ELASTO-PLASTIC STIFFNESS MATRIX OF ROCK MASS ELEMENT WITH SOFT CLAY STRATA AND ITS APPLICATION①

Li, Zhenzi Sun, Zongqi

Central-South University of Technology, Changsha, Hunan, 410083, China

ABSTRACT

The fault element is used to handle soft clay strata in a rock mass. The formulas of elasto-plastic stiffness matrix for the fault element are derived using the constitutive relationship between plastic increment stress and strain. A numerical example of a circular tunnel with soft clay strata in the rock medium are examined.

Key words: rock mass element fault elasto-plastic stiffness matrix

1 INTRODUCTION

Determining the elasto-plastic stiffness matrix for a rock mass with soft clay strata is an important problem for non-linear finite element calculation. In this paper, the fault element is used to handle soft clay strata in a rock mass. The formulas for the elasto-plastic stiffness matrix for the fault element using the constitutive relationship between plastic increment stress and strain. A numerical example of a circular tunnel with soft clay strata in a rock medium are examined. A numerical example showes that the existence of soft clay strata is unfavourable for the stability of the surrounding rock.

2 ELASTO-PLASTIC STIFFNESS MATRIX OF THE FAULT ELEMENT

Soft clay strata and surrounding rock are media of two different kinds. In Ref. [2], soft clay strata are handled using boundary elements. Soft clay strata is handled using finite elements in this paper.

The fault element is illustrated in Fig.1. The relationship between the plastic increment stress and strain are

$$\{d\sigma\} = [D_{ep}]\{d\varepsilon\} = ([D]-[D_p])\{d\varepsilon\} \tag{1}$$

and

$$[D_p] = \frac{[D]\{\frac{\partial F}{\partial \sigma}\}\{\frac{\partial F}{\partial \sigma}\}^T[D]}{A + \{\frac{\partial F}{\partial \sigma}\}^T[D]\{\frac{\partial F}{\partial \sigma}\}} \tag{2}$$

where $[D_p]$ is the plastic stiffness matrix of the fault element and is a function of stress and is related to the field function F; $[D]$ is elastic stiffness matrix

$$A = -\frac{1}{d\lambda}\frac{\partial F}{\partial H}dH \tag{3}$$

H—hardening parameter;

$d\lambda$—constant;

$A=0$ for the perfect plastic rock media

A unified expression of the yield criterions given in Ref. [2] is

$$F=\beta_1\sigma_m^2+\alpha_1\sigma_m-K+\sigma_0^n \tag{4}$$

where

$$\sigma_m=(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})/3$$

$$\sigma=\sqrt{I_2}/g(\theta_\sigma)$$

$$I_2=[(\sigma_{xx}-\sigma_{yy})^2+(\sigma_{yy}-\sigma_{zz})^2+(\sigma_{zz}-\sigma_{xx})^2+(\sigma_{xy}^2+\sigma_{yz}^2+\sigma_{zx}^2)]/6$$

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$g(\theta_\sigma)$—The functions of yield curves for θ_σ on π plane;

θ_σ— Lode angle in stress space. When $\beta_1=0$, $\alpha_1=3\alpha$

$n=1, g(\theta_\sigma)=1$

The Drucker-Prager criterion is derived

$$F=\alpha I_1+\sqrt{I_2}-K \tag{5}$$

$$\left.\begin{aligned}\alpha&=\sin\varphi/\sqrt{9+3\sin^2\varphi}\\ K&=3C\cdot\cos\varphi/\sqrt{9+3\sin^2\varphi}\end{aligned}\right\} \tag{6}$$

where C and φ are the cohesion and friction angle of the rock respectively.

For the fault element

$$F=\alpha_1\sigma_{yy}+[(1-4\alpha_1^2/3)(\sigma_{xy}^2+\sigma_{yz}^2)]^{1/2} \tag{7}$$

For the plane strain problerm

$$F=\alpha_1\sigma_{xy}+\sigma_{xy}(1-\frac{4}{3}\alpha_1^2)^{1/2} \tag{8}$$

Then, we have

$$\frac{\partial F}{\partial\sigma}=[\frac{\alpha_1}{3}+\frac{\alpha_1}{6\sqrt{I_2}}(\sigma_{xx}-\sigma_m)+\frac{1}{2I_2}\sigma_{xy}(\sigma_{yy}-\sigma_m)]^T \tag{9}$$

$$[D]\{\frac{\partial F}{\partial\sigma}\}=\frac{E}{2(1+\mu)(1-\mu-2\mu^2)\sqrt{I_2}}\times[M_1\ \ M_2\ \ M_3]^T \tag{10}$$

$$\{\frac{\partial F}{\partial\sigma}\}^T[D]=\frac{E}{2(1+\mu)(1-\mu-2\mu^2)\sqrt{I_2}}\times[M_1\ \ M_2\ \ M_3] \tag{11}$$

here

$$\left.\begin{aligned}M_1&=C_{11}[\frac{2}{3}\alpha_1\sqrt{I_2}+(\sigma_{xx}-\sigma_m)]+C_{12}[\frac{2}{3}\alpha_1\sqrt{I_2}+(\sigma_{yy}-\sigma_m)]\\ M_2&=1C_{22}[\frac{2}{3}\alpha_1\sqrt{I_2}+(\sigma_{xx}-\sigma_m)]\ 1+C_{22}[\frac{2}{3}\alpha_1\sqrt{I_2}+(\sigma_{yy}-\sigma_m)]\\ M_3&=2C_{33}\sigma_{xy}\end{aligned}\right\} \tag{12}$$

Substituting eqs. (9), (10)and (11)into eq. (2), we then have

$$[D_p]=\frac{E}{(1+\mu)(1-\mu-2\mu^2)}\begin{bmatrix}d_{11}&d_{12}&d_{13}\\ d_{21}&d_{22}&d_{23}\\ d_{31}&d_{32}&d_{33}\end{bmatrix} \tag{13}$$

where

$$\left.\begin{aligned}d_{11}&=M_0M_1^2\\ d_{12}&=d_{21}=M_0M_1M_2\\ d_{22}&=M_0M_2^2\\ d_{23}&=d_{32}=M_0M_2M_3\\ d_{31}&=d_{13}=M_0M_1M_3\\ d_{33}&=M_0M_3^2\\ M_0&=1/\{M_1[\frac{2}{3}\alpha_1\sqrt{I_2}+(\sigma_{xx}-\sigma_m)]+M_2[\frac{2}{3}\alpha_1\sqrt{I_2}+(\sigma_{yy}-\sigma_m)]+2M_3\sigma_{xy}\}\end{aligned}\right\} \tag{14}$$

The elastic stiffness matrix of the element can be written as

$$[D]=\frac{E}{(1+\mu)(1-\mu-2\mu^2)}\begin{bmatrix}c_{11}&c_{12}&c_{13}\\ c_{21}&c_{22}&c_{23}\\ c_{31}&c_{32}&c_{33}\end{bmatrix} \tag{15}$$

where

$$\left.\begin{aligned}C_{11}&=(1-\mu^2)\\ C_{12}&=C_{21}=\mu(1+\mu)\\ C_{22}&=1-\mu^2\\ c_{33}&=2(1-\mu-2\mu)\\ c_{13}&=C_{31}=C_{23}=C_{32}=0\end{aligned}\right\} \tag{16}$$

Substituting eqs. (13) and (15) into eq. (1), elasto-plastic stiffness matrix of the fault element is derived

$$[D_{ep}]=\frac{E}{(1+\mu)(1-\mu-2\mu^2)}\begin{bmatrix}b_{11}&b_{12}&b_{13}\\ b_{21}&b_{22}&b_{23}\\ b_{31}&b_{32}&b_{33}\end{bmatrix} \tag{17}$$

where $b_{ij}=c_{ij}-d_{ij}$ (i, j = 1, 2, 3) (18)

The elasto-plastic stiffness matrix of the fault element in terms of the global system of coordinates is derived through transformation

$$[D_{ep}]'=[T][D_{ep}][T]^T \tag{19}$$

For sofe clay strata element

$$[T]=\begin{bmatrix} \cos^2\theta & \sin 2\theta \\ -\sin 2\theta/2 & \cos 2\theta \end{bmatrix} \qquad (20)$$

where θ is the angle between the axis of the fault element and x axis; it's positive direction is assumed to be a counter-clockwise rotation.

3 NUMERICAL EXAMPLE

A circular tunnel with two parallel soft claystrata in a rock medium is illustrated in Fig.1.

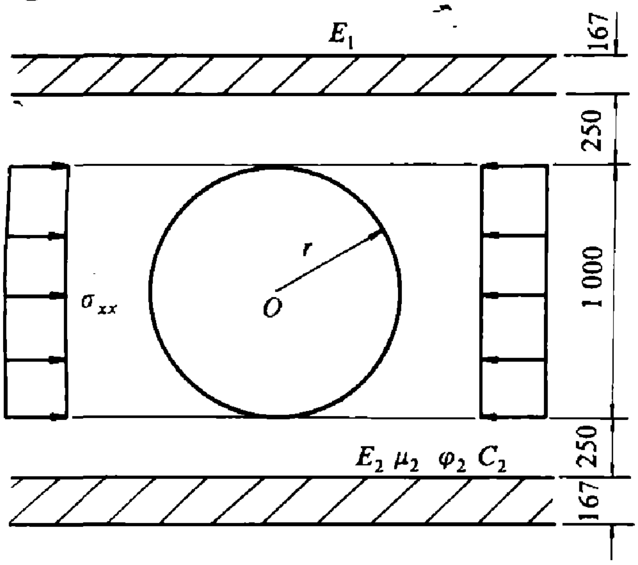

Fig. 1 A circular tunnel with soft clay strata

$\sigma_{xx}=1$ Mpa, elastic modulus of the rock material $E_2=1{,}000$ Mpa, Possion ratio $\mu_2=0.3$, the cohesion $C_2=0.25$ MPa, the friction angle $\varphi_2=35°$, the elastic modulus of the soft clay strata material $E_1=10$ MPa. Using the symmetry condition, a quarter of the tunnel is analyzed. Calculation results are shown in Fig.2. It is clear that the computing value is in agreement with the results of non-linear BEM[3].

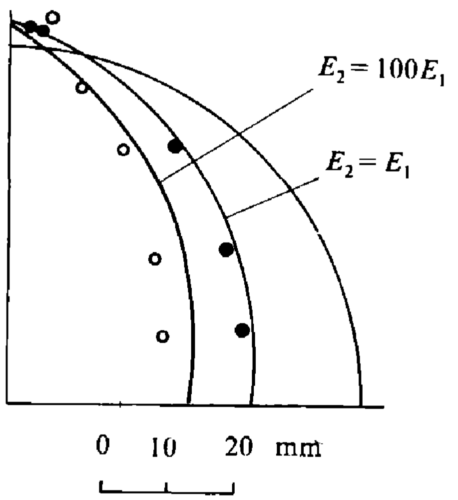

Fig. 2 Boundary displacement

4 CONCLUSIONS

From above exposition and calculation, we know that the computing formulas and method in this paper are feasible.

The calculated values are in agreement with the results of non-linear BEM.[3]. Using a similar method, the elasto-plastic matrix of the fault element in the heterogeneous case can also be obtained.

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