Vol.2 №.1 TRANSACTIONS OF NFsoc February 1992

REFLECTION AND REFRACTION OF STRESS WAVES AT A STRUCTURAL WEAKNESS PLANE IN ROCK MASS①

Li, Xibing Lai, Haihui Gu, Desheng
Central South University of Technology, Changsha 410083, China

ABSTRACT

In the case of wave interactions with structural weakness planes in rook mass the results of reflection and refraction of waves at entirely cohesive boundaries have been used all along. But, oblique incidences of compressive stress waves at the structural weakness plane may result in relative slippage of rocks on both sides of the plane. The authors have derived the reflection and refraction coefficients for plane stress waves generated by blasting incidents on the structural weakness plane in rook mass according to its special boundary condition. The formula of energy dissipation and some computational results at the structural weakness plane are obtained. On the above basis, some problems about rook and soil engineering are discussed

1 INTRODUCTION

In blasting rook, shock waves generated by detonating explosives rapidly attenuate with the increasing distance of transmission and they are transferred into stress waves[1] by the time they travel 15 times the radius of the charge. But, natural rock mass is not homogeneous, and various weakness planes with different structures and different geological mechanism, such as, fractures, joints, and cracks, are full of it. These structural weakness planes seriously hinder and affect the propagation of stress waves in rock mass. Thus to investigate the characteristics of both reflection and refraction of stress waves through structural weakness planes is of great interest in several fields , such as rock blasting, anti-seismic, and geophysical prospecting[2, 3, 4]. In this aspect some reseachers have done much work. Howerer, in those works the boundary conditions were adequately modeled by the perfectly bonded assumptions[3–5]. In fact, the cohesive force of preexisting weakness in rock mass such as faults and joints is very small, generally less than 0.1 MPa, and even some of the weakness planes are free of it[6–7]. When the compressive stress waves generated by an explosion[8] are obliquely incident to the structural weakness plane it is possible that the tangential components of the stress waves incident to it overcome its cohesive force and result in the relative slipping of rocks on both sides of the plane. Up to now, much less attention has been paid to the situation in which the appropriate boundary conditions involve friction energy dissipation and relative slipping. It is the purpose of this paper to obtain the

①This project was sponsored by the National Natural Science Foundation under Grant No. 58974150
②Dr. ③Posfessor

reflection, refraction coefficients and energy dissipation of the compressive stress waves in the above situation.

2 REFLECTION AND REFRACTION WITH FRICTIONAL SLIPPING

2.1 *Exact Coefficients of Both Reflection and Refraction*

We consider a structural weakness plane at its face z = 0 as shown in Fig. 1. The parameters of rocks on both sides of it are ρ, C_p, C_s, and ρ_1, C_{p1}, C_{s1}, where ρ, C_p, and C_s denote density, longitudinal wave speed, and shear wave speed respectively. Consider the plane strain problem of a harmonic plane compressive wave. Let Φ, Ψ be a longitudinal compressive wave potential and a shear wave potential, which are respectively incident on the plane z = 0 at the angle θ and γ measured from the normal to the interface (see Fig.1). Associated with those waves are a set of reflected longitudinal and shear waves with potentials Φ' and Ψ', a set of longitudinal and shear waves that are transmitted into the media of $z<0$ with potentials Φ_1'', Ψ_1''. Let W_l, W_t, V_{ll}, V_{lt} represent the four unknown ratios of potentials respectively: Φ_1''/Φ'' the longitudinal transmission coefficient, Ψ_1''/Φ'', the shear transmission coefficient, Φ'/Φ'', the longitudinal reflection coefficient, and Ψ'/Φ'', the shear reflection coefficient. Adopting a notation similar to Brek- hovskikh's[10] we represent the waves in $z \geqslant 0$ as follows:

$$\Phi = (\Phi' e^{j\alpha z} + \Phi'' e^{-j\alpha z}) \cdot e^{j(\xi x - \omega t)} \tag{1}$$

$$\Psi = (\Psi' e^{j\beta z} + \Psi'' e^{-j\beta z}) \cdot e^{j(\xi x - \omega t)} \tag{2}$$

In the rock mass of $z \leqslant 0$, because there are not reflected waves, the waves can be represented as:

$$\Phi_1 = \Phi_1'' e^{-j\alpha_1 z} e^{j(\xi x - \omega t)} \tag{3}$$

$$\Psi_1 = \Psi_1'' e^{-j\beta_1 z} e^{j(\xi x - \omega t)} \tag{4}$$

where $\alpha = K_l \cos\theta$, $\beta = K_T \cos\gamma$,

$\alpha_1 = K_{l1} \cos\theta_1$, $\beta_1 = K_{T1} \cos\gamma_1$,

$\xi = K_l \sin\theta = K_T \sin\gamma = K_{l1} \cos\theta_1$

$= K_{T1} \cos\gamma_1$

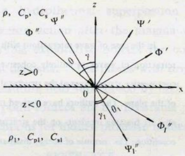

Fig.1 Schematic diagram of the refraction and reflection of waves at a structural weakness plane

Thus, K_l, K_{l1} represent longitudinal wave numbers for wave vectors making acute angles θ, θ_1 with the normal to the interface, and K_T, K_{T1} represent shear wave numbers for shear waves with the angles γ, γ_1.

In rock mass, the relationships among displacements, stresses, and wave potentials can be represented through the following equations.

$$\left.\begin{aligned} U_x &= \frac{\partial\Phi}{\partial x} - \frac{\partial\Psi}{\partial z} \\ U_z &= \frac{\partial\Phi}{\partial z} + \frac{\partial\Psi}{\partial x} \\ U_y &= 0 \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} \sigma_z &= \lambda\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_z}{\partial z}\right) + 2G\frac{\partial U_z}{\partial z} \\ \tau_{zx} &= G\left(\frac{\partial u_x}{\partial z} + \frac{\partial U_z}{\partial x}\right) \end{aligned}\right\} \tag{6}$$

where λ, G are the Lame constants, and they are related to the elastic wave speeds by

$$\rho C_p^{\ 2} = \lambda + 2G, \qquad \rho C_s^{\ 2} = G$$

The below boundary conditions at the frictionally bonded structural weakness plane permitting relative slipping of rocks require that

$$U_z(x,o,t) = U_{z1}(x,o,t) \quad (7)$$

$$\sigma_z(x,o,t) = \sigma_{z1}(x,o,t) \quad (8)$$

$$\tau_{zx}(x,o,t) = \tau_{zx1}(x,o,t) \quad (9)$$

$$\tau_{zx}(x,o,t) = -\sigma_z(x,o,t)\mathrm{tg}\varphi \quad (10)$$

where φ is the frictional angle of the plane

In view of the above equations, we can obtain the following equations

$$\alpha(\Phi' - \Phi'') + \xi(\Psi' + \Psi'') = -\alpha_1\Phi_1'' + \xi\Psi_1'' \quad (11)$$

$$G[\beta(\Psi' - \Psi'') + p(\Phi' + \Phi'') = G_1(-\beta_1\Psi_1'' - p_1\Phi_1'') \quad (12)$$

$$G[\alpha(\Phi' - \Phi'') - p(\Psi' + \Psi'')] = G_1(-\alpha_1\Phi_1'' + p_1\Psi_1'') \quad (13)$$

$$(-\alpha\Phi_1'' + \rho\Psi_1'') = (\beta_1\Psi_1'' + p_1\Phi_1'')\mathrm{tg}\Phi \quad (14)$$

Where

$$p = (\xi^2 - K_T^2/2)\xi^{-1} = -K_T\cos 2\gamma / 2\sin\gamma$$

While we only consider a compressive stress wave incident on the plane, Ψ'' is equal to zero, and the equations (11)~ (14) are simplified and rewritten as

$$\alpha(V_{ll} - 1) + \xi V_{lt} = -\alpha_1 W_1 + \xi W_t \quad (15)$$

$$-p(1 + V_{ll}) + \beta V_{lt} = -(G_1/G) \times (\beta_1 W_t + p_1 W_l) \quad (16)$$

$$\alpha(V_{ll} - 1) + pV_{lt} = (G_1/G) \times (-\alpha_1 W_l + p_1 W_t) \quad (17)$$

$$(p_1\mathrm{tg}\varphi + \alpha_1)W_l + (\beta_1\mathrm{tg}\varphi - p_1)W_t = 0 \quad (18)$$

According to equation(18), W_l, the longitudinal transmission coefficient, can be expressed as

$$W_l = [(p_1 - \beta_1\mathrm{tg}\varphi)]W_t / (p_1\mathrm{tg}\varphi + \alpha_1) = m_1 W_t \quad (19)$$

Equations (17) and (15) yield,

$$V_{lt} = [\alpha_1(1 - G_1/G)m_1 + (p_1 G_1/G - \xi)] \cdot W_t / (p - \xi) = n_1 W_t \quad (20)$$

$$V_{ll} = (-\alpha_1 m_1 + \xi - \xi n_1)W_t / \alpha + 1 \quad (21)$$

After substitution of these expressions into equation (16) we obtain for the shear transmission coefficient, W_t

$$W_t = 2p / \left[G_1(\beta_1 + p_1 m_1)/G + \beta n_1 + p\alpha_1 m_1/\alpha + p\xi(n_1 - 1)/\alpha\right] \quad (22)$$

2.2 *Numerical Analytic Example*

From the expressions (19)~(22) we know that W_l, W_t, V_{ll}, V_{lt}; the reflection and refraction coefficients, depend on the incident angle of compressive waves, the specific acoustic resistance of rocks, the shear wave speed in rocks, and the frictional angle of the structural weakness plane. We have written a computational program to calculate the reflection and refraction coefficients in line with their expressions. The results of numerical analyses are shown in Fig. 2~6 in which the four ratios of wave potentials: W_l, W_t, V_{ll}, V_{lt}, are plotted as a function of the incident angle for different frictional angles and for given rock parameters respectively.

2.3 *Limiting Cases*

We consider the case for which rocks on both sides of the structural weakness plane become the same. This limit is obtained for equations (19)~(22) by setting

$$\rho_1/\rho = 1 \qquad C_{p1}/C_p = 1 \qquad C_{s1}/C_s = 1$$

so that equations (19)~(22) become

$$V_{ll} = \frac{(C_s/C_p)^2\dfrac{\sin 2\theta}{\cos 2\gamma} - \mathrm{tg}\varphi}{\mathrm{ctg}2\gamma + (C_s/C_p)^2\dfrac{\sin 2\theta}{\cos 2\gamma}} \quad (23)$$

$$W_t = \frac{tg\varphi - (C_s / C_p)^2 \frac{\sin2\theta}{\cos2\gamma}}{1 + tg2\gamma \frac{\sin2\theta}{\cos2\gamma}(C_s / C_p)^2} = V_{lt} \quad (24)$$

$$W_l = \frac{ctg2\gamma + tg\varphi}{ctg2\gamma + (C_s / C_p)^2 \frac{\sin2\theta}{\cos2\gamma}} \quad (25)$$

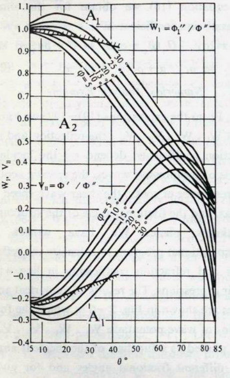

Fig.2 The ratios of the refracted, reflected longitudinal wave potentials to the incident compression wave potentials; W_l, V_{ll} for the case of $\rho_1 / \rho = 0.8$, $Cp_1 / Cp = 0.8$, $C_{s1} / Cp_1 = C_s / Cp = 0.6$

(A_1—region of stick; A_2—region of slipping)

In the further limit in which rocks on both sides of the structural weakness plane can slip freely along it and it is frictionless, we obtain from equations (23)~(25) by setting $tg\varphi = 0$

$$V_{ll} = (\sin2\theta \cdot \sin2\gamma) / (C_p / C_s)^2 \times$$
$$\cos^2 2\gamma + \sin2\theta \sin2\gamma \quad (26)$$

$$W_t = [-(C_p / C_s)\sin2\theta\cos2\gamma] / (C_p / C_s)^2$$
$$\cos^2 2\gamma + \sin2\theta \cdot \sin2\gamma = V_{lt} \quad (27)$$

$$W_l = (C_p / C_s)^2 \cos2\gamma / [(C_p / C_s)^2$$
$$\cos2\gamma + \sin2\theta \cdot \sin2\gamma] \quad (28)$$

The equations (26)~(28) are the same as the results derived in Reference [9]

On the other hand, the relations between the reflection, refraction coefficients of stresses and those of potentials can be expressed as

$$\sigma_t / \sigma_i = W_l \cdot \rho_1 / \rho \quad (29)$$

$$\sigma_r / \sigma_i = V_{ll} \quad (30)$$

$$\tau_t / \sigma_i = W_t \cdot \rho_1 / \rho \quad (31)$$

$$\tau_r / \sigma_i = V_{lt} \quad (32)$$

Thus, after the ratios of potentials have been obtained we can easily know the reflection and refraction coefficients of stress.

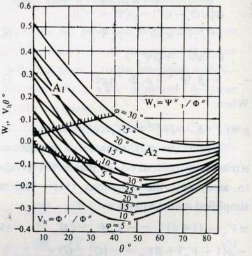

Fig.3 The rations of the refracted, reflected transverse wave potentials to the incident compressive wave potential: W_t, V_{lt} for the case of $\rho_1 / \rho = 0.8$, $Cp_1 / Cp = 0.8$, $C_{s1} / Cp_1 = C_s / Cp = 0.6$

3 THE CRITERION OF SLIPPING AND FRICTIONAL ENERGY DISSIPATION

The transformations , reflections, and

refractions occur at boundaries because of changes in physical properties occurring across them, it is impossible for the materials on both sides of an entirely cohesive boundary to slip when a compressive stress wave arrives at it. Part of the energy will be transmitted and part will be reflected. Energy does not dissipate at the boundary . The energy flow of the incidence will be equal to the total reflected energy flow plus the total refracted energy flow in a certain time. For the structural weakness planes in rock mass permitting relative slipping, there are two possibilities as follows when a compression stress wave is obliquely incident on it.

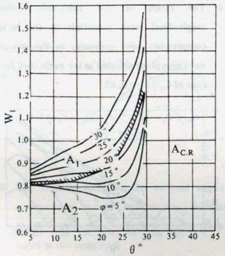

Fig.4 The ratios of the refracted, reflected longitudinal wave potentials to the incident compression wave potential W_I, V_{II}, for the case of $\rho_1 / \rho = 2$, $C_{p1} / C_p = 2$, $C_{s1} / C_{p1} = C_s / C_p = 0.6$

(A_{CR}—region of total reflection)

(1) $\tau(x, o, t) < \sigma(x, o, t) \cdot tg\varphi$. For this case, rocks on both sides of the structural weakness planes can not slip relatively and there is no energy occurred dissipation at the plane. Consequently, its reflection and refraction waves correspond to the results of waves through a perfectly bonded boundary.

(2) $\tau(x, o, t) > \sigma(x, o, t) \cdot tg\varphi$. In this case, rocks on both sides of the plane will slip relatively, and a part of the energy is dissipated at the plane. So the reflection and refraction of waves at the plane accompany the energy dissipation. On this basis, we conclude that the criterion for judging if rocks on both sides of the structural weakness plane slip relatively is whether there is energy absorption at the plane. Let E_{pi} be the incident energy flow of a stress wave entering into the structural weakness plane of unit area,

$$E_{pi} = \frac{1}{2} \rho \Phi''^2 \omega^2 / C_p \qquad (33)$$

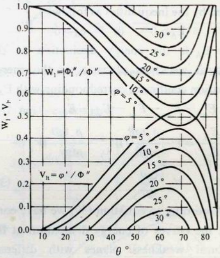

Fig. 5 The ratios of the refracted, reflected longitudinal wave potentials to the incident compression wave potential at the structural weakness plane situated in the same rock for the case of $C_s / C_p = 0.6$

Correspondingly , the refracted longitudinal energy flow, the refracted transverse energy flow, the reflected longitudinal energy flow, the reflected transverse energy flow, and their ratios to the incident energy flow, are respectively

$$\left.\begin{aligned} E_{pt} &= \frac{1}{2}\rho_1 \Phi_1''^2 \frac{\omega^2 \cos\theta_1}{C_{p1}\cos\theta} \\ e_{pt} &= \frac{\rho_1 \mathrm{tg}\theta}{\rho \mathrm{tg}\theta_1} W_l^2 \end{aligned}\right\} \quad (34)$$

$$\left.\begin{aligned} E_{st} &= \frac{1}{2}\rho_1 \Psi_1''^2 \frac{\omega^2 \cos\gamma_1}{C_{s1}\cos\theta} \\ e_{st} &= \frac{\rho_1 \mathrm{tg}\theta}{\rho \mathrm{tg}\gamma_1} W_t^2 \end{aligned}\right\} \quad (35)$$

$$E_{pr} = \frac{1}{2}\rho\Phi'^2 \omega^2 / C_p, \quad e_{pr} = V_{ll}^2 \quad (36)$$

$$\left.\begin{aligned} E_{sr} &= \frac{1}{2}\rho\Psi'^2 \frac{\omega^2 \cos\gamma}{C_s \cos\theta} \\ e_{sr} &= \frac{\mathrm{tg}\theta}{\mathrm{tg}\gamma} \cdot V_{lt}^2 \end{aligned}\right\} \quad (37)$$

The criterion of relative slippage can be expressed by the inequalities as follows

$$E_A > 0 \quad \text{or} \quad e_A > 0$$

where E_A is the energy dissipation at the structural weakness plane, e_A is the energy dissipation coefficient corredponding with E_A.

$$E_A = E_{pi} - E_{pt} - E_{st} - E_{pr} - E_{sr} \quad (38)$$

$$e_A = \frac{E_A}{E_{pi}} = 1 - \frac{\rho_1 \mathrm{tg}\theta}{\rho \mathrm{tg}\theta_1} W_l^2 - \frac{\rho_1 \mathrm{tg}\theta}{\rho \mathrm{tg}\gamma_1} W_t^2 -$$

$$V_{ll}^2 - V_{lt}^2 \frac{\mathrm{tg}\theta}{\mathrm{tg}\gamma} \quad (39)$$

According to equation (39) we have computed the curves of energy dissipation at the structural weakness planes with different frictional angles. In Fig. 7 the rock parameters are $\rho_1/\rho = 0.8$, $C_{p1}/C_p = 0.8$, and $C_s/C_p = C_{s1}/C_{p1} = 0.6$. Fig. 8 is corresponding to the case for which rocks on both sides of the structural weakness plane become the same and their ratio of shear wave speed to longitudinal wave speed is equal to 0. 6. Meanwhile, we have also determined a range of incident angles resulting in relative slippage (see Fig. 9—Fig. 10).

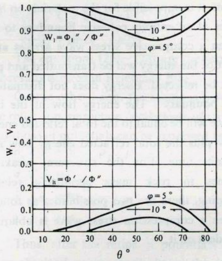

Fig. 6 The ratios of the refracted, reflected longitudinal wave potentials to the incident compression wave potential at the structural weakness plane situated in the same rock for the case of $C_s/C_p = 0.45$

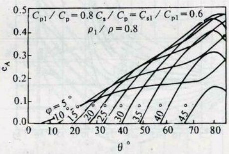

Fig.7 The relations between the incident angles of compression waves and energy dissipation coefficients at the structural weakness plane with different frictional angles

4. CONCLUSIONS AND DISCUSSION

From the above analyses and our computations, we have reached the follwing conclusions.

(1) The reflection, refraction coefficients,

and energy dissipation values of compression waves at various structural weakness planes with frictional slipping can be obtained according to the corresponding equations derived in this paper and the computational program. Then, we can further determine the range of the incident angles $[\theta_1, \theta_2]$ resulting in relative slippage of rocks on both sides of the structural weakness plane, on the conditions of different rock paramters and structural plane features in the light of the criterion of frictional slipping. For the case for which the incident angles are less than θ_1 or larger than θ_2, the reflection, and refraction of waves at the structural weaknesss planes must be recomputed on the basis of the perfectly bonded boundary conditions.

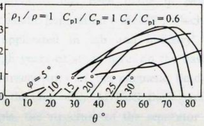

Fig.8 The relations between the incident angles of compression waves and energy dissipation coefficients at the structural weakness plane situated in the same rock

(2) While the regions of incident angles is $\theta<\theta_1$ or $\theta>\theta_2$, a structural weakness plane in rock mass is regarded as a perfectly bonded boundary because rocks on both sides of it do not slip relatively. In the limit which rocks on both sides of a structural weakness plane become the same the reflection will not occur when a compression wave arrives at it. The propagation of compression waves through the structural weakness plane is similar to that in a homogenous media.

(3) It is shown in Fig. 9~10 that the range of incident angles at which rocks on both sides of a structural weakness plane slip relatively depends on the friction angle of the plane and rock parameters.As the angle of friction is increased the range of the incident angles resulting in slipping diminishs until it vanishes entirely at a critical friction angle: φ_c

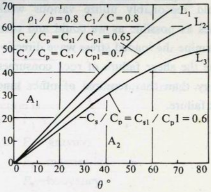

Fig9 The critical incident angles resulting in the relative slipping of rocks on both sides of the structural weakness plane

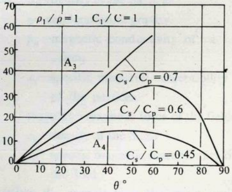

Fig.10 The critical incident angles resulting in the relative slipping of rocks on both sides of the structural weakness plane situated in the same rock

(4) So far as both the anti-seismic and

shockproof in soil and rock engineering are concerned, we must carefully investigate the feature of some large-scale weakness planes in rock mass and various vibration sources, and then take measures to reinforce the weakness planes along which rocks may slip or to eliminate various dangerous vibration sources. In rock blasting and fragmentation, on the other hand, in order to improve the efficiency of rock fragmentation and spare energy we should reasonably utilize various weakness planes as possible as we could and correctly determine the loaded stress wave direction because the shear failure of rock consumes less energy than that required of other kinds of rock failure.

REFERENCES

1 Liu, Dianzhong. Blasting Processing of Rock and Ore, Beijing: Metallurgical Industry Press, 1980 (in Chinese)
2 Holmbery,Roger and Rustan,Agne. Rock Fragmentation by Blasting, TECE–tryck AB, Lulea, Sweden, 1983
3 Tooley,R.D..Geophysics, 1965: 30: 552
4 Koefoed,O., et al.. Geophysics. 1980: 45(8): 1254
5 Wang, Wenlong.Drilling and Blasting, Beijing: Coal Industry Press, 1984 (In Chinese)
6 Li, Zhonglin. Geomechanis in Mine, Beijing: Metallurgical Industry Press, 1987 (in Chinese)
7 Augenheister, G..L · B · V / Ib, Springer–Verlay Bertin, 1982
8 Starfield,A.M. ,Pugliese,J. M.. Int. J. Rock Mech. Min. Sci., 1968,5(1) 65
9 Rinehart,J.S..Stress Transients in Solids, Hyper Dynamics, Santafe, New Mexico, 1975
10 Brekhovskikh,L.M. Maksimocich,L.. Waves in layered Media, 2d ed., New York: Academic Press, 1980