

## Elastic-plastic properties of thin film on elastic-plastic substrates characterized by nanoindentation test

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Received 9 October 2009; accepted 31 January 2010

**Abstract:** To characterize the elastic-plastic properties of thin film materials on elastic-plastic substrates, a simple theory model was proposed, which included three steps: dimensionless analysis, finite element modeling and data fitting. The dimensionless analysis was applied to deriving two preliminary nondimensional relationships of the material properties, and finite element modeling and data fitting were carried out to establish their explicit forms. Numerical indentation tests were carried out to examine the effectiveness of the proposed model and the good agreement shows that the proposed theory model can be applied in practice.

**Key words:** elastic-plastic thin film; nanoindentation test; dimensionless analysis; finite element analysis

### 1 Introduction

Thin films are ubiquitous in modern technology, from integrated circuits to thermal barrier coatings, due to its high performance, density and smaller overall size. However, in practical applications, there are many difficulties. For instance, thin films often experience very high stresses during service which can lead to distortion of the device, deformation, fracture or decohesion of the film and degradation of the film due to enhanced diffusion or corrosion; and all these could eventually affect the functionality of the thin film/substrate layered system. Furthermore, the microstructural aspects of materials in the form of thin films can be quite different from those of the same materials in bulk form which can lead to unexpected mechanical behavior[1–2]. Therefore, in order to understand, predict and improve the reliability of devices containing thin films, it is necessary to characterize the mechanical properties of thin films.

Nanoindentation test has been successfully used to evaluate the mechanical properties of thin films due to its capability of deforming materials on a micro or nano-scale and measuring their elastic-plastic properties in situ[3–4]. However, most previous studies about

depth-sensing nanoindentation technique demonstrate that, to extract the mechanical properties of the coatings or films from the indentation test, we should limit the indentation depth less than 10%–20% of the film thickness in order to avoid the influence of the substrate[5–6]. As we know, during nanoindentation experiments, especially when the indentation depth is 100–1 000 nm, size-scale-dependent indentation effects is inevitable[7–9]. Thus, for material system with very thin film, it becomes extremely difficult to obtain the intrinsic mechanical properties of the film by the indentation test when the indentation depth is too shallow. The indentation depth has to be larger than 10%–20% of the film thickness in this situation and the effects of elastic-plastic deformation of the substrate should be considered.

Recently, as for an elastic-plastic film coated on an elastic substrate system, MA et al[10] and ZHAO et al[11] proposed the techniques which can extract the film elastic-plastic properties considering the influence of the elastic substrate from one conical indentation test using the finite element method (FEM). Whereafter, as for an elastic-plastic film coated on an elastic-plastic substrate system, based on the work of MA et al[10] and ZHAO et al[11], LIAO et al[12] proposed a technique which also

**Foundation item:** Projects(50531060, 10525211, 10828205) supported by the National Natural Science Foundation of China; Project(10525211) supported by National Science Found for Distinguished Young Scholars of China; Project(076044) supported by the Cultivation Fund of the Key Scientific and Technical Innovation Project, Ministry of Education of China

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DOI: 10.1016/S1003-6326(10)60653-X

considered the substrate effects to measure the elastic-plastic properties of thin film materials. In all these studies, dimensional analysis was carried out to derive three preliminary nondimensional relationships among total work, reversal work, residual penetration of indentation test and elastic-plastic properties of thin film, respectively. In order to obtain the values of total work and reversal work, it first needs to integrate the loading and unloading curves. Then, by finite element and data fitting, explicit forms of these three nondimensional relationships were established. Because these techniques need integrate the loading and unloading curves and solve three equations, and they are a little complicated for engineering applications.

In this work, a new theory model was proposed to measure the elastic-plastic properties of thin film materials on elastic-plastic substrates directly from force—displacement curve of nanoindentation test. Like the method proposed by LIAO et al[12], this theory model also considers the substrate effects and includes three steps: dimensional analysis, finite element modeling and data fitting. But compared with those proposed by previous studies, it needn't integrate the loading and unloading curves, and just concludes two nondimensional equations. So, this study provides a simple framework to extract elastic-plastic properties of elastic-plastic film by considering the influence of the elastic-plastic substrate from one indentation test that is widely applicable.

## 2 Theory model

Considering a rigid, conical indenter indenting into an elastic-plastic film coated on an elastic-plastic substrate (see Fig.1), both the film and substrate materials can be characterized by a power law relation defined as

$$\varepsilon = \begin{cases} \sigma / E, & \sigma \leq \sigma_y \\ (\sigma_y / E) / (\sigma / \sigma_y)^{1/n}, & \sigma > \sigma_y \end{cases} \quad (1)$$

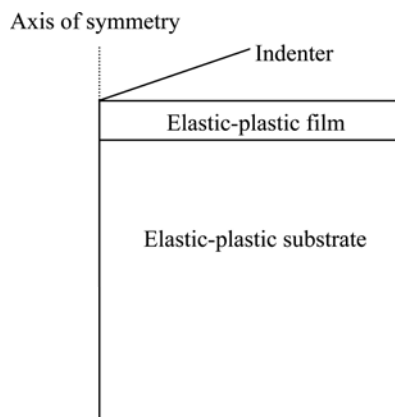


Fig.1 Schematic of normal indentation on film/substrate system

where  $\sigma_y$  is the initial yield stress  $n$  is the power hardening exponent; and  $E$  and  $\nu$  are elastic modulus and Poisson ratio, respectively. The tensile behavior is generalized to multiaxial stress states by assuming isotropic hardening and using the von Mises yield surface. For thin film the parameters are denoted as  $\sigma_{yf}$ ,  $n_f$ ,  $\nu_f$  and  $E_f$ , while for substrate they are denoted as  $\sigma_{ys}$ ,  $n_s$ ,  $\nu_s$  and  $E_s$ . A typical loading—unloading curve is given in Fig.2, where  $P_m$ ,  $h_m$  and  $h_r$  are the maximum indentation load, maximum indentation depth and residual penetration, respectively.

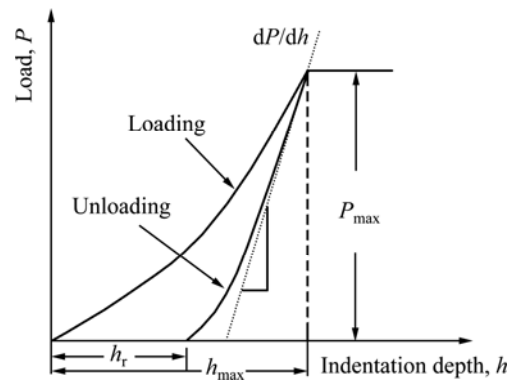


Fig.2 Typical  $P-h$  response of elastic-plastic material to nanoindentation test

Our purpose is to measure the elastic-plastic properties of thin film materials. In other words,  $\sigma_{yf}$ ,  $n_f$ ,  $\nu_f$  and  $E_f$  are the parameters that we don't know and should be obtained in the present work. Previous studies demonstrated that the Poisson ratio  $\nu$  is not an important factor in the indentation experiment, and for most engineering materials  $\nu \approx 0.3$ [13–15]. In addition, because the indentation is not completely rigid, we defined a reduced elastic modulus[16], that is

$$\frac{1}{E_r} = \frac{1 - \nu_f^2}{E_f} + \frac{1 - \nu_i^2}{E_i} \quad (2)$$

where  $E_r$  is the reduced elastic modulus,  $E_i$  and  $\nu_i$  are the elastic modulus and Poisson ratio of the indenter[16].  $E_r$  and film stiffness  $S$  have the following relationship:

$$S = \left. \frac{dP}{dh} \right|_{h=h_{\max}} = \frac{2}{\sqrt{\pi}} E_r \sqrt{A} \quad (3)$$

It also can be rewritten as

$$E_r = \frac{\sqrt{\pi}}{2} \cdot \frac{S}{\sqrt{A}} \quad (4)$$

where  $A$  is the contact area. Since the value of  $S$  can be calculated from the unloading curve as shown in Fig.2,  $E_r$  can be easily established by Eq.(4). Once  $E_r$  is known, the elastic modulus of the film  $E_f$  can be obtained with Eq.(2). Hence, there are only two material parameters

(i.e.  $\sigma_{yf}$  and  $n_f$ ) that we cannot establish indirectly from the indentation test.

In order to extract the left two parameters from the force—displacement curve of nanoindentation test, a theory model was developed.

## 2.1 Dimensional analysis

It is considered that each type of material has respective force—displacement curve. The following power function is adopted to fit the loading curve as shown in Fig.1.

$$P = P_m \left( \frac{h}{h_m} \right)^x \quad (5)$$

Dimensional analysis reveals that the maximum indentation load  $P_m$  and index  $x$  should be a function of all the independent parameters ( $E_f$ ,  $\sigma_{yf}$ ,  $n_f$ ,  $E_s$ ,  $\sigma_{ys}$ ,  $n_s$ ,  $t_f$ ,  $h_m$ ). Here,  $h_f$  is the thickness of films. Therefore, we have

$$P_m = \varphi(\sigma_{yf}, n_f, E_f, \sigma_{ys}, n_s, E_s, h_m, h_f) \quad (6)$$

$$x = \varphi(\sigma_{yf}, n_f, E_f, \sigma_{ys}, n_s, E_s, h_m, h_f) \quad (7)$$

By applying the Buckingham-II theorem and choosing the dimensionless quantities that unknown parameters are normalized by known quantities, the following dimensionless forms can be obtained:

$$\frac{P_m}{E_s h_m^2} = \Phi \left( \frac{\sigma_{yf}}{E_s}, n_f, \frac{E_f}{E_s}, \frac{\sigma_{ys}}{E_s}, n_s, \frac{h_f}{h_m} \right) \quad (8)$$

$$x = \Psi \left( \frac{\sigma_{yf}}{E_s}, n_f, \frac{E_f}{E_s}, \frac{\sigma_{ys}}{E_s}, n_s, \frac{h_f}{h_m} \right) \quad (9)$$

In order to reduce the number of parameters of Eqs.(8) and (9),  $h_m/t_f$  is assumed to be a constant value of 1/2. Moreover, because substrate properties  $\sigma_{ys}/E_s$  and  $n_s$  in this work are considered to be known, Eqs.(8) and (9) can be simplified as

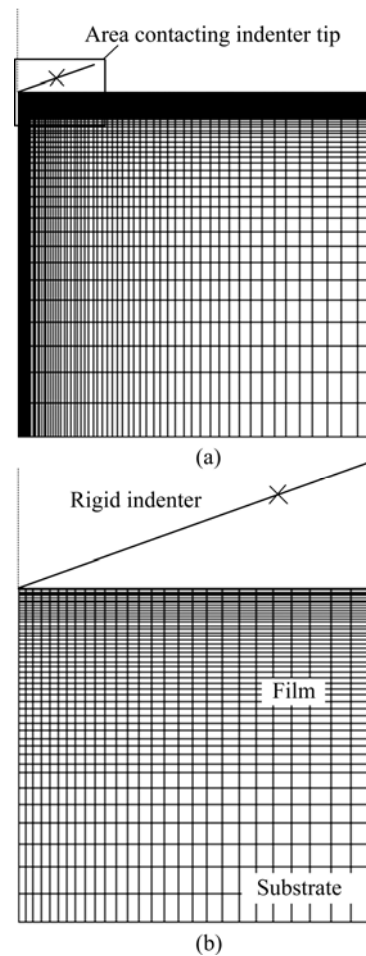
$$\frac{P_m}{E_s h_m^2} = \Phi \left( \frac{\sigma_{yf}}{E_s}, n_f, \frac{E_f}{E_s} \right) \quad (10)$$

$$x = \Psi \left( \frac{\sigma_{yf}}{E_s}, n_f, \frac{E_f}{E_s} \right) \quad (11)$$

## 2.2 Finite element modelling

The FEM simulations were carried out using the commercial code ABAQUS [17] on Dell workstations. Due to the fact that the conical indenter was assumed to be axisymmetric, Fig.3 shows the mesh design for axisymmetric calculations. The semi-infinite film/substrate system of the indented solid is modeled using 9900 four-noded, bilinear axisymmetric quadrilateral elements, where a refine mesh near the

contact region and a gradually coarser mesh further from the contact region have been designed to ensure numerical accuracy. The mesh was well-tested for convergence and was determined to be insensitive to far-field boundary conditions. A rigid conical surface with half apex angle of 70.3 ° was used to model the widely used Berkovich indenter. In this study, the rigid indenter is assumed to have a perfectly sharp tip. Coulomb's friction law is used between contact surfaces with a friction coefficient of 0.1 and the friction is a minor factor in indentation[4, 18].



**Fig.3** Finite element mesh of axisymmetric indentation: (a) Overall mesh design for conical indentation calculation; (b) Detailed illustration of area that directly contacting indenter tip

Both film and substrate are assumed to be isotropic. Generally, the initial yield and elastic modulus for metal or alloy are 0.03–1.1 GPa and 40–210 GPa respectively, while  $n_f$  typically varies between 0 and 0.5. Therefore, large deformation finite element computations are carried out for combinations of elastic-plastic properties that encompass the wide range of film material parameters. The input material properties of film and substrate are listed in Table 1.

**Table 1** Input material properties for finite element modelling

Material	Elastic modulus/GPa	Poisson ratio	Yield stress/MPa	Hardening exponent
Elastic-plastic film	50–250	0.3	100–2 000	0.1–0.5
Substrate	210	0.3	500	0.1

### 3 Results and discussion

#### 3.1 Dimensionless functions

With the finite element model, large amounts of calculations are performed to study the effects of each dimensionless parameters on the right of Eqs.(10) and (11) on the indentation load  $P_m$  and index  $x$ . Lots of data are obtained at the same time. Finally, Eqs.(12) and (13) can be established by fitting the finite element simulation results:

$$\frac{P_m}{E_s h_m^2} = \left[ A + B \left( \frac{E_f}{E_s} \right) + C \left( \frac{E_f}{E_s} \right)^2 + D \left( \frac{E_f}{E_s} \right)^3 + E \left( \frac{E_f}{E_s} \right)^4 \right] \cdot \left[ \ln \left( \frac{E_s}{\sigma_{yf}} \right) \right]^{A_1 + B_1 \left( \frac{E_f}{E_s} \right) + C_1 \left( \frac{E_f}{E_s} \right)^2 + D_1 \left( \frac{E_f}{E_s} \right)^3} \quad (12)$$

where the coefficients  $A, B, C, D, E, A_1, B_1, C_1$  and  $D_1$  are functions of  $n_f$  and

$$\begin{aligned} A(n_f) &= -22.67575 + 286.9729n_f - 311.177n_f^2; \\ B(n_f) &= 256.36276 - 2507.81365n_f + 2729.16662n_f^2; \\ C(n_f) &= -654.76278 + 7032.7634n_f - 7705.27925n_f^2; \\ D(n_f) &= 691.90809 - 7807.4869n_f + 8609.36025n_f^2; \\ E(n_f) &= -248.94658 + 2890.7299n_f - 3201.29175n_f^2; \\ A_1(n_f) &= -2.73663 + 10.1163n_f - 9.7785n_f^2; \\ B_1(n_f) &= 3.45032 - 59.4961n_f + 57.7581n_f^2; \\ C_1(n_f) &= -6.0827 + 93.2563n_f - 94.80463n_f^2; \\ D_1(n_f) &= 2.52924 - 40.1131n_f + 40.73125n_f^2. \end{aligned}$$

$$x = \left[ A_2 + B_2 \left( \frac{E_f}{E_s} \right) + C_2 \left( \frac{E_f}{E_s} \right)^2 + D_2 \left( \frac{E_f}{E_s} \right)^3 \right] \cdot \left[ \ln \left( \frac{E_s}{\sigma_{yf}} \right) \right]^{A_3 + B_3 \left( \frac{E_f}{E_s} \right) + C_3 \left( \frac{E_f}{E_s} \right)^2 + D_3 \left( \frac{E_f}{E_s} \right)^3 + E_3 \left( \frac{E_f}{E_s} \right)^4} \quad (13)$$

where the coefficients  $A_2, B_2, C_2, D_2, A_3, B_3, C_3, D_3, E_3$  also are functions of  $n_f$  and

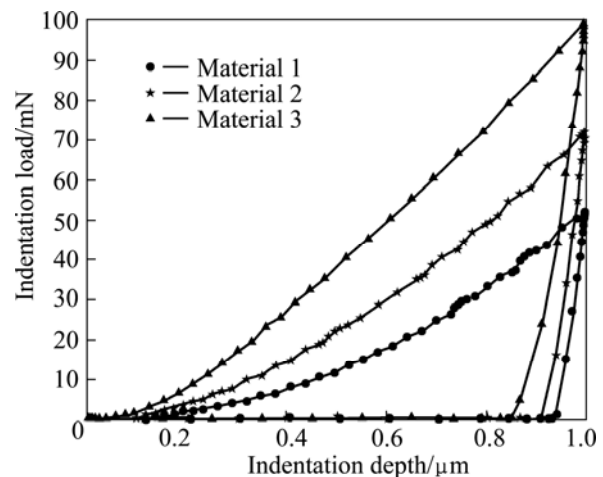
$$\begin{aligned} A_2(n_f) &= 1.3968 - 4.01885n_f + 5.95963n_f^2; \\ B_2(n_f) &= -3.45747 + 27.86185n_f - 39.742n_f^2; \\ C_2(n_f) &= 4.92756 - 46.1173n_f + 64.6765n_f^2; \\ D_2(n_f) &= -2.05404 + 19.39815n_f - 26.12088n_f^2; \\ A_3(n_f) &= 2.06616 - 18.55825n_f + 30.25837n_f^2; \end{aligned}$$

$$\begin{aligned} B_3(n_f) &= -13.47168 + 153.4997n_f - 251.26338n_f^2; \\ C_3(n_f) &= 38.02442 - 424.29397n_f + 692.02775n_f^2; \\ D_3(n_f) &= -41.81394 + 465.2059n_f - 755.47387n_f^2. \end{aligned}$$

Once the explicit form of the dimensionless is established, it is easy to identify the film material parameters  $E_f, \sigma_{yf}$  and  $n_f$ . The detailed steps are as follows: 1) an experimental or numerical indentation test can be performed to obtain the  $P$ — $h$  curve; 2) according to Eqs.(2)–(4), the elastic modulus of film  $E_f$  can be measured; 3) by fitting the  $P$ — $h$  curve, the indentation load  $P_m$  and index  $x$  can be determined; 4) by substituting indentation load  $P_m$  and index  $x$  to Eqs.(12) and (13) and solving these two equations, the elastic-plastic properties of the film ( $\sigma_{yf}$  and  $n_f$ ) can be characterized.

#### 3.2 General numerical examination of theory model

Numerical indentation tests are performed to examine the effectiveness of the theory model. The elastic-plastic properties of the substrate ( $E_s, \sigma_{ys}, n_s$ ) are given as:  $E_s=210$  GPa,  $\sigma_{ys}=500$  MPa and  $n_s=0.1$ , and three different film material combinations ( $E_f, \sigma_{yf}, n_f$ ) |input with thickness of 2  $\mu\text{m}$  coated on the substrate (i.e., material 1, material 2, material 3) are used as the input in FEM simulations, with  $h_m=1/2t_f$ . Numerical indentation tests are carried out on these materials, the  $P$ — $h$  curves can be obtained, as shown in Fig.4. With the theory model, we can extract the set of film material properties ( $E^*, \sigma_{yf}, n_f$ ) from the  $P$ — $h$  curves, as shown in Fig.4. The identified material properties and those of the input material properties are listed in Table 2 to examine the effectiveness of the proposed algorithm. From Table 2, it can be seen that the identified film material properties are very close to those of the input data.



**Fig.4** Indentation load—depth curves of thin films deposited on elastic-plastic substrate

**Table 3** Comparisons of material properties identified by theory model proposed and input material properties

Film material	Input material properties			Identified material properties		
	$E_f$ /GPa	$\sigma_{yf}$ /MPa	$n_f$	$E_f$ /GPa	$\sigma_{yf}$ /MPa	$n_f$
Material 1	80	500	0.1	79.5	500	0.1
Material 2	130	1000	0.2	129.1	998.6	0.2
Material 3	210	1800	0.4	207.8	1 799.5	0.4

## 4 Conclusions

1) A theory model which effectively measures the elastic-plastic properties of thin films on elastic-plastic substrates from the force—displacement curves of the nanoindentation test was presented. It is worth noting that, compared with previous studies, this model becomes much easier for practical applications. This is because in the theory model proposed here we only need to establish two nondimensional functions, but three nondimensional functions needed to be established in previous studies.

2) The effectiveness of the theory model is verified through numerical indentation. The current analysis does not take into account of any size effects associated with plastic strain gradients. When the aforementioned assumptions are satisfied, the method in this work can be used for measuring the elastic-plastic properties of a film on an elastic-plastic substrate, which is also useful for very thin films.

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(Edited by LI Xiang-qun)