THEORY OF STOCHASTIC MEDIUM AND ITS APPLICATION IN SURFACE SUBSIDENCE DUE TO EXCAVATION①

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ABSTRACT

Research on the stochastic theory and its application have been conducted in China for 40 years. This paper emphasizes on the basic theory of stochastic medium and its practice in predicting the ground movements and deformations induced by underground and open pit mining, near surface excavation of tunnel and so on.

Key words: stochastic medium ground surface movement rock deformation mining and excavation

1 INTRODUCTION

In the past 40 years an extensive research program has been conducted in China to predict the ground surface movements and deformations induced by underground mining and near surface excavation. The aim is to minimize damage to surface structures such as buildings, railways and rivers from mining and underground excavation of railway, storage and other large spaces.

Recently, an increasing need for improving surface transportation and ecological aspects has led to increasing use of underground space for railways, storage, garages and shopping centers. These underground services are now regarded as an essential part of life in modern cities, and are placed close to the surface for low cost and convenience of use. However, caving into these spaces is liable to damage existing surface structures and services. In some cases the potential surface damage was estimated to be so great that the planned underground excavation project was pressed for either changing or cancelling.

With the accumulation of engineering experience, a theoretical approach has been developed for predicting ground surface movements, deformations and damage due to underground excavation. This approach based on a stochastic method has been used for the past 30 years by design institutes and companies to design the extraction of coal seams beneath buildings, railways and rivers. it has also been used to design underground railways.

For a quantitative approach to any mechanical phenomena and its effects, it is necessary to understand the physico-mechanical nature of the stressed body. The intrinsic properties of an actual body existing in nature may be very complex. However, when making a detailed analysis of the mechanical behaviour of a body, it is necessary to idealize the actual body as if it is composed of a certain ideal medium. In the past hundred years, several kinds of idealized medium have been used in rock

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mechanics to model the real rock masses. Among those frequently used are continuous medium, and discontinuous medium.

Because of jointing, the rock mass can be considered as a structure composed of a large number of rock block, which are different in size and shape but closely locked together. The degree of freedom of a single rock block is too large for classical mechanics to be able to define the motion trajectories of rock particles precisely. In the middle of the 1950's, taking this fact into account that the movement of a fractured rock mass is governed by a great number of known and unknown factors, Litwiniszyn, J. suggested a new method to compute the rock mass motion[1]. According to Litwiniszyn, a rock can be considered as a stochastic medium and its motion problem can be solved by the stochastic method.

In the past 30 years, this method has been undergoing continual improvement by experiments and has been widely applied to coal mining in Poland, China and USA.

Based on the stochastic medium concept a series of solutions for rock movement calculations in different geological and extraction conditions have been obtained. The solutions have been used in mining practice and underground construction to solve excavation problems under buildings, railways and rivers. Satisfactory comparisons have been made between theory and practice.

2 BASIC EQUATION FOR ELEMENTARY SUBSIDENCE BASIN

According to stochastics, we can divide an underground excavation into infinitesimal excavation elements. The effect due to the original excavation will be equal to the sum of the effects caused by these infinitesimal excavation elements. An excavation with infinitesimal unit thickness, width and length ($d\xi$, $d\zeta$, $d\eta$) is called an elementary excavation. The subsidence basin caused by an elementary excavaton is called an elementary basin. The subsidence and horizontal displacement of any point in the elementary basin is called the elemntary subsidence (W_e) and elementary horizontal displacement (U_e).

A rectangular cooridinate system is chosen with vertical axis z directed upward from the elementary excavation. Based on probability analysis, the occurrence of motion of a rock mass element over the elementray excavation may be a random event which takes place with a certain probability. If the rock mass is isotropic in the horizontal plane, then the probability density function will be continuous and symmetrical about the axis z. The occurrence of the event that subsidence occurs in an infinitesimal area $ds = dx\ dy$ at the horizon z with point A (x, y, z) at its centre, is equivalent to the occurrence of two events composed of a subsidence in the horizontal strip dx through A and simultaneously a subsidence in the horizontal strip dy through A (Fig. 1). Mathematically we can write the probability separately for these two events by the function $f(x^2)$ dx and $f(y^2)$ dy respectively, where f is the probability density function. The probability for the simultaneous occurrence of these two events is

$$p(ds) = f(x^2)dx f(y^2)dy = f(x^2)f(y^2)ds \qquad (1)$$

Through the origin o, a new set of rectangular coordinate axes (x_1, y_1, z) is chosen such that the coordinates of point A are (x_1, y_1, z). Using the new coordinates the probability will be:

$$p(ds_1) = f(x_1^2)dx_1 f(y_1^2)dy_1 = f(x_1^2)f(y_1^2)ds_1 \qquad (2)$$

Based on the fact that the probability

$p(\mathrm{d}s)$ does not change with the selection of the coordinate system, if the excavation elementary area $\mathrm{d}s=\mathrm{d}s_1$, and point A does not change, then

$$f(x^2)f(y^2)=f(x_1^2)f(y_1^2) \quad (3)$$

If the axis ox_1 passes through the point A, then

$$x_1^2=x^2+y^2 \text{ and } y_1=0 \quad (4)$$

Inserting Eq. (4) into Eq. (3) gives

$$f(x^2)f(y^2)=f(x^2+y^2)f(0)=Cf(x^2+y^2) \quad (5)$$

Differentiating Eq. (5) yields

$$f(y^2)\frac{\mathrm{d}f(x^2)}{\mathrm{d}(x^2)}=C\frac{\partial f(x^2+y^2)}{\partial(x^2+y^2)}$$

$$f(x^2)\frac{\mathrm{d}f(x^2)}{\mathrm{d}(y^2)}=C\frac{\partial f(x^2+y^2)}{\partial(x^2+y^2)}$$

finally

$$\frac{1}{f(x^2)}\cdot\frac{\mathrm{d}f(x^2)}{\mathrm{d}(x^2)}=\frac{1}{f(y^2)}\cdot\frac{\mathrm{d}f(y^2)}{\mathrm{d}(y^2)} \quad (6)$$

Both sides of Eq. (6) must equal a constant K, thus

$$\left.\begin{aligned}\mathrm{d}f(x^2)/\mathrm{d}(x^2)=Kf(x^2)\\ \mathrm{d}f(y^2)/\mathrm{d}(y^2)=Kf(y^2)\end{aligned}\right\} \quad (7)$$

Solving the differential Eq. (7) and considering the condition that as x and y approaches $\pm$ infinity then $p(\mathrm{d}s)=0$ gives

$$f(x^2)=q(z)\exp[-\pi x^2/r^2(z)]$$

$$f(y^2)=q(z)\exp[-\pi y^2/r^2(z)]$$

Hence

$$p(\mathrm{d}s)=q^2(z)\exp[-\frac{\pi}{r^2(z)}(x^2+y^2)]\mathrm{d}x\mathrm{d}y \quad (8)$$

Then the three dimensional density function will be

$$f(x,y,z)=q^2(z)\exp[-\frac{\pi}{r^2(z)}(x^2+y^2)] \quad (9)$$

where $q(z)$ and $r(z)$ are the coefficients dependent on the z axis. The above density function governs the geometric law for distribution of subsidence of rock particles due to the elementary excavation. As the elementary excavation is a component of the original excavation, which is large enough to cause the rock mass above the goaf move, then the elementary subsidence must be factual in this case. By equating the subsidence distribution of the elementary basin with the density function gives

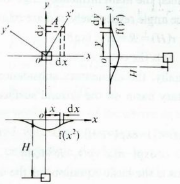

Fig. 1 Influence of elementary excavation

$$W_e(x,y,z,t)=q^2(z)\exp[-\pi(x^2+y^2)/r^2(z)]\times \mathrm{d}\xi\mathrm{d}\zeta\mathrm{d}\eta \quad (10)$$

From (10), the volume of the elementary basin will be

$$V_e=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} q^2(z)\exp[-\pi(x^2+y^2)/r^2(z)]\times \mathrm{d}\xi\mathrm{d}\zeta\mathrm{d}\eta\mathrm{d}x\mathrm{d}y \quad (11)$$

The subsidence basin develops with time as the overlying rock mass presses onto the goaf at a rate governed by

$$\mathrm{d}V_e/\mathrm{d}t=c(1-V_e) \quad (12)$$

where c is a coefficient. Under the condition $t=0$, $V_e=0$ and $t\to\infty$, $V_e=\mathrm{d}\xi$, $\mathrm{d}\zeta$, $\mathrm{d}\eta$, the solution to Eq. (12) is

$$V(t)=[1-\exp(-c\,t)]\mathrm{d}\xi\mathrm{d}\zeta\mathrm{d}\eta \quad (13)$$

Substituting (11) into (13), gives

$$q^2(z)=-[1-\exp(-c\,t)]/r^2(z) \quad (14)$$

Substituting (14) into (10), produces

$$W_e(x,y,z,t)=[1-\exp(-c\,t)]/r^2(z)\times \exp[-\pi(x^2+y^2)/r^2(z)\times \mathrm{d}\xi\mathrm{d}\zeta\mathrm{d}\eta \quad (15)$$

For two dimensional problem, the length of elementary excavation is infinite in the y axis direction. Integration of Eq. (15) gives

$$W_e(x,z,t) = [1-\exp(-c\,t)] / r(z) \times \exp[-\pi x^2 / r^2(z)] \mathrm{d}\xi \mathrm{d}\eta \quad (16)$$

After Knothe, St. [2] the coefficients R and β are called the main influence range and main influence angle respectively and are related by

$$r(H) = R = H / \tan\beta \quad (17)$$

where H is the excavation depth.

Finally, the elementary subsidence in the elementary basin on the ground surface is given by:

$$W_e(x,y,t) = [1-\exp(-c\,t)] / R^2 \times \exp[-\pi(x^2+y^2) / R^2] \mathrm{d}\xi \mathrm{d}\eta \quad (18)$$

This is the basic equation for the analysis. By integrating the ground surface subsidence caused by a rectangular excavation will be (Fig. 2):

$$W_e(x,t) = [1-\exp(-c\,t)] / R \times \exp[-\pi x^2 / R^2] \mathrm{d}\xi \mathrm{d}\zeta \mathrm{d}\eta \quad (19)$$

where $W(\xi, \zeta)$ is the roof subsidence

Like the ground surface subsidence horizontal displacement of the ground surface is also generated due to the underground elementary excavation. In the study of horizontal displacement, it is assumed that the rock mass is incompressible at any moment, and for a plane strain state

$$E_x + E_y + E_z = 0, \text{ and } E_y = 0 \quad (21)$$

thus

$$\partial U_e / \partial x + \partial W_e / \partial z = 0 \quad (22)$$

where E_x, E_y, E_z are the strains of rock mass in x, y and z direction respectively. Solving Eq. (22) yields

$$U_e = -\int \partial W_e(x) / \partial z \mathrm{d}x + c \quad (23)$$

Inserting Eq. (16) into Eq. (23) and assuming the boundary conditions $x \to \pm\infty$, $U_e = 0$, produces the expression for the horizontal displacement on the ground surface in the elementary subsidence basin

$$U_e(x,t) = -2\pi Bx[1-\exp(-c\,t)] / R^2 \times \exp(-\pi x^2 / R^2) \mathrm{d}\xi \mathrm{d}\eta \quad (24)$$

where B is the horizontal displacement coefficient.

$$B = 1 / 2\pi \cdot \partial(z) / \partial z = U_{e\,\max} / W_{e\,\max} \quad (25)$$

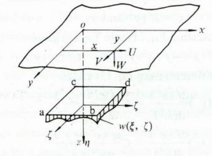

Fig. 2 Two-dimensional extraction

3 GROUND SUREACE MOVEMENTS AND DEFORMATIONS DUE TO NEAR-SURFACE TUNNELING

An example is shown in Fig. 3 of a long horizontal tunnel of depth H with arbitrary cross-section. It is clear that the problem can be reduced to that of a plane strain state.

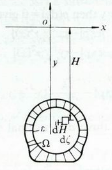

Fig.3 Tunnel excavation

Consider an extreme case where the underground tunnel has totally collapsed. The maximum subsidence will be reached after an infinite time ($t \to \infty$) following the total collapse. A sufficiently large area of excavation can be treated as being composed of infinite numbers of infinitesimal elementary excavations $dxdy$ which have the same influence on the surface. The equation for the final surface subsidence caused by elementary excavation can be obtained from Eq. (19) for $t \to \infty$ as

$$W_e(x,H) = \frac{1}{r(\eta)} \exp[-\frac{\pi}{r^2(\eta)} x^2] d\xi d\eta \quad (26)$$

The ground surface subsidence due to the total collapse (area Ω can be obtained using the principle of superposition and considering that $r(\eta) = \eta \cot\beta$, that is

$$W(x) = \iint_{\Omega} \tan\beta / \eta \exp[-\pi \tan^2\beta \times (x-\xi)^2 / \eta^2] d\xi d\eta \quad (27)$$

The total collapse of the tunnel represents the worst case producing subsidence outside the acceptable limits, it also provides the upper estimate of maximum ground surface subsidence.

Experience in underground engineering has shown that, when the tunnel is excavated and supported correctly only small movements develop in the surrounding rock. Hence, ground surface movement depends on the nature and extent of the convergence over the cross-section of the working. After excavation, the area of cross-section Ω converges to ω and the amount of surface subsidence is equal to the difference between subsidence due to excavation area Ω and ω. The difference is

$$W(x) = W_{\Omega}(x) - W_{\omega}(x) = \iint_{(\Omega-\omega)} \tan\beta / \eta \times \exp[\pi \tan^2\beta (x-\xi)^2 / \eta^2] d\xi d\eta \quad (28)$$

If a tunnel of circular cross-section with the initial radius, a converges to b, after a long period of time, then the surface subsidence can be obtained from Eq. (28)[5]

$$W(x) = \tan\beta \{ \int_A^B \int_C^D \exp[-\pi \tan^2\beta / \eta^2 \times (x-\xi)^2] d\xi d\eta - \int_E^F \int_G^K \times \exp[-\pi \tan^2\beta (x-\xi)^2 / \eta^2] d\xi d\eta \quad (29)$$

where $A = H - a$; $B = H + a$

$C = -\sqrt{a^2 - (H-\eta)^2}$

$D = \sqrt{a^2 - (H-\eta)^2}$

$E = H - b$, $F = H + b$

$G = -\sqrt{b^2 - (H-\eta)^2}$

$K = \sqrt{b^2 - (H-\eta)^2}$

The performance of the above equation is compared with the observations from a case history described in ref. [5]. In this case the tunnel is 17.4 m below surface with a crossection of 4.5 m (height) × 5.7 m (width). The area of convergence is 0.76 m^2. The parameter β which characterizes the behavior of the rock mass is determined by comparison of the area of subsidence basin cross-section with that of the convergence cross-section. The assumption implies that both are equal. For this case $\tan\beta = 1.37$, a comparison between the calculated curve and measured values is shown in Fig. 4.

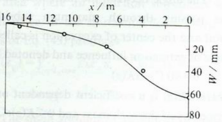

Fig.4 Comparison between the caculated (curve) and measured (circle) subsidence

The horizontal displacement of the ground surface is determined from

$$U(x) = (B\,H / \tan\beta) dW(x) / dx \quad (30)$$

The horizontal strain of the groundsurface is given by

$$E_x(x) = \mathrm{d}U(x) / \mathrm{d}x \qquad (31)$$

4 GROUND SURFACE MOVEMENTS AND DEFORMATIONS DUE TO UNDERGROUND MINING

Practice and model experiments[4] have shown that, when mining an inclined coal seam, the center of the elementary subsidence basin shifts from the point above the center of the excavation B to a new point A. The tilt of the subsidence basin does not change the symmetry of the basin shape (Fig. 5).

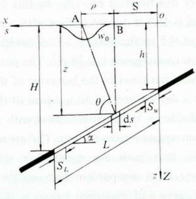

Fig. 5 Extraction of an inclined coal seam

The angle between the horizontal and the line passing through maximum subsidence point and the center of excavation is called the angle of extraction influence and denoted by

$$\theta = 90\,(^\circ) - \alpha K(z) \qquad (32)$$

where $K(z)$ is a coefficient dependent on the properties of the rock mass and $0 < K(z) < 1$

From Fig. 5

$$\rho(z) = z / \tan[90\,(^\circ) - \alpha K(z)] \qquad (33)$$

Applying Eq. (20) to the two-dimensional case and substituting $x - \rho(z)$ for x, produces the elementary subsidence basin for inclined strata as $t \to \infty$

$$W_e(x) = [r(z-Z)]^{-1} \exp\{-\pi / r^2(z-Z) \times [x - S - \rho(z-Z)]^2\} \mathrm{d}\xi \mathrm{d}\eta \qquad (34)$$

Thus from Eq. (23) the elementary horizontal displacement will be

$$U_e(x) = \frac{r(z-Z)}{2\pi} \frac{\partial r(z-Z)}{\partial z} \frac{\partial W_e(x)}{\partial x} - \frac{\partial \rho(z-Z)}{\partial z} W_e(z) \qquad (35)$$

Integrating Eq. (34) gives the surface subsidence due to the extraction of an inclined coal seam with extraction width L maximum and minimum extraction depth H and h.

$$W(x) = W_{max} \tan\beta \int_A^B (h + S\tan\alpha)^{-1} \times \exp\{-\pi[x - S - (h + S\tan\alpha)\cot\theta] \times \tan\beta / h + S\tan\alpha)^2\} \mathrm{d}S \qquad (36)$$

where $A = S_1\cos\alpha \quad B = (L - S_2)\cos\alpha$

Combining Eq. (36) and (35), gives the surface horizontal displacement as

$$U(x) = -BW_{max} \int_A^B \frac{2\pi\tan^2\beta}{(h + S\tan\alpha)^2} [x - S - (h + S\tan\alpha)\cot\theta] \times \exp\{\frac{\pi[x - S - (h + S\tan\alpha)\cot\theta]^2}{(h + S\tan\alpha)^2 \tan^{-2}\beta}\} \times dS - W(x)\cot\theta \qquad (37)$$

The maximum possible subsidence is

$$W_{\max} = W_{\theta,\max}\sin\theta = M_\theta \eta \sin\theta \qquad (38)$$

where $W_{\theta\,\max}$ and M_θ are the maximum ground surface displacement and effective coal seam thickness in the direction of θ.

If $\quad M_\theta = M / \cos[(1-K)\alpha]$

then $\quad W_{max} = M\,\eta\cos(k\,\alpha) / \cos[(1-k)\alpha] \qquad (39)$

The inclination of ground surface is

$$T(x) = \frac{\mathrm{d}W(x)}{\mathrm{d}x} = -W_{max} \int_A^B \frac{2\pi\tan^3\beta}{(h + S\tan\alpha)^3} \times [x - S - (h + S\tan\alpha)\cot\theta] \times exp\{-\frac{\pi\tan^2\beta}{(h + S\tan\alpha)^2}[x - S - (h + S\tan\alpha)\cot\theta]^2\} \mathrm{d}S \qquad (40)$$

Ground surface curvature is given by

$$K(x)=\frac{\mathrm{d}T(x)}{\mathrm{d}x}=-W_{max}\int_A^B\frac{2\pi\tan^3\beta}{(h+S\tan\alpha)^3}\times$$
$$\{1-\frac{2\pi\tan^2\beta}{(h+S\tan\alpha)^2}[x-S-(h+S\tan\alpha)cot\theta]^2\}\times$$
$$\exp\{-\frac{\pi\tan^2\beta}{(h+S\tan\alpha)^2}\times[x-S-(h+S\tan\alpha)\cot\theta]^2\}\,\mathrm{d}S \quad (41)$$

The horizontal strain of ground surface is

$$E_x(x)=\mathrm{d}U(x)/\mathrm{d}x$$
$$=-BW_{max}\int_A^B\frac{2\pi\tan^2\beta}{(h+S\tan\alpha)^2}\times$$
$$\{1-\frac{2\pi\tan^2\beta}{(h+S\tan\alpha)^2}[x-S-(h+S\tan\alpha)\cos\theta]\}\exp\{-\frac{\pi\tan^2\beta}{(h+S\tan\alpha)^2}\times$$
$$[x-S-(h+S\tan\alpha)\cot\theta^2\}\mathrm{d}S-T(x)\cot\theta \quad (42)$$

5 GROUND SURFACE MOVEMENTS AND DEFORMATION DUE TO OPEN PIT EXCAVATION

Vertical and horizontal movement beyond the perimeter of an open pit will be resulted if the development is deep and extensive. The surface point always moves towards the center of the open pit. The surface movements are not only great in relation to the stability of the open pit slope and the safety of mining operation but also are of significance to the stability of structures.

Recently, some studies on factors contributing to the surface movements have carried out. Particular attention has been paid to prediction of vertical and horizontal movements and horizontal strain and careful assessment of their influence on existing structure in the adjacent area.

A small amount of excavation leads to a small amount of movement on the surface, and the resultant movements of a point are given by the sum of movements caused by the excavation of different areas.

In the following analysis only the two-dimensional problem is considered. Let W be the subsidence on surface, and subtituing $r^{-2}(z)=h^2(z)\pi^{-1}$ into Eq. (10)gives

$$W_e=h(z)\exp[-h^2(z_0)(x-x_0)^2]/\sqrt{\pi} \quad (43)$$

Fig. 6 shows the cross-section through the open pit where H is the excavation depth. the plan equation for the top level of the open pit is $z=0$, for the bottom is $z=-H$, and for the slope is $z=x\tan\alpha$.

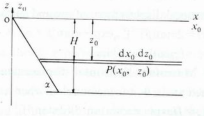

Fig. 6 Schematic section of open pit

Consider a point $P(x_0,\ z_0)$ in the excavation area where an induction subsidence has taken place in the horizontal level $z=z_0$. In terms of Eq. (43) produces

$$\triangle W(z_0)=\int_{-z_0\cot\alpha}^{\infty}\triangle z_0\pi^{-1/2}h(z_0)x\times\exp[-h^2(z_0)(x-x_0)^2]\mathrm{d}x_0 \quad (44)$$

The small amount of increment of the distribution function of subsidence $\triangle W(z_0)$ can be considered to be a result of the small amount of induction subsidence.

Bearing in mind the range of the main influence, Eq. (44) may be presented in the following form

$$W(x)=\int_{-H}^{o}\int_{-z_o\cot\alpha}^{\infty}\mathrm{h}(z_o)/\sqrt{\pi}\times$$
$$\exp[-h^2(z_o)(x-x_o)^2]\mathrm{d}x_o\mathrm{d}z_o \quad (45)$$

After some operations Eq. (45) becomes

$$W(x)=\int_{-H}^{o}\int_{a}^{b}\tan\beta/z_0\exp$$
$$[-z_2^{-2}\pi\tan^2\beta(x-x_0)^2]\mathrm{d}x_0\mathrm{d}z_0 \quad (46)$$

Using of the main influence range gives

$$h(z_0)=-z_0\tan\beta^{-1} \quad (47)$$
$$h(z_0)=-\sqrt{\pi}z_0^{-1}\tan\beta \quad (48)$$

Substituting Eq. (48) into Eq. (46) gives

$$W(x)=0.5\int_{-H}^{0}\{1-\mathrm{erf}[z_0^{-1}\sqrt{\pi}\tan\beta\times$$
$$(x+z_0\cot\alpha]\}\mathrm{d}z_0 \quad (49)$$

Maximum subsidence is only attained at $x=0$, it is meaningless when $x>0$,

$$W_{max}=0.5H[1-\mathrm{erf}(\sqrt{\pi}\tan\beta\cot\alpha)] \quad (50)$$

Solving the Eq. (23) we can obtain the horizontal displacement of ground surface

$$U(x)=2\pi\tan\beta)^{-1}\int_{-H}^{0}\exp[-\pi\tan^2\beta/z_0^{-2}\times$$
$$(z_0\cot\alpha+x_0)^2]\mathrm{d}z_0 \quad (51)$$

Maximum horizontal displacement is attained at $x=0$, it is meaningless when $x>0$.

$$U_{\max}=H\exp(-\pi\cot^2\alpha\tan^2\beta)(2\pi\tan\beta)^{-1} \quad (52)$$

As to other cases, the subsidence W and horizontal displacement U can also be obtained using Eq. (49) and (51), only upper and lower limits must be rewritten according to the boundary conditions.

The common excavation form of open pit is shown in Fig. 7 where the respective slope angles on the two sides are α_1 and α_2. The depth of open pit is H and the width at the button is L. In this case the subsidence of ground surface is determined by

$$W(x)=\int_{-H}^{0}\int_{A}^{B}\tan\beta/z_0\times$$
$$\exp[-\pi\tan^2\beta/z_0^2(x-x_0)^2]\mathrm{d}x_0\mathrm{d}z_0 \quad (53)$$

where $A=-z_0\cot\alpha_1$;

$B=H\cot\alpha_1+L+(z_0+H)/\tan\alpha_2$

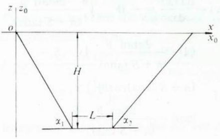

Fig. 7 Schematic section of an open pit

The horizontal displacement of the ground surface is given by

$$U(x)=\int_{-H}^{0}\int_{A}^{B}\tan\beta/z_0^2(x-x_0)\times$$
$$\exp[-z_0^{-2}\pi\tan^2\beta(x-x_2)^2\mathrm{d}x_0\mathrm{d}x_0 \quad (54)$$

The horizontal strain of the ground surface is given by

$$E(x)=\mathrm{d}U(x)/\mathrm{d}x \quad (55)$$

6 CONCLUSION

Mining and underground engineering show that the theory of the stochastic medium can be used in practice for solving the problems of prediction of the ground surface movement and deformation.

In mining design. it is essential to determine the characteristic parameters of rock mass. That is the angle of main influence.

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