact safety and crashworthiness analysis by using the proposed elasto-plastic constitutive model. It is also proposed that besides phenomenological approach, further study on metal foam constitutive model should take sub-cellular level deformation mechanism into account.

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泡沫铝合金动态弹塑性本构关系的研究

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摘 要：提出一个多参数的非线性弹塑性唯象本构模型，该模型能够全面地描述泡沫铝合金的典型三阶段变形特征，即线弹性阶段、应力平台阶段和密实化阶段。考虑到密度(相对密度)是泡沫铝这类多孔材料性能表征的最重要参数，在对泡沫铝合金进行各种应变率下的单向压缩实验基础上，确定模型中的参数与相对密度的函数表达式，从而，该模型能系统地描述相对密度、应变率效应对其动态力学行为的影响。模型预测结果和实验结果的对比验证了该模型的可靠性。研究结果可为吸能缓冲及防护结构的优化设计提供技术参考。

关键词：弹塑性本构模型；泡沫铝合金；应变率效应；能量吸收

(Edited by FANG Jing-hua)