A NEW SIMULATING ALGORITHM FOR BALL MILLS —— CONVERSION COEFFICIENT METHOD①

Chen Bingchen②
Northeast University of Technology, Shenyang 110006, China

Xiao Wencai③
Chengdu Measuring Centre of Rocks and Minerals, Chengdu 610081, China

ABSTRACT

The conversing relation among the Bond Work Index W_I, specific output of mill and the energy efficiency of grinding were developed using dimensional analysis. The relations of which were proved and the value of the coversion coefficient ξ was also obtained using a new installed device for measuring grindability of ores. With the new simulating algorithm, the mill specifications, output and size distribution of griding products of industrial mills can be calculated based on laboratory measuring results. This algorithm is more accurate and simpler than any of the others.

Key words: grindability, conversion coefficient method, grinding constant

1 INTRODUCTION

There are two methods for ball mill calculation at present: 1) the conventional algorithm; 2) the simulating algorithm. Each has its own limitations and shortcomings.

The conventional algorithm for the choice of mill, for the calculation of output and for parameters, is first used to measure the grindability of ore in laboratory, and then multiply it by a series of revision coefficients because of the different conditions of mills in laboratory and in industry. Ore grindability is the "grinding constant" for mill selection and calculation, which may vary according to specific grinding conditions. At present, three kinds of grinding constants are widely used: 1) Volume constant or specific output of mill q (t / m³h); 2) Power constant, such as Bond Work Index W_I (kWh / t), and energy efficiency e (t / kWh); 3) Area constant, such as the Thompson index[1, 2].

The conventional algorithm is simple and easy, but it is impossible to obtain the size distribution of the ground product. Besides, there will be significant error with the onventional algorithm method because of the different revision coefficients (such as feed, size distribution and mill diameter etc.) used for different methods.

In order to solve the problems of the conventional algorithm method, the simulating algorithm was put forward in the 1960s[2,3]. With this method, the breakage function B, selection function S, residence time distribution function RTD and classification function C

①Manuscript received Nov.15, 1991. The research supported by the Doctoral Foundation of the National Education Committee; ②Professor; ③MSc for closed grinding circuit should be determined first; then the simulating calculation can be carried out in accordance with the grinding dynamics or the dynamic balance models.

Though the size distribution and other parameters can be calculated with this method, the following shortcomings remain: 1) it is very difficult to determine B and S because they are closely related to ore properties and operation factors; 2) the simulating algorithm is more complicated than the conventional algorithm but without higher accuracy.

In this paper, we present a newly developed simulating algorithm based on the conventional algorithm.

2 THEORY OF THE CONVERSION COEFFICIENT ALGORITHM

The work index W_I, specific output q and energy efficiency e describe ore grindability from different aspects separately. From the interrelation of W_I, q and e, a new simulating algorithm can thus be developed.

From dimensional analysis:

$$[q / e] = [\mathrm{t} / \mathrm{m^3 h}] / [\mathrm{t} / \mathrm{kWh}] = [\mathrm{kW} / \mathrm{m^3}]$$

$$[q / W_I] = [\mathrm{t} / \mathrm{m^3 h}] / [\mathrm{kWh} / \mathrm{t}] = [\mathrm{kW} / \mathrm{m^3}]$$

Therefore, the ratio between q / e or q / W_I is a dimensionless constant that can be obtained from the definition and dimensional analyis of W_I, q, e as follow[4–6]:

$$W_I = W_x / (10 / \sqrt{P_x} - 10 / \sqrt{F_x}) \quad (1)$$

$$q = Q(\beta - \alpha) / V \quad (2)$$

$$e = Q(\beta - \alpha) / N \quad (3)$$

Where, W_x in equation (1) means the specific energy consumption, and

$$W_x = N / Q \quad (4)$$

Then: $q / e = N / V = \xi$ (5)

$$q \cdot W_I = (\frac{N}{V})(\beta - \alpha) / (10 / \sqrt{P_x} - 10 / \sqrt{F_x}) = K\xi \quad (6)$$

$$K = (\beta - \alpha) / (10 / \sqrt{P_x} - 10 / \sqrt{F_x}) \quad (7)$$

Where, F_x and P_x represent the sizes (μm) of the feed, and the product respectively 80% of which passes through a screen; Q is the capacity of the mill with new feed (t / h); V is the effective volume of the mill (m^3); α, β are the contents of the feed and product of mill respectively (decimal); N is mill power (kW).

If the values of ξ and K are known, any grinding constants, such as W_I, q or e, can be obtained with the eqs. (5), (6).

If the effective power (N_e) of the mill is regarded as the mill power (N) it can be calculated as follows[4]:

$$N_e = \lambda \Delta V \sqrt{D} \cdot \mathrm{f}(\varphi, \psi) \quad (8)$$

Where, λ is the mill type coefficient, for grate mill $\lambda = 1.16$; for overflow mill $\lambda = 1.0$; Δ is the bulk density of the grinding medium (t / m^3); f (φ, ψ) is the power coefficient and a dual function, in which φ is the fraction of mill volume loaded with balls, ψ is the fraction of critical speed. Substituting eq. (8) into eq. (5), then

$$\xi = N_e / V = \lambda \Delta_D \mathrm{f}(\varphi, \psi) \quad (9)$$

If a standard mill with diameter D_o is selected and its operating parameters Δ_o, φ_o, ψ_o are given the standard conversion coefficient ξ_o can be obtained. Therefore, the conversion coefficiet ξ_o for any mill ($D_x \times L_x$) can be calculated as follows:

$$\xi_{X_o} = \lambda_x \xi_o (D_x / D_o)^{0.5} (\Delta_x / \Delta_o) \quad (10)$$

When $\Delta_x \approx \Delta_o$, eq. (10) can be rewritten as

$$\xi_{Xo} = \lambda_x \xi_o (D_x / D_o)^{0.5} \quad (11)$$

$$\xi_x = \lambda_x \xi_o (D_x / D_o)^{0.5} (\Delta_x / \Delta_o) \times f(\varphi_x, \psi_x) / f(\varphi_o, \psi_0) \quad (12)$$

Where, λ_x is the mill type coefficient; $f(\varphi_x, \psi_x)$, and $f(\varphi_o, \psi_o)$ are the power coefficients for nonstandard and standard grinding respectively.

The similar power coefficient $K_{\varphi,\psi}$ can be calculated with the following eqs:

$$K_{\varphi,\psi} = f(\varphi_x, \psi_x) / f(\varphi_o, \psi_o) \quad (13)$$

$$\xi_x = \xi_X \cdot K_{\varphi,\psi} \quad (14)$$

Thus if the mill (305× 305mm) and the test conditions for measuring the Bond ball mill work index are taken as standard, the standard conversion coefficient ξ_o can be determined.

3 EXRERIMENTAL VERIFICATION

With a new testing set which consists of the ball mill (305× 305mm) and a computer control power measuring instrument, the effective power consumption, and the grinding constants with a standard of q_o, W_{Io} and e_o can be obtained.

The standard testing conditions are as follows: ball mill (305× 305mm) with smooth linear, the effective volume $V_o = 0.02227\ m^3$; the rotation speed $n_o = 70r / min$; the fraction of critical speed $\psi_o = 91.2\%$; the mass of loaded balls is 20.26kg with a given distribution; the fraction of mill volume loaded balls $\varphi = 18.9\%$; the bulk density of balls $\Delta_o = 4.81t / m^3$.

eqs. (5) and (6) were proved as follows:

1) The ball mill work index W_{Io} and specific capacity q_o of three different iron ores were measured under standard conditions.

2) The effective power of ball mill (305× 305mm) was determined while the work index was being measured. Through experiments, the effective power was measured as $N_o = 121.06 \times 10^{-3}$kW. Thus

$$\xi_o = N_{e_o} / V_o = 5.435\ kW / m^3$$

3) q_o, e_o and W_{Io} can be calculated from the following eqs. respectively.

$$q_o = (60 \times n_0 \times G_o) / (V_o \times 10^6) = 4.2 \times 10^{-3} G_o / V_o \quad (15)$$

$$e_o = (60 \times n_0 \times G_o) / (N_{eo} \times 10^6) = 4.2 \times 10^{-3} G_o N_{eo} \quad (16)$$

$$W_{Io} = W_{x0} / (10 / \sqrt{P_x} - 10 / \sqrt{F_x}) = (N_{eo} / Q_o) / (10 / \sqrt{P_x} - 10 / \sqrt{F_x}) = \frac{N_{eo}}{4.2 \times 10^{-3} G_o} \frac{\beta - \alpha}{\frac{10}{\sqrt{P_x}} - \frac{10}{\sqrt{F_x}}} \quad (17)$$

Where, G_o is the ore grindability determined from standard Bond work index testing (g / r); W_{xo} is the specific power consumption (kWh / t); W_{Io} is the work index of the mill (305× 305mm) (kW / t). The value of the work index W_I can be calculated as:

$$W_I = 49.04 / [P^{0.32} G_o^{\ 0.32} (10 / \sqrt{P_x} - 10 / \sqrt{F_x})] \quad (18)$$

The values of the conversion coeffcient ξ_o with work index mill (305× 305mm) and those ξ_o derived from eq. 5 were listed in Table 1. The relative mean deviation between these two coefficients only 1. 7%

An empirical formula eq. (19) is then derived from Eq. 7.

$$K = -1.515 + 0.533 \ln P_i \quad (19)$$

If the controlling screen opening P_i is known, the constant K_i can be obtained from eq. (19) . And the deviation of K_iS from equations (7) and (19) is less than 1.5%, Eq (19) is suitable for feed size F_o of standard procedure which is about 6 mesh only, and the proportional constant K_i for other feed sizes can be calculated from eq. (20)

$$K_i = (2237 / F_x)^{0.14} (-1.514 + 0.533 \ln P_i) \quad (20)$$

Then:

$$W_{io} = N_{eo} / (4.2 \times 10^{-3} G_o)(2237 / F_x)^{0.14} (-1.54 + 0.533 \ln P_i) \quad (21)$$

From $q_o w_o = K\xi_o$ the conversion coefficient ξ_2 is calculated and listed in Table 2, the mean deviation between ξ_{o2} and the actual determined conversion coefficient is 1.92%.

Thus the close relations among W_I, q and e were verified.

Table 1 The measuring results of three kinds of ores

No.	F_{80} (μm)	controlling screen opening(μm)	P_{80} (μm)	grind-ability G(g/r)	total power of mill (W)	effective power of mill (W)	ξ_0 (kW/m^3)	α (%)	β (%)
1	2800	280	177	2.043	252.26	123.15	5.529	20.43	100.00
2	2800	224	157	1.984	248.63	119.52	5.366	20.00	100.00
3	2800	180	139	1.852	252.56	123.45	5.543	18.84	100.00
4	2800	154	116	1.609	250.96	121.85	5.471	15.40	100.00
5	2800	135	95	1.449	251.00	121.89	5.473	13.62	100.00
6	2800	90	64	1.160	248.92	119.81	5.379	10.49	100.00
7	2800	77	53	1.008	248.59	119.48	5.364	5.46	100.00
			mean value		250.42	121.31	5.447		
8	2185	180	157	3.688	250.00	120.89	5.428	27.54	100.00
9	2185	150	111	2.776	247.30	118.19	5.307	23.30	100.00
10	2185	90	76	2.111	251.30	122.19	5.486	19.39	100.00
			mean value		249.53	120.42	5.407		
11	2225	180	143.5	2.748	249.88	120.77	5.422	21.58	100.00
12	2225	125	98.5	2.012	253.17	124.06	5.570	17.62	100.00
13	2225	90	71.4	1.587	247.67	118.56	5.323	14.97	100.00
			mean value		250.24	121.13	5.439		
		Total mean value			250.17	121.06	5.435		

1–7 Donganshan Fe Ore ; 8–10 Dagushan Fe Ore ; 11–13 Baotou Fe Ore

Table 2 The calculated values ξ_0 from testing results of various ore

ore type	controlling screen opening(μm)	q_0 (t/m^{3}h)	e_0 (t/kWh)	W_{I0} (kWh/t)	$\xi_{01} = q_0 / e_0$ (kW/m^3)	$(\xi_0 - \xi_{01} / \xi_0)$ ×100%	$\xi_{02} = q_0 W_{I0} / K$ (kW/m^3)	$\xi_0 - \xi_{01} / \xi_0$ ×100%
	280	0.386	0.070	21.036	5.514	−1.45	5.479	−0.81
	224	0.385	0.068	19.967	5.368	1.28	5.343	1.69
	180	0.349	0.068	20.140	5.540	−1.93	5.632	−3.62
Donganshan	154	0.304	0.055	21.186	5.627	−1.69	5.528	−1.71
	125	0.274	0.050	21.170	5.480	−0.83	5.503	−1.25
	90	0.219	0.041	21.135	5.341	1.73	5.260	3.22
	77	0.189	0.035	23.037	5.400	0.04	5.463	−0.52
		mean value			5.545		5.458	
	180	0.704	0.128	9.681	5.495	−1.01	5.422	0.24
Dagushan	125	0.580	0.099	10.575	5.572	1.16	5.278	2.89
	90	0.408	0.073	11.905	5.554	−2.19	5.427	0.15
		mean value			5.474		5.876	
	180	0.525	0.096	13.176	5.490	−1.01	5.516	−1.49
Baotou	125	0.384	0.068	15.202	5.639	−3.75	5.512	−1.42
	90	0.303	0.056	15.569	5.389	0.85	5.336	1.82
		mean value			5.506		5.455	
	Total mean value				5.478	1.70	5.430	1.92

4 SIMULATING PROCEDURE FOR INDUSTRIAL MILLS

4.1 *Calculation on Capacity of Industrial Mill*

By means of the grinding energy constant e_o, the simulated calculation of energy efficiency from laboratory measured results up to industrial operations is relatively simple, because it only contains the grindability coefficient K_o and the size modifying coefficient K_{FP}. The formula for calculating energy efficiency is:

$$e_x = e_o K_g K_{FP} \quad (22)$$

Here, if the ores ground with laboratory and industrial mills are the same, $K_g = 1.0$; but for K_{FP} there are quite a few calculation methods . Based on our study we believe that eq. (23) tallies better with actual condictions.

$$K_{FP} = \left(\frac{P_x F_x}{P_o F_o}\right)^{0.5} \frac{F_o^{0.5} - P_o^{0.5}}{F_x^{0.5} - P_x^{0.5}} \frac{\beta_x - \alpha_x}{\beta_o - \alpha_o} \quad (23)$$

Where, F_x, P_x, F_o, P_o represent the feed and product sizes (80% passing through screen) of industrial and standard mill respectively; β_x, α_x, β_o and α_o are the percentages of given products and feed sizes.

The value of $f(\varphi,\psi)$ should be calculated first when the ξ_x of an industrial mill is calculated from eq. 12. From eq. 9, $f(\varphi_o, \psi_o)$ can be derived as follows:

$$f(\varphi_o, \psi_o) = N_{eo} / (\lambda_o V \sqrt{D_o}\, \Delta_o) = 2.044$$

The ξ_o is known as $\xi_o = 5.435\text{kW} / \text{m}^3$, thus:

$$\xi_x = \lambda_x 5.435\left(\frac{D_x}{0.305}\right)^{0.5}\left(\frac{\Delta_X}{4.81}\right)\frac{f(\varphi_x, \psi_x)}{2.044}$$

$$= 4.815\lambda_x\left(\frac{\Delta_X}{4.81}\right)\sqrt{D_x}\, f(\varphi_x, \psi_x) \quad (24)$$

When the bulk density Δ_x of medium for the industrial mill is 4. 81:

$$\xi_x = 4.815\lambda_x \sqrt{D_x}\, f(\varphi_x, \psi_x) \quad (25)$$

The value of $f(\varphi_x, \psi_x)$ for either φ_x or ψ_x has been derived[1]. Therefore, the specific capacity q_x and the capacity (Q_x) dealing with new feed for the industrial mill can be calculated as:

$$q_x = \xi_x e_x = \xi_x e_o K_G K_{FP} \quad (26)$$

$$Q_x = \frac{q_x V_x}{\beta_x - \alpha_x} = \frac{e_x \xi_x V_x}{\beta_x - \alpha_x} \quad (27)$$

Where V_x is the effective volume of the industrial mill(m^3) .

To make the calculation results more coincidental with practical use, the mean Q_x can be calculated with q_{xi} and Q_{xi} based on multiple sizes, such as:

$$\bar{Q}_x = \frac{1}{n}\sum_{i=1}^{n} Q_{xi} \quad (28)$$

Where Q_{xi} is thé mill capacity of single size i; n is the number of size classes.

4.2 *Calculation of the Size Distribution of Products*

From eqs. (7) and (20), the size of the mill products is:

$$P_x = \{10[(\beta_x - \alpha_x)(2237/F_x)^{-0.14} \times (0.533\ln P_i - 1.515)^{-1} + 10/\sqrt{F_x}\,]^{-2}\}^2 \quad (29)$$

From eqs. (26) and (27):

$$Q_x = (e_o K_{FP} V_x \xi_x) / (\beta_x - \alpha_x) \quad (30)$$

thereafter

$$K_{FP} = (\beta_x - \alpha_x) Q_x / (e_o V_x \zeta_x) \quad (31)$$

From eqs. (23) and (31), another eq. for P_x is:

$$P_x = F_x \left\{\frac{e_o V_x \xi_x F_x^{0.5}}{(\beta_o - \alpha_o) Q_x} \frac{F_o^{0.5} - P_o^{0.5}}{(F_o P_o)^{0.5}} + 1\right\}^{-2} \quad (32)$$

From eqs. (7) and (20), the value β_x can be obtained from the following eq.

$$\beta_x = \alpha_x + [(\frac{2237}{F_x})^{0.14}(0.533\ln P_i - 1.515)] \times (10/\sqrt{P_x} - 10/\sqrt{F_x}) \quad (33)$$

In design and practical production, the values of V_x, F_x and Q_x are usually known, ξ_x can be obtained from eq. (25); and Q_x(or Q_x) from eqs. (27) or (28); consequently, the P_x can be obtained from eq. (32) and the β_x from eq. (33).

5 SIMULATING EXAMPLE

The grindability parameters of Donganshan iron ore measured with new instruments are listed in Table 1. The operating parameters and the product size distribution of the primary mill in this system are shown in Tables 3 and 4 respectively.

Simulating procedure is ($P_1 = 77\mu m$, for example) as follows:

1) from eq. (16), the e_{01} of the ball mill (305×305mm) is:

$$e_{01} = 4.2 \times 10^{-3} \times (1.002 / 100.48 \times 10^{-3})$$
$$= 0.0352 \text{ t / kwh}$$

2) The size modifying coefficient K_{FP}.

As for mill (305×305mm):

$F_x = 14700\mu m$, $\beta_x = 27.0\%$, $\alpha_x = 1.24\%$,

$P_1 = 77\mu m$, from eq. (29):

$$P_{xi} = \{10 \times [(0.27 - 0.124)(\frac{2237}{14700})^{-0.14} \times$$
$$(0.533 ln77 - 1.515)^{-1} + \frac{10}{\sqrt{14700}}]^{-1}\}^2$$
$$= 397.6\mu m$$

From eq. (23):

$$K_{FP1} = (\frac{14700 \times 397.6}{2300 \times 53})^{0.5} \times$$
$$(\frac{\sqrt{2300} - \sqrt{53}}{\sqrt{14700} - \sqrt{296.6}})(\frac{0.27 - 0.0124}{1 - 0.546})$$
$$= 0.758$$

3) According to eq. (22):

$$e_{xi} = e_{01} K_{FP} = 0.0352 \times 0.758$$
$$= 0.0267 \text{t / kWh}$$

4) According to eq. (25)

$$\xi_x = 4.815 \times 1 \times 2.719 \times \sqrt{3.0}$$
$$= 22.678 \text{ kW / m}^3$$

5) According to eq. (26) q_x and Q_{x1} of the industrial mill are:

$$q_{xi} = \xi_x e_{xi} = 0.0267 \times 22.678$$
$$= 0.606 \text{ t / m}^3\text{h}$$

6) According to the preceding step, the mean Q_x can be calculated and its results are listed in Table 5.

The actual capacity of the mill (3200×3450mm) equals 60.4 t / h. The deviation from the calculated value is 2.65%

7) With the known data the value P_{x1} and β_{x1} can be calculated from eqs. (32) and (33) respectively. The results are shown in Table 6 and Fig 1. The simulating results with new algorithm are close to the actual data.

Table 5 Mill output of different size categories

$F_i(\mu m)$	Q_{xi}(t / h)	error[$(Q_{xi}-Q_{xi}) / Q_{xi}$](%)
77	57.4	23.8
90	60.9	−3.57
125	58.6	0.34
154	62.3	−5.95
180	59.4	−1.02
224	58.4	0.68
280	54.3	7.65
mean value	58.8	4.03

When the actual capacity of the industrial mill is compared with the simulated results based on various algorithms (see Table 7) , the result of the conversion coefficient algorithm is the most accurate.

6 SUMMARY

All the current simulating algorithms for ball mills have their shortcomings, yet the new conversion coefficient algorithm recommended in this paper can calculates both the capacity of an industrial mill and the size distribution of the grinding product. Besides, it requires less testing work and is of high accuracy.

Table 3 Construction and operation parameters of 5th system mill in Donganshan plant

mill type	dimensions $D \times L$(m)	D_1 (m)	V_1 (m^3)	ψ (%)	φ (%)	Δ (t/m^3)
overflow	3.2×3.45	3.0	24.39	75	43	4.8

Table 4 Size distribution of products for 5th system in Donganshan plant, cumulative wt.%

product size, μm	−280	−224	−180	−154	−125	−90	−77
feed of mill	2.86	2.53	2.23	2.03	1.77	1.39	1.24
overflow of classifier	58.8	52.7	47.0	43.0	38.3	31.0	27.0

Table 6 Comparison of simulated and measured results of size distribution of grinding products

P_i (μm)	−280	−224	−180	−154	−125	−90	−77
β− measured (%)	58.8	52.7	47.0	43.0	38.3	31.0	27.0
β− simulated (%)	52.87	50.96	46.22	44.35	37.16	31.26	25.67
relative errors(%)	10.09	3.30	1.66	−3.14	2.98	−0.84	4.93

Table 7 Comparison of mill capacity Q with different calculating algorithms

Mine	algorithm	Q_{act} (t/h)	Q_{cal} (t/h)	$(Q_{act}-Q_{cal})/Q_{act}$ (%)
Donganshan	conversion coefficent method	60.4	58.8	2.65
	volume method	60.4	68.5	13.08
	Bond method	60.4	62.3	3.15
	general energy effective method	60.4	47.8	20.86
Dagushan	conversion coeffcient method	43.6	43.1	1.15
	volume method	43.6	50.1	14.91
	Bond method	43.6	42.6	2.30
	general energy effective method.	43.6	33.2	23.85

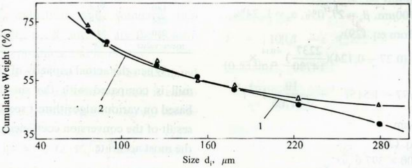

Fig. 1 The curves of size distribution based on measured and calculated results

1, calculated result 2, measured result

All the current simulating algorithms for ball mills have their shortcomings, yet the new conversion coefficent algorithm recommended in this paper can calculate with better accuracy.

REFERENCE

1 Chen Bingchen. The Principle of Grinding. Metallurgical Industry Press, 1989.

2 Mular A L, *et al.* Design and Installation of Comminution Circuits. New York: Port City Press, 1982.

3 Xiao Wencai. Master Thesis. Northeast Univecsity of Technology, 1989, 12.

4 Chen Bingchen. In: China National Scientific and Technological Information Center (eds.): The Proceedings of the Developent of Fine Ore Dressing. 1987

5 Справочник По ОбогаⅢ ению Руд, РазделⅣ. Москва: 〈НЕДРА〉, 1982

6 The Editorial Committee of the Handbook of Design in Mineral Processing. The Handbook of Design in Mineral Processing. Beigjing: Metallurgical Industry Press, 1988.