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# Constitutive description for casting magnesium alloy involving void evolution

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**Abstract:** In order to investigate the effect of microvoids on the mechanical behavior of casting magnesium alloy, a spherical void−cell model of the material was presented. The velocity and strain fields of the model were obtained from the assumption that the material matrix is homogeneous and incompressible. The hardening and softening functions, which respectively reflect the deformation-hardening and void-softening behaviors of the material, were presented and introduced to an endochronic constitutive equation for describing the mechanical behavior of the material including microvoids. The corresponding numerical algorithm and finite element procedure were developed and applied to the analyses of the elastoplastic response and the porosity of casting magnesium alloy ZL102. The computed results show satisfactory agreement with experimental data.

**Key words:** casting magnesium alloy; spherical void; void volume fraction; elastoplasticity; constitutive model

## 1 Introduction

Due to lightweight, being easily shaped and recycled, casting magnesium alloys are predestined for light-weight constructions of components in automotive industry, for example, steering wheels, door structures and oil sumps[1−2]. However, because the solidified casting magnesium alloys will contract, they inevitably contain a certain amount of microscopic voids. As the microscopic voids grow under loading, the walls or ligaments between the voids thin down and ductile fracture may occur due to the coalescence of the voids[3−4]. The investigations on the evolution of the voids and its effect on the mechanical properties of the material are significant to make more efficient use of the materials. Great effort has been made in the analyses of the void evolution and the corresponding mechanical properties of materials[3−7]. It has been found that the microvoids usually distribute randomly in materials, and void growth plays an important role in material properties[7−9]. A feasible method used frequently for the investigation on the void evolution and the material properties involving void evolution is based on the analysis of a representative void−cell model of materials. In this aspect, GURSON[9] made a pioneering contribution. He assumed that the void−matrix aggregate of a ductile material including microvoids could be represented with circular-cylindrical or spherical void−cell models. With the models the constitutive relationship of materials was obtained. TVERGAARD and NEEDLEMAN[10] extended Gurson model to account for the final failure of materials at a realistic value of void volume fraction. FAN et al[11] put forward a formulation of constitutive relations to describe the mechanical behavior of void-damaged materials based on a finite element analysis. SIRUGUET and LEBLOND[12] modeled the effect of the inclusions on the void growth in a porous ductile metal. BAASER and GROSS[13] analyzed the growth of the microvoids in a crack tip of a ductile material loaded by a remote $K_I$ field. HORSTEMEYER et al[14] investigated the internal-state-variable rate equations in continuum framework to model void nucleation, growth, and coalescence in a cast Al-Si-Mg alloy. In this work, the mechanical behavior of a casting magnesium alloy containing numerous microvoids was investigated through the method combining microscopic and macroscopic analyses. Firstly, a spherical void−cell model of the material was presented. Based on the analysis on the model, the

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deformation-hardening and void-softening functions were given. Then, they were introduced to an endochronic elastoplastic constitutive equation in order to obtain the constitutive description involving void evolution of the material. Lastly, the corresponding finite element procedure was developed. It was applied to investigate the elastoplastic behavior and the porosity of casting magnesium alloy ZL102.

## 2 Constitutive description and void evolution equation

Fig.1 shows a representative volumetric element (RVE) of a casting magnesium alloy material, in which many microvoids are included. $V_s$ is solid volume of the element; $V_v$ is the void volume and $V$ is the total volume. Furthermore, the RVE can be simplified as a spherical void−cell model as shown in Fig.2[9]. The radii of the void and the cell are $a$ and $b$, respectively. The radius of an arbitrary point in the matrix of the void−cell model is $r$. Suppose the matrix of the model is homogeneous and incompressible. The global axis 3 coincides with the local spherical axis and the deformation is axisymmetrical. In the case, there are following relationship for the microscopic strain field $\dot{E}_{ij}$ .

$$\dot{E}_{11}=\dot{E}_{22},\ \dot{E}_{ij}=0,(i\neq j) \tag{1}$$

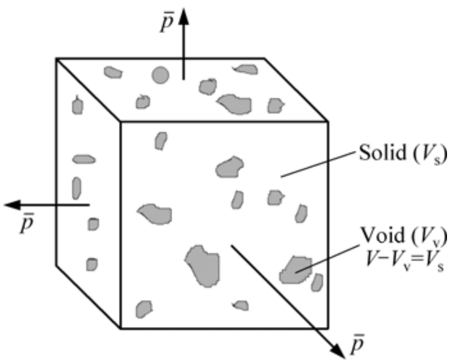


**Fig.1** Material element with voids

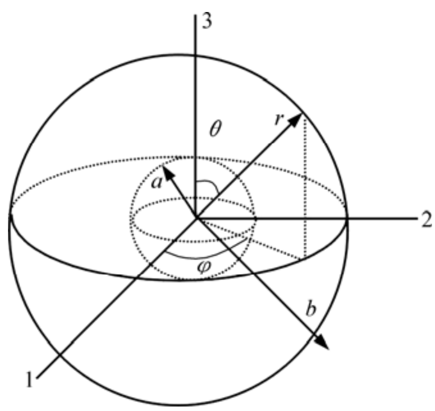


**Fig.2** Spherical void-cell model

Then the boundary velocity field can be obtained in a spherical coordinate system:

$$v_r=b(\dot{E}_{11}\sin^2\theta+\dot{E}_{33}\cos^2\theta),$$

$$v_\theta=b(\dot{E}_{11}-\dot{E}_{33})\sin\theta\sin\varphi,\ v_\varphi=0 \tag{2}$$

The corresponding microscopic strain-rate field $\dot{\varepsilon}_{ij}$ in the matrix can be expressed as

$$\dot{\varepsilon}_r=\frac{\partial v_r}{\partial r},\ \dot{\varepsilon}_\theta=\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r}{r},$$

$$\dot{\varepsilon}_\varphi=\frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}+\frac{v_\theta}{r}ctg\theta+\frac{v_r}{r},$$

$$\dot{\varepsilon}_{r\theta}=\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r}+\frac{1}{r}\frac{\partial v_r}{\partial\theta} \tag{3}$$

Furthermore, the microscopic intrinsic time measure is denoted by $\zeta$, which is defined as the Euclidean norm of the deviatoric increment of microscopic strain[15]. Due to the assumption of incompressibility of the matrix, one has

$$\mathrm{d}\zeta=(\mathrm{d}\varepsilon'_{ij}\,\mathrm{d}\varepsilon'_{ij})^{1/2}=(\mathrm{d}\varepsilon_{ij}\mathrm{d}\varepsilon_{ij})^{1/2} \tag{4}$$

where $\mathrm{d}\varepsilon'_{ij}$ is the increment of microscopic deviatoric strain. The intrinsic time scale can be defined as

$$\mathrm{d}z=\mathrm{d}\zeta/[F(\zeta)\eta(f)] \tag{5}$$

where $F(\zeta)$ is the hardening function which reflects the hardening behavior of the material subjected to plastic deformation; $\eta(f)$ is the softening function which reflects the softening behavior of the material due to void growth during plastic deformation; $f$ is current void volume fraction of the material. For simplicity, $F(\zeta)$ and $\eta(f)$ are given the following simple forms without considering the strain-rate effect:

$$F(\zeta)=1+\beta_1\zeta^{\gamma_1},\ \eta(f)=1+\beta_2 f^{\gamma_2} \tag{6}$$

where $\beta_1$, $\beta_2$, $\gamma_1$ and $\gamma_2$ are material constants, which can be determined from the curve of a uniaxial experiment. Substituting Eq.(5) into the incremental form of endochronic elastoplastic constitutive equation[16], which can describe the constitutive behavior of materials without a yield surface with favorable precision, yields the following incremental constitutive equation involving void evolution:

$$\Delta s_{ij}^{(r)}=s_{ij}^{(r)}-s_{ij}^{(r)}(z_n)=$$
$$k_r\sum_{r=1}^{3}[C_r\Delta e^{\mathrm{p}}-\alpha_r s_{ij}^{r}(z_n)\Delta\zeta/F(\zeta)\eta(f)] \tag{7}$$

where

$$\Delta e_{ij}^{\mathrm{p}}=\mathrm{d}e_{ij}-\mathrm{d}s_{ij}/2G,\ k_r=\frac{1-\exp(-\alpha_r\Delta z)}{\alpha_r\Delta z} \tag{8}$$

$s_{ij}$ denotes the microscopic deviatoric stress; $z_n$ is the intrinsic time scale after $n$th increments of loading and $s_{ij}^r(z_n)$ represents $r$th component of $s_{ij}$ at $z_n$; $C_r$ and $\alpha_r$ ($r$=1, 2, 3) are material constants, and $G$ is the elastic shearing modulus. By setting

$$A=\sum_{r=1}^{3}k_rC_r,\ B_{ij}=\sum_{r=1}^{3}-k_r\alpha_r s_{ij}^{(r)}(z_n) \tag{9}$$

one can derive the following expression of the incremental form of the endochronic constitutive equation involving void evolution:

$$\Delta s_{ij}=A\Delta e_{ij}^{\mathrm{p}}+B_{ij}\Delta z \tag{10}$$

If the effect of the voids is not considered ($f$=0, $\eta(f)$=1), one can prove that Eq.(10) reduces to the incremental form of the constitutive equation given by PENG and FAN[16]. Moreover, the Chaboche's constitutive law for back stress can also be obtained as a special case when $\eta(f)$=1 and $F(\xi)$ is constant.

Assuming homogeneous matrix, the homogenization principle[9] can be used in the transition between microscopic and macroscopic quantities. Letting $\Phi$ and $\phi$ be the macroscopic and the microscopic potential functions, respectively, the macroscopic stresses can be expressed as[9]

$$\sum_{ij}=\frac{\partial\Phi}{\partial E_{ij}}=\frac{1}{V}\int_{V_{\mathrm{m}}}\frac{\partial\varphi}{\partial E_{ij}}\mathrm{d}V=\frac{1}{V}\int_{V_{\mathrm{m}}}s_{kl}\frac{\partial\varepsilon_{kl}}{\partial E_{ij}}\mathrm{d}V \tag{11}$$

Substituting Eqs.(3) and (10) into Eq.(11), the macroscopic constitutive equation of casting magnesium alloy can be obtained.

The void volume fraction may change during material deformation, which is contributed by both the growth of existing voids and the nucleation of new voids, i.e.,

$$\dot f=\dot f_{\mathrm{growth}}+\dot f_{\mathrm{nucleation}} \tag{12}$$

Keeping in mind that the matrix is incompressible, the increment of the void volume fraction due to the growth of void can be given by

$$\dot f_{\mathrm{growth}}=\frac{\dot V_v}{V}=(1-f)\dot\varepsilon_{kk} \tag{13}$$

The new voids, nucleated either by cracking of the particles or by decohesion of the particle/matrix interface, can be described with the following equation

$$\dot f_{\mathrm{nucleation}}=\varLambda\dot\zeta \tag{14}$$

where $\varLambda$ follows a normal distribution with mean value $\zeta_N$ and standard deviation $s_N$. $\varLambda$ is given as

$$\varLambda=\frac{f_N}{s_N\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\zeta-\zeta_N}{s_N}\right)^2\right] \tag{15}$$

where $s_N$=0.1; $f_N$=0.4; $\zeta_N$=0.2. The addition of the void nucleation term would more fully reflect the effect of the void evolution on material behavior and improve the predictive ability of the constitutive relation.

## 3 Application and verification

The corresponding numerical algorithm and FE procedure were developed based on the presented constitutive equation. They were applied to the analyses of the relationship between the stress and strain as well as the porosity of the notched cylindrical specimen of casting magnesium alloy ZL102. The size of the specimen is shown in Fig.3. The upper right quarter of the specimen was taken for the analyses due to the symmetry of the problems (Fig.4). The eight-node isoparametric element with 2×2 Gaussian points was adopted. The axial displacement was imposed at the end of the specimen with the incremental step of 0.02 mm. Fig.5 shows the contours of the void volume fraction at different applied strains. It can be seen from Fig.5 that the voids firstly occur at the notch root where the strain is relatively large. The porosity takes its maximum in the region near to the notch root and decreases with the increase of the distance away from the notch root. Figs.6 and 7 show the distributions of the axial stress and the

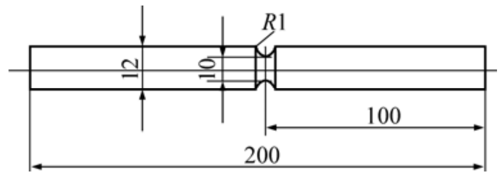


**Fig.3** Cylindrical tensile specimen with notch (Unit: mm)

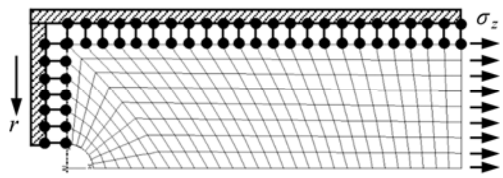


**Fig.4** Finite element mesh of specimen

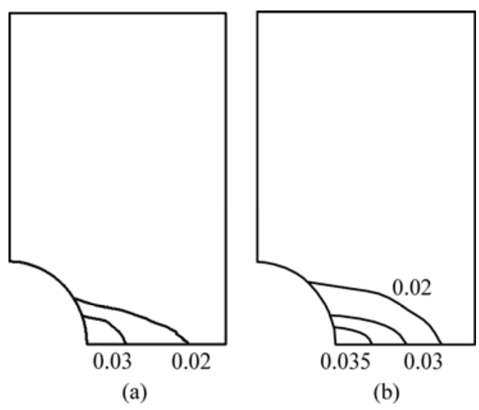


**Fig.5** Contours of void volume fraction: (a) $\varepsilon_{\mathrm{a}}$=0.014; (b) $\varepsilon_{\mathrm{a}}$=0.026

porosity along the notch line, where $d$ is the distance from the surface to the center of the smallest cross section of the specimen. Fig.6 shows that the maximum axial stress arises at the notch root, but the maximum stress considering voids is larger than that without considering voids. Fig.7 shows that the maximum porosity also appears at the notch root of the specimen and agrees well with the experimental results.

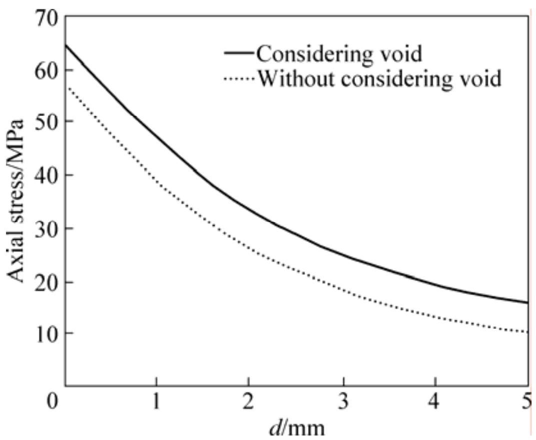


**Fig.6** Distributions of axial stresses along notch line

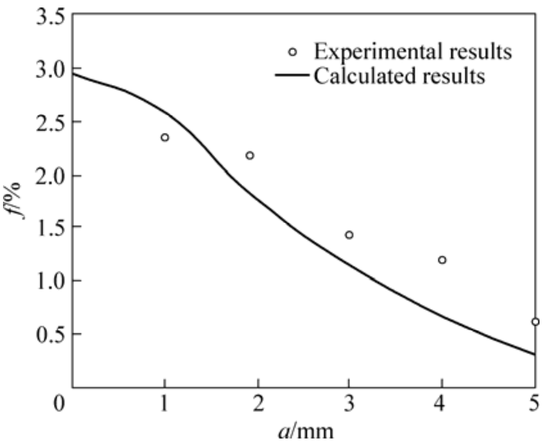


**Fig.7** Distribution of void volume fraction along notch line

## 4 Conclusions

1) The representative volumetric element of the casting magnesium alloy material including many microvoids is simplified as a spherical void−cell model. The microscopic velocity and strain fields of the model are given.

2) The intrinsic time scale of the material that can reflect the deformation-hardening and the void−softening behaviors of the material is presented.

3) The presented intrinsic time is introduced to endochronic elastoplastic constitutive equation, and the constitutive description of casting magnesium alloy considering void evolution is obtained.

4) The corresponding numerical algorithm and FE procedure are developed. They are applied to the analyses of the relationship between the stress and strain as well as the porosity of the notched cylindrical specimen of casting magnesium alloy ZL102. The analytical results are in agreement with experimental data.

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