NONLINEAR PROGRAMMING PROBLEMS IN MINE VENTILATION NETWORKS AND THEIR SOLUTIONS①

Xie, Xian Ping Zhao, Zicheng
Kunming Institute of Technology, Kunming 650093, China

ABSTRACT

Converting the balance equation of the branch of a mine ventilation network into an equivalent nonlinear programming problem, this paper proves that the total sum of the energy loss in every branch will be a minimum when the airflow distribution in the networks is in a balanced state. The energy means of solving the network equations by nodal methods is also noted, and a theorem for the unique existence of the solution for a network balance equation is give. An example is used to explain these conclusions.

Key words: *nonlinear programming mine ventilation energy airflow Distribution*

1 ANALYSIS OF ENERGY IN AN AIRFLOW DISTRIBUTION NETWORK

Let us consider a network including f fans. The airflow of every fan in the network is Q_{fi}, and the pressure H_{fi}. Then the total sum of energy which goes into the network from all the fans is

$$E = \sum_{i=1}^{f} Q_{fi} H_{fi} \tag{1}$$

If the discharge in branch i is Q_i, the head loss h_i, the discharge at node j is q_j (i.e. the airflow leakage at node j), the pressure at this node H_j, then the total energy E_1 which lost due to branch resistances in the airflow distribution network and the total energy E_2 which is transfered into the nodes can be respectively expressed as

$$E_1 = \sum_{i=1}^{N} Q_i H_i \tag{2}$$

$$E_2 = \sum_{j=1}^{J} Q_j H_j \tag{3}$$

where N is the number of branches and J is the number of nodes.

When the airflow distribution is in a balanced state in the branch network, according to the law of conservation of energy, we get

$$E = E_1 + E_2 \tag{4}$$

From the above equation, we can see that the energy E can be divided into two parts. One part is the resistance and frictional losses. The other part consists of losses due to tunnel fractures, vacant places, caving zones, heading faces and so on.

The discharge method and the nodal method are usually used to solve the branch network balance equation[1]. In this paper, we convert the balance equation of the branch network into an equivalent nonlinear programming problem to prove that the total sum of the energy losses in every branch is a minimum when the airflow distribution in the network is in a balanced state. We also point out the use of the nodal method for solving the network equations and give a theorem for the unique existence of the solution for a network balance equation.

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2 NONLINEAR PROGRAMMING PROBLEMS IN THE AIRFLOW DISTRIBUTION NETWORK

2.1 *The Calculation of the Network Balance Equation by the Discharge Method*

For convenience, we only consider the case of a single fan. Let us choose a network consisting of N branches, J nodes, and M independent loops. Jth is considered to be the node of the fan and is taken as the reference node.

If the essential connected matrix of the branch network is B and the essential return matrix C, they can be expressed as

$B=[b_{ij}]_{(J-1)}\times N,\ C=[C_{ij}]_M\times N$

The symbols to represent the matrices of airflows, head losses and leakages are as follows:

$Q=[Q_1, Q_2, \cdots\cdots, Q_N]$,

$h=[h_1, h_2, \cdots\cdots, h_N]$,

$q=[q_1, q_2, \cdots\cdots, q_{J-1}]$

When a network is in a balanced state, and the flow direction of every branch is determined, the following equation[2] is obtained

$$BQ=q \tag{5.1}$$

$$Ch=0 \tag{5.2}$$

$$h_i=R_iQ_i^2 \tag{5.3}$$

where R_i is the resistance coefficient of branch i. The equalities (5) are the essential relations and can be used to solve the airflow distribution in the network by the discharge method. According to equality (5.1), the solution for the equation (5) is the branch discharge. If the relation expressing the equality between the equivalent loop discharge and branch discharge is used to replace equality (5.1), the solution for the equation (5) is the loop discharge. We can use the above methods to solve the branch equation of the branch network and the loop equation. The energy E_1 lost due to branch resistance is determined by the following equation:

$$E_1=\sum_{i=1}^{N} R_iQ_i^3 \tag{6}$$

The first law is that the solution of balance equation (5) of the branch network is equivalent to that of the following nonlinear programming problem:

$$\min G=\frac{1}{3}\sum_{i=1}^{N} R_iQ_i^3 \tag{7.1}$$

$$\text{S.T. } BQ=q \tag{7.2}$$

With respect to the objective function G of equation (7. 1), there are the following equations

$$\frac{\partial G}{\partial Q_i}=R_iQ_i^2=h_i \tag{8}$$

and

$$\frac{\partial^2 G}{\partial Q_i^2}=2R_iQ_i \tag{9}$$

The Hazen matrix of Q about G is

$$\nabla^2G(Q)=\mathrm{diag}[2R_1Q_1, 2R_2Q_2, \cdots, 2R_NQ_N] \tag{10}$$

It is seen that $\nabla^2G(Q)$ is a diagonal matrix. As mentioned, every diagonal element is then greater than zero. Therefore, it is a positive symmeric matrix. Also the control expressed by equality (7. 2) is linear, so equation (7) is a convex programming problem. If the equation (7) has a solution, the solution must be unique and can be expressed as

$$Y=[Y_1, Y_2, \cdots\cdots, Y_{J-1}]^T \tag{11.1}$$

$$g_i=\sum_{i=1}^{N} b_{ij}Q_{j-qi} \tag{11.2}$$

Let Q^* be the optimal solution. Because equation (7) is a convex programming problem, the Kuhn-Tucker condition holds at Q^*, i.e. there is Y^*, which makes the following equality tenable[3]:

$$\nabla G(Q^*)-\sum_{i=1}^{J-1} Y_i^*\ \nabla_{g_i}(Q_i^*)=0 \tag{12}$$

From equality (8), we can get

$$\nabla G(Q^*)=h^* \tag{12a}$$

From equality (11), we have

$$\nabla_{g_i}(Q)=[b_{i1}, b_{i2}, \cdots\cdots, b_{iN}]^T \tag{13}$$

$$\sum_{i=1}^{J-1} Y_i^*\ \nabla_{g_i}(Q_i^*)=B^TY^* \tag{14}$$

Putting equalities (14) and (12a) into equality (12), we can get

$$h^{*}=B^{T}Y^{*} \tag{15}$$

Equality (15) is the condition with the solution of equation (7) must satisfy, because the solution of the Kuhn–Tucker condition (12) is the optimal solution of equation (7). Multiplying by the return matrix on each side of the equality (15) we have

$$Ch^{*}=B^{T}Y^{*} \tag{16}$$

As we know, the relation matrix and the return matrix of a graph are orthogonal, i.e. $CB_T=0$[4]. From equality (16), we obtain

$$Ch^{*}=0 \tag{17}$$

Equality (17) shows that $h*$ satisfies the equation (5.2). Putting all the above mentioned relations together, the first law is proved. If Lagrange's function $L(Y, Q)$ is constructed by equality (7), i.e.

$$L(Y,Q)=\frac{1}{3}\sum_{i=1}^{N} R_i Q_i^3 + Y^T(BQ-q) \tag{18}$$

then

$$\nabla L(Q)=h-B^{T}Y \tag{19}$$

$$\nabla L(Y)=BQ-q \tag{20}$$

As mentioned above, the existence of Q^* and Y^* at the optimal solution of the equation (7) makes the equations (5.1) and (15) tenable, i.e. $\nabla L(Q^*)=0$; $\nabla L(Y^*)=0$. It is seen that Y^* is actually the Lagrange multiplier of equality (18).

Now let us assume that the pressure at every node is H_j, and can be expressed as

$$H=[H_1, H_2, \cdots, H_{J-1}]^T \tag{21}$$

When the branch network is in a balanced state, we get

$$h=B^{T}H \tag{22}$$

From equalities (15) and (22), we conclude that the Lagrange multiplier Y_J^* is the pressure H_j at node $j\,(j=1, 2, \cdots\cdots, J-1)$.

The second law is that, for a network including a single fan, the solution we get when using the discharge method to solve a network equation makes E_1 minimum. In other words, the balanced state of a network is a a state when E_1 is minimum. From the above discussion, the result is obviously tenable. What we must show is that the minimum is considered in the calculation,which amends the closed error in a loop and transforms the network from an unbalanced state to a balanced state.

Based on the above discussion, we conclude that there is some solution for balanced equation (5) of a branch network, the solution is unique.

2.2 *The Calculation of a Branch Network*

Assuming the symbol $N^{\#}$ as the number of the node, we have

$$H^{\#}=\{H_j | j\in N^{\#}\}$$

When we use the nodal method in the calculation of a branch network, the balance equation for the branch network can be expressed as

$$T_j=\sum_{i\in A_j} r_{ji}\,(H_j-H_i)^c + q_j=0 \quad {}_{j\in N^{\#}} \tag{23}$$

In the above, A_j is a set of symbols of nodes next to the node j, r_{ji} a parameter of branch j–i, c a constant, which can be decided by the hydraulic formula. Usually, c is obtained by the spot measurement or a model test.

The third law is that the solution of the balance equation branch network (23) is equivalent to that of the following uncontrolled, optimum problem:

$$\min W=\sum_{t=1}^{N} R_t (H_{t1}-H_{t2})^{1+c} + (1+c)\sum_{j=1}^{J} q_j H_j \tag{24}$$

where R_t is a parameter of the branch and has the same meaning as c, and t_1 and t_2 are the start-node and end–node symbols.

From equality (24), we get

$$\frac{\partial W}{\partial H_j}=(1+c)\Big(\sum_{i\in A_j} r_{ji}(H_j-H_i)^c+q_j\Big)$$

$$=(1+c)T_j \quad (j\in N^{\#}) \tag{25}$$

Because W is a continuous function of $H^{\#}$, at the place of the optimized solution of equality (24), there exist j, $N^{\#}$, and $c+1>0$. So at the place of the optimized solution of equality (24), it should be the case that

$$T_j = \sum_{i\in A_j} r_{ji}(H_j - H_i)^c + q_j = 0 \quad (26)$$
$$j\in N^{\#}$$

Equality (26) is a balance equation of the branch network (23). From equality (25), we can obatin

$$\nabla W(H^{\#}) = (1+C)J(H^{\#}) \quad (27)$$

in which, $J(H^{\#})$ is the Jacobi matrix about $H^{\#}$ of the branch network equation (23). It is a positive, definite symmetric matrix. Therefore, $\nabla^2 W(H^{\#})$ is also a positive, definite matrix and is unique. Obviously, it is also the unique solution of equality (23). Putting all the above relations together, the third law is proved. The objective function of equality (24) can be expressed as

$$W = (1+c)E_2 + E_1 \quad (28)$$

According to equality (28) and the above discussion, we have a forth law: when we use the nodal method in the calculation of the branch network, i.e. when we use the method to amend the closed loop discharge error at a node and make the network turn from an unbalanced state to a balanced state, the network in the balanced state makes $E_1+(1+c)E_2$ minimum. What we should say is that the minimum of $E_1+(1+c)E_2$ is considered in the whole calculation of the branch network by using the nodal method.

3 EXAMPLE

As an example, consider the network of Fig.1. The given data are listed in Table 1. The problem is to determine the natural splitting in every branch. This problem is as follows.

$$G = \frac{1}{3}\sum_{i=1}^{6} R_i Q_i^3 - (AQ_1 + \frac{1}{2}BQ_1^2 + \frac{1}{3}CQ_1^3) \quad (29)$$

$$Q_4 - Q_5 - Q_6 = 0 \quad (30.1)$$

$$Q_2 + Q_6 - Q_3 = 0 \quad (30.2)$$

$$Q_3 + Q_5 - Q_1 = 0 \quad (30.3)$$

By a linear conversion, the above solution is equivalent to the following uncontrolled optimum problem:

$$\min G = \frac{1}{3}[R_1Q_1^3 + R_2Q_2^3 + R_3Q_3^3 + R_4(Q_1-Q_2)^3 + R_5(Q_1-Q_3)^3 + R_6(Q_3-Q_2)^3] - (AQ_1 + \frac{1}{2}BQ_1^2 + \frac{1}{3}CQ_1^3) \quad (31)$$

The Newton-Raphson method is an iterative method for solving a general system of nonlinear equations. The computational results are summarized in Table 2.

Table. 1

Branch Number	1	2	3	4	5	6
Resistance $/ N\cdot S^2\cdot M^{-8}$	0.53	0.49	0.36	0.32	0.76	1.20

Operating discharge of the fan $Q_1 = 45.03\ M^3/S$
Type of the fan 4−72−11 No.20
Coefficient of H−Q curve of the fan $A=1920$, $B=7.92$, $C=-0.366$

Table.2

Branch Number	1	2	3	4	5	6
Discharge $/ m^3\cdot S^{-1}$	45.03	20.93	26.03	24.05	19.00	5.05

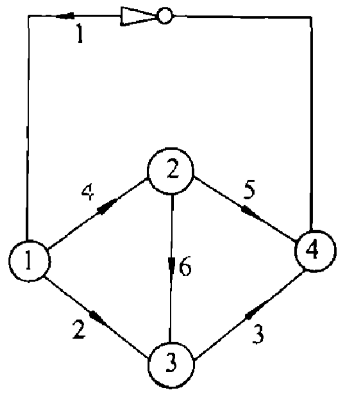

Fig. 1 Example Network

4 CONCLUSIONS

(1) When the discharge method is used in the calculation of a branch network, the solution of the balance equation will make E_1 minimum;

(2) When the nodal method is used in the

(*To be continued on page* № 96)

Key words: deep-drawing ratio, rectangular workpieces, formation limit.

Chang, Zhihua; Huang, Shangyu; Jiang, Kuihua (Wuhan Institute of Technology, Wuhan 430070, China). Trans NFsoc (Chinese edition), April 1993, 3(2): 44–47. ISSN 1004–0609.

61–0009 Integrated Treatment Process for the Utilization of Slags from Aluminum Smelting. The formiation of aluminum slags in aluminum smelting and aluminum casting was studied. A new economical and contamination-free process for the recovery of aluminum slags was proposed.

Key words: integrated treatment process, aluminum smelting, slags.

Wang Qiang, Kos B* (Jilin University of Technolog, Changchun 130025, China; * FOCON Casting Company, Austria). Trans NFsoc (Chinese edition), April 1993, 3(2): 90–92. ISSN 1004–0609.

83–0002 Computer Controlled Ultrasonic Concentration Meter of Liquid Ammonia. The computer controlled ultrasonic concentration meter for ammonia liquid consists of a personal computer, I / O interface, high speed time counter, spike pulse generater, ultrasonic wave generater, receiver amplifier, time delayer, gate trigger, voltage converter, order converter, thermometric circuit, AD converter, non- intrusive ultrasonic transducer and platinum resistor. According to the relationship between ultrasonic velocity, ammonia liquid concentration and temperature, the meter realized the on-line concentration measurement of ammonia liquid. 4 graphs, 1 table, 4 refs.

Key words: ultrasonic wave, velocity, ammonia liquid, concentration on-line measurement

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(Continued from page №91)

calculation of a branch network, the solution of balance equation makes $E_1+(1+c)E_2$ minimum;

(3) If the branch network equation has any solution, the solution must be unique. In this paper, we also convert the branch network (5) into an equivalent nonlinear programming problem expressed by equation (7). A gradient method can be used to solve this problem.If the balance equation of branch network (23) is turned into the equivalent optimized problem without any control, we can use the Newton-Raphson method to obtain a solution. Whether we can use other methods to obtain a solution for the uncontrolled, optimized problem in order to simplify the discussion will require further analysis.

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