THEORETICAL STUDY ON ALEKSANDROWSKI'S METHOD OF THE GRAPHICAL DETERMINATION OF PRINCIPAL STRESS DIRECTIONS FROM THE FAULT SLIP DATA SETS ①

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ABSTRACT

A graphical method for determining the principal stress distribution of a triaxial stress state from a fault slip state was proposed by Aleksandrowski in 1985, based on Arthaud's concept of plane movement, Aleksandrowski's method, however, is only valid for the cases in which the values of the stress ratios(C) are considered to be ∞, 10, 2, 1. 1 and 1. Whether the method is applicable for general cases of all values of C has not yet been confirmed. In this paper. Aleksandrowskis' method is tested using a numerical derivation from spatial geometric analysis, and it is revealed that this method is correct for all values of stress ratios other than $C=\infty$, 10, 2, 1. 1, and 1, i. e $-\infty<C<\infty$.

Keywords: fault slip principal stress direction graphical determination numerical derivation.

1 INTRODUCTION

In structural geology, the dynamic analysis of a fault system is based on Anderson's model(1942). According to his analysis, there is a conjugate set of fault planes inclined 45 degrees to the direction of maximum compression stress and containing the direction of the intermediate one. For frictional materials, the two planes are inclined at acute angles to the direction of maximum compression stress, and the fault slip vectors are always perpendicular to that of the intermediate one. However, in many cases, faults often show oblique slip and are usually unable to separate into different conjugate sets as accounted by Anderson's model. Bott(1959) proposed a dynamic method which assumes that faults slip in the direction of resolved shearing stress on the fault plane. Based on his classic work, considerable efforts have been directed toward formulating both an inverse method of determining a reduced stress tensor (Etchecoper *et al.* 1981, Angeliger 1984, Gephart and Forsyth 1984, Reches 1987) and graphical techniques (Arthaud 1969, Reches 1983, Aleksandrowski 1985, Krantz 1988) for determining the directions of principal stresses. In this paper, Aleksandrowski's method is discussed and tested theoretically by numerical derivation from spatial geometric analysis.

2 BRIEF REVIEW

Arthaud (1969) proposed a graphical method for determining the orientations of principal stresses based on the analysis of movement planes which are defined as the planes containing the normals to the fault planes and the direction of slickenside lineations. Carey (1976) suggested that the method can only be applied to the slickenside lineation populations that originated

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in radial stress fields. Aleksandrowski (1985) computed the directions of maximum shearing stress on fault planes for several tens of variably oriented fault planes and for several different principal stress ratios according to the formula derived by Bott (1959):

$$\operatorname{tg}\theta = \frac{n}{lm}[m^2 - (1-n^2)\frac{\sigma_z - \sigma_x}{\sigma_y - \sigma_x}] \qquad (1)$$

and
$$C = \frac{\sigma_z - \sigma_x}{\sigma_y - \sigma_x} \qquad (2)$$

where θ is the pitch of the maximum shearing stress; l, m and n are the direction cosines of the unit normal to a fault plane; σ_x, σ_y and σ_z the principal stresses.

After ploting the traces of the poles to the movement planes on a stereogram, Aleksandrowski found that only for the cases of $C=\infty$ and $C=1$, do the M−planes intersect at one point, and this conclusion conforms with Arthaud's method. In other words, Arthaud's method can only be applied in the radial stress field corresponding to cases of extreme states of stress, i. e. $C=1$ and $C=\infty$. For $C=1.1$, 2 and 10, the movement planes which intersect at one point CIP correspond only to the cases that the poles to fault planes should lie in a great circle containing the direction of one principal stress. (Fig.1) Aleksandrowski extended these special cases to general ones ($1<C<\infty$) and concluded that if (a) a group of the normals to fault planes is arranged along a great circle, and (b) this great circle contains the direction of a principal stress σ, then the movement planes of these faults intersect at a point which lies in another great circle perpendicular to the principal stress σ. His theory is based only on a few examples and graphical operations, but whether the method can be extended to general cases has not yet been tested.

Therefore in the following discussion we present the theoretical derivation to test whether Aleksandrowski's method could be applicable for **all C values (that is, $-\infty<C<\infty$) other than $C=\infty$, 10, 2, 1.1 and 1.**

3 THEORETICAL DERIVATION

For simplification, two assumptions are made: (1) the x and y axes of the coordinate system are horizontal, and the z axis is vertical and (2) $\sigma_z=\sigma_1$, $\sigma_y=\sigma_2$, $\sigma_x=\sigma_3$ (Fig. 2), while $C=(\sigma_1-\sigma_3)/(\sigma_2-\sigma_3)$, where $-\infty<C<\infty$. Thus, equations for the fault planes can be represented as

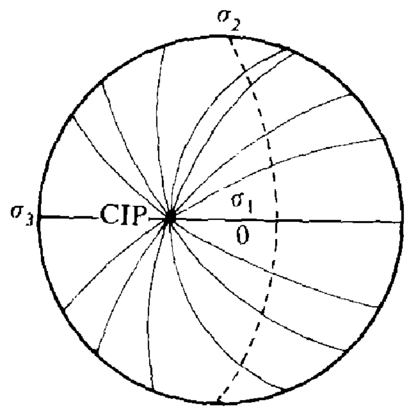

Fig.1 Movement plane patterns for $C=2$ drawn separately for groups of the normals(dots) to related fault planes arranged along a great circle(dashed line) containing σ_z (after Aleksandrowski).

$$\left.\begin{array}{l} l_1x+m_1y+n_1z=0 \\ l_2x+m_2y+n_2z=0 \\ \cdots\cdots\cdots\cdots\cdots \\ l_tx+m_ty+n_tz=0 \\ \cdots\cdots\cdots\cdots\cdots \\ l_rx+m_ry+n_rz=0 \end{array}\right\} \qquad (3)$$

where l_t, m_t and n_t ($t=1\cdots, \cdots, r$) are direction cosines of unit normal N_t to the t-th fault

Assuming that a plane P is parallel to the z axis(σ_1), the unit normal to the P is defined as

$$F=\{a, b, c\}=\{a, b, o\} \qquad (4)$$

where a, b and c are the direction cosines of F Thus the equation for P can be expressed as

$$ax+by=0 \qquad (5)$$

If the normals to the fault plane represented in Eq. (3) is distributed on P, it follows that

$$\{l_t, m_t, n_t\}\cdot\{a, b, o\}=0 \qquad (6)$$

or
$$l_ta+m_tb=0 \qquad (7)$$

where the left−hand side of Eq.(6) is the scalar product of N_t and F. According to Jaeger and Cook(1979), the slip vector on a fault plane is

$$S_t = \{l_t[m_t^2(\sigma_2-\sigma_1)-n_t^2(\sigma_1-\sigma_2)], \\ m_t[n_t^2(\sigma_3-\sigma_2)-l_t^2(\sigma_2-\sigma_1)], \\ n_t[l_t^2(\sigma_1-\sigma_3)-m_t^2(\sigma_3-\sigma_2)]\} \quad (8)$$

Since each M−plane contains both the normal to the fault plane and the slickenside lineation on the fault plane, the unit normal to each M−plane is

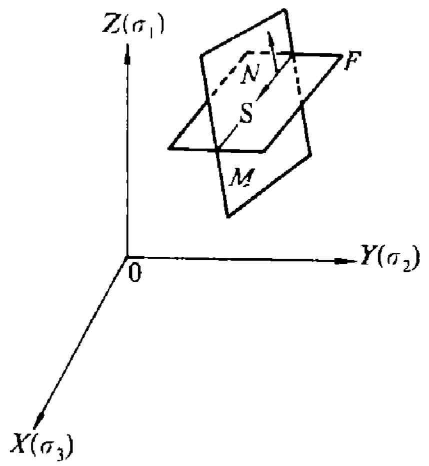

Fig. 2 The orientations of the fault (*F*) and movement plane (M) with respect to the coordinate system of principal stress axes. The normal to the fault and slip direction on it are marked *N* and *S* respectively

$$M_t = S_t \times N_t \quad (9)$$

or

$$M_t = \begin{vmatrix} i & j & k \\ l_t & m_t & n_t \\ s_1 & s_2 & s_3 \end{vmatrix}$$
$$= m_t n_t(\sigma_2-\sigma_3)i + n_t l_t(\sigma_3-\sigma_1)j + l_t m_t(\sigma_1-\sigma_2)k$$
$$= \{m_t n_t(\sigma_2-\sigma_3), n_t l_t(\sigma_3-\sigma_1), l_t m_t(\sigma_1-\sigma_2)\} \quad (10)$$

where i, j and k are the unit vector of x, y and z respectively and s_1, s_2 and s_3 are components of S_t.

From equation 10 we can see that if $\sigma_1=\sigma_2(C=1)$, all the M−planes pass through z axis, and if $\sigma_2=\sigma_3(C=\pm\infty)$, all the M−planes pass through the x axis. The patterns of M−planes show good agreement with the arrangements raised by Carey (1976) as to the applicability of Arthaud's method.

From Eq. (7), we get

$$\frac{l_t}{m_t} = -\frac{b}{a} \quad (11)$$

Substituting Eq. (11) into Eq. (10) and rearranging it yields

$${}_t = \{m_t n_t(\sigma_1-\sigma_3), -(b/a)m_t n_t(\sigma_3-\sigma_1) \\ -(b/a).m_t^2(\sigma_1-\sigma_2)\} \quad (12)$$

We know if some planes intersect on a line, the normals to the planes are necessarily perpendicular to the line and vice versa. From Eq. (12), we can easily find that M_t is perpendicular to vector

$$L = \{\frac{b}{a}(\frac{\sigma_1-\sigma_3}{\sigma_2-\sigma_3}), 1, 0\} \quad (13)$$

Thus all the M−planes intersect on a line. We should note that the vector L is perpendicular to z axis, so the intersection is parallel to the $x-y$ plane.

The above analysis indicates that Aleksandrowski's conclusions are obviously appropriate for $-\infty < C < \infty$. However, we usually first plot the traces of M−planes on stereogram then infer the directions of principal stresses. A question arises as to, if a group of M−planes intersect at a point, whether the conditions, (a) and (b) mentioned above, are necessarily satisfied. In order to solve the problem, we assume that the three movement planes intersect at a point. From Eq. (12), we get

$$\left.\begin{aligned} &m_1 n_1(\sigma_2-\sigma_1)x + n_1 l_1(\sigma_3-\sigma_1)y + \\ &l_1 m_1(\sigma_1-\sigma_2)z = 0 \\ &m_2 n_2(\sigma_2-\sigma_1)x + n_2 l_2(\sigma_3-\sigma_1)y + \\ &l_2 m_2(\sigma_1-\sigma_2)z = 0 \\ &m_3 n_3(\sigma_2-\sigma_1)X + n_3 l_3(\sigma_3-\sigma_1)y + \\ &l_3 m_3(\sigma_1-\sigma_2)z = 0 \end{aligned}\right\} \quad (14)$$

In order to obtain non−zero solutions for x, y and z, the determination must follows

$$\begin{vmatrix} m_1n_1(\sigma_2-\sigma_1) & n_1l_1(\sigma_3-\sigma_1) & l_1m_1(\sigma_1-\sigma_2) \\ m_2n_2(\sigma_2-\sigma_1) & n_2l_2(\sigma_3-\sigma_1) & l_2m_2(\sigma_1-\sigma_2) \\ m_3n_3(\sigma_2-\sigma_1) & n_3l_3(\sigma_3-\sigma_1) & l_3m_3(\sigma_1-\sigma_2) \end{vmatrix} = 0 \quad (15)$$

or

$$\begin{vmatrix} m_1n_1 & n_1l_1 & l_1m_1 \\ m_2n_2 & n_2l_2 & l_2m_2 \\ m_3n_3 & n_3l_3 & l_3n_3 \end{vmatrix} = 0 \quad (16)$$

Assuming $l_tm_t \neq 0 \quad (t=1, 2 \text{ and } 3) \quad (17)$

yields

$$\begin{vmatrix} n_1/l_1 & n_1/m_1 & 1 \\ n_2/l_2 & n_2/m_2 & 1 \\ n_3/l_3 & n_3/m_3 & 1 \end{vmatrix} = 0 \quad (18)$$

or

$$(l_1n_2-l_2n_1)(n_3m_2-m_3n_2)/l_1m_3-(n_2m_1-n_1m_2)\times (l_2n_3-n_2l_3)/l_3m_1=0 \quad (19)$$

It is obvious, if three M-planes intersect at a line, then the direction cosines of the normals to the related faults must follow Eq. (19). We should note, however, that the normals do not necessarily lie on a plane . In contrast, if the normals are located on a plane , then they must obey the determinant

$$\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0 \quad (20)$$

Assuming

$n_t \neq 0 (t=1, 2 \text{ and } 3), \quad (21)$

we have

$$\begin{vmatrix} l_1/n_1 & m_1/n_1 & 1 \\ l_2/n_2 & m_2/n_2 & 1 \\ l_3/n_3 & m_3/n_3 & 1 \end{vmatrix} = 0 \quad (22)$$

or $(l_1n_2-l_2n_1)(n_3m_2-m_3n_2)-(n_2m_1-n_1m_2)(l_2n_3-n_2l_3)=0 \quad (23)$

Substituting Eq.(23) into eq.(19) yields

$l_1m_3=l_3m_1, \quad (24)$

or $l_1/m_1=l_3/m_3 \quad (25)$

Changing the positions of the rows in the determinants of Eq.(15) and Eq.(20), we get

$l_1/m_1=l_2/m_2 \quad (26)$

or $l_1/m_1=l_2/m_2=l_3/m_3. \quad (27)$

Eq. (27) resembles Eq. (11), which shows that the plane containing the normals passes through the z axis.

The foregoing analysis indicates that movement planes intersecting on a line can not guarantee that the normals to the related fault planes lie in a single plane (Fig. 3). But , if the normals to the related faults do lie in a plane, then the plane must contain the direction of principlal stress σ, and the intersecting line of M-planes must be parallel to another plane perpendicular to the σ accordingly, in analysing a CIP, we must check whether three or more normals to the faults related to the M-plane lie in a great circle on a stereogram. If not, we cannot infer a principal stress from it.

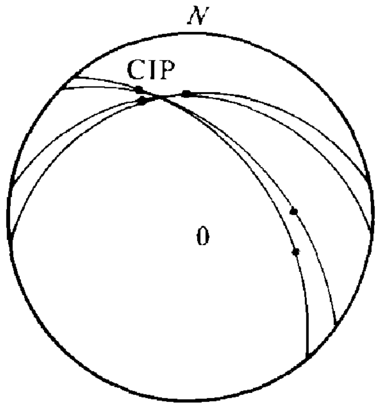

Fig. 3 The normals (dots) to fault planes do not lie in a great circle, although the related M-planes intersect at one line ($C=2$, after Aleksandrowki).

4 DISCUSSION AND CONCLUSIONS

Although the numerical derivation is based on the assumption that a principal stress is vertical, the results are still correct for the rotating principal stresses, because the rotation of principal stresses will not change the inherent relationships between the fault planes, slickenside and principal stresses.

We have proved that Aleksandrowski's method, which supplements Arthaud's method of analyzing the movement planes, is universally correct, and we have given Alcksandrowski's method theoretical support. Alcksandrowski's method is appropriate for both the general stress state ($-\infty < C < \infty$) and mechanically heterogeneous media which contains pre–existing planes, such as joints, older faults or bedding surfaces.

However, the graphical method, as Alcksandrowski has pointed out, is time cosuming. This may be overcome by a computer program in preparation by means of the quantitative representations of both the planes and lines, which we present in this paper .

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