DESCRIPTION OF G–S PARTICLE-SIZE DISTRIBUTION OF ROCK COMMINUTION WITH FRACTAL GEOMETRY①

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ABSTRACT

The fractal model of rock comminution is presented with Mandelbrot's fractal geometry. The results show that it is difficult for those with only a linear similarity ratio to fit practical situations. The comminution probability of the central part should be considered so the geometric meaning of the constant in Gaudin-Schuhmann's distribution function can be explained more clearly.

Key words: fractal dimension, comminution engineering, particle-size distribution

1 FRACTAL MODEL IN PROCESS OF ROCK COMMINUTION

Recently, fractal geometry has been mainly used to study objects of self-similarity. "Self-similarity" is characteristic by which an object can be decomposed into parts, each of which belongs to the whole by a similitude. Stochastic self-similarity in the rock fracture process has been recognized. The multitude of rock segments by comminution is self-similar for minute particles under suitable dilations and large particles resemble each other in construction and shape[1].

In fractal geometry, the most important exponent which can characterize fractal quality is called fractal dimension D, which is defined as

$$D = \lg[N(r)] / \lg(r^{-1}) \qquad (1)$$

Where r is the similarty ratio; $N(r)$ is the number of parts at the similarity ratio.

The continual fracture process of a pyramid is regarded as the model of rock or mineral comminution. This model is illustrated in Fig 1. A pyramid with a casual size is referred to as an initiator. This initiator is divided into four smaller pyramidic elements which are easily destroyed and one central part which is hard to be fragmented further. Each of these smaller elements is redivided into another four smaller elements and another smaller central one. The process is repeated without end. As illustrated in Fig 1, the smaller parts follow the same shape as the whole, and size is reduced at the same ratio r in every direction. This fits perfectly with the self-similar requirements of the system.

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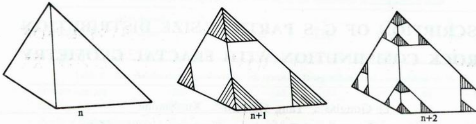

Fig. 1 The fractal model of the process of rock comminution

In the fractal model, suppose the central part is not fragmented at all. The size k of a pyramid can be divided into two, three or four equal segments, etc. Each fragmention produces one centre and four smaller pyramids with a size k / 2, k / 3, k / 4, ···

According to the definition of fractal dimension $r = 1/2,\ 1/3,\ 1/4,\ \cdots,\ N = 4$. Therefore $D = \lg[N(r)] / \lg(r^{-1}) = 2,\ 1.26,\ 1,\ \cdots 0$.

The fractal dimension value D is over 2.0 as shown in eguation (5) when $\alpha = 0.7$—1.0 according to reference[2].

If only four angles of the initiator are destroyed, the largest value of D is 2. Therefore it is understood that the centres in the fractal model are partly destroyed during comminution. Let the probability of a centre destroyed be P.

When linear similar ratio r is 1 / 3, the number of the smaller pyramids into which one centre can be divided equals to

$$(r^{-1})^E - 4 = 23$$

where E is the topological dimension, the value of which is 3 for the pyramidic model.

According to the assumption that the central part is comminuted with the probability P. On average, the number of the pyramids into which a central part is divided is $23P$.

So the total number of the reduced-scale parts from the initiator for one comminution is

$$N = [(r^{-1})^E - 4]P + 4$$

since $E = 3$

$$D = \lg[N(r)] / \lg r^{-1}$$

so we have following formula

$$D = \lg[(r^{-3} - 4)P + 4] / \lg(r^{-1}) \quad (2)$$

2 PARTICLE-SIZE DISTRIBUTION

The fractal model illustrates how fragmentation can result in a fractal distribution. The model is illustrated in Fig.1. Let the largest size of resource pyramid be k, x_n be referred to the particle-size after n-times division, the particle-size is in reduced order according to the shape of the whole figure (initiator) by means of the linear similarity ratio r.

Therefore $x_n = r^n k \quad (n = 1, 2, 3, \cdots)$

We have $n = \lg(x_n k^{-1}) / \lg r$

On the other hand, taking account of the particle volume y_n whose particle-size is below x_n, the process of fractal model development shown in Fig. 1 can be described as follows:

$$y_1 = r^E N$$

$$y_2 = (r^E N)^2$$

$$\cdots$$

$$y_n = (r^E N)^n$$

We put

$$N^n = N^{\lg(\frac{x_n}{k}) / \lg r} = [(x_n / k)^{1/\lg r}]^{\lg N}$$

$$= (x_n / k)^{-\lg N / \lg(r^{-1})}$$

From equation (1) we have $N^n = (x_n / k)^{-D}$

The relation between volume and size is provided by

$$y_n = (r^n)^E N = (x_n / k)^E (x_n / k)^{-D} = (x_n / k)^{E-D}$$

So we obtain the ideal particle-size distribution formula for fractal model as follows:

$$y_n = (x_n / k)^{E-D} \quad (3)$$

After rock is comminuted, there exists a certain distribution function of its production. One of the well known is Gaudin-Schuhman's distribution function, which provided empirically a useful description of the applicable statistical distribution.

$$y_n = (x_n k^{-1})^{\alpha} \quad (4)$$

where y_n — the percentage of volume;

x_n — the particle-size, mm;

k — the characteristic value of particle- size distribution; as $x_n = k$, we have $y_n = 100\%$, that is to say, k is the largest particle size;

α — the exponent of particle-size distribution function.

If we compare the G−S particle-size distribution function (4) with the equation (3)which is formulated from the fractal model, we find that the fractal model shown in Fig. 1 satisfies the G−S distribution function, and we can also identify the geometric meaning of particle-size distribution exponent α, which is connected with fractal dimension D as:

$$\alpha = E - D \quad (5)$$

3 THE EMPIRICAL RELATION

Some of the experiments were carried out in our laboratory in a piston press so as to comminute iron ore and rock material by varying the pressure; then the products were screened and analyzed by statistical method[3,4]

Generally, the test results of rock and iron ore fracture products follow the G−S distribution function perfectly. But for different materials the parameters k and α in formula (4) vary with the materials.

iron ore: $y_n = (x_n / 23.18)^{0.36}$

limestone: $y_n = (x_n / 12.44)^{0.46}$

We put the test results into the lg−lg plot as illustrated in Fig. 2. Since both are perfect linear lines, they satisfy the exponential distribution.

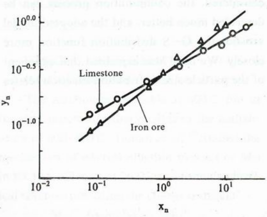

Fig.2 Two particle-size distribution of piston press products (particle size: 20— 30mm; depth of material bed: 50mm; pressure: 152MPa)

Using formula (5), we can obtain the fractal dimension values of the particle-size distribution of two kinds of material:

$D_{iron} = 2.64$

$D_{limestone} = 2.54$

If we choose r to be $1/3$, then:

$P_{iron} = 0.617$;

$P_{limestone} = 0.534$

4 CONCLUSION

One of the major requirements in rock mechanics is an accurate description of the discontinuity structure of rock masses and of the way in which the rock fragmentation processes are affected[5]. The fractal model which described the process of rock comminution can connect the comminution products with its process. It is a new method for studying rock comminution. Some basic ideas about how to describe the rock comminution processes with the fractal model have been given in this paper. The result shows that it is difficult for the model only with a linear similarity ratio to fit a practical situation. After the comminution probability of the central part is considered, the comminution process can be descrided much better, and the adopted model satisfies the G—S distribution function more closely. We have also explained that exponent of the particle-size distribution function relates to the fractal dimension D in the fractal model.

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