THE INTEGRAL OF THE INVERSE FUNCTION OF Φ FOR AN ANALYTICAL SOLUTION TO THE COMPRESSION OF A THIN WORKPIECE [1]

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ABSTRACT

An analytical solution of the unit pressure on a thin workpiece under compression has been obtained by using the inverse function of Φ to the integral $\int_0^{l} \Phi\, dx$. Its result is basically the same as the prevailing numerical formula $\int_0^{l} \Phi dx = \Sigma\Phi_i \cdot \Delta X_i$. However, the new integral is simpler and more convenient to use.

Key words: slip line field thin workpiece inverse function analytical solution

1 INTRODUCTION

The calculation of the pressure for a slip line field in the compression of a thin workpiece between perfectly rough, rigid parallel platens was investigated originally by Prandtl[1], and later by Hill [2]. However, recently published textbooks show that the conventional numerical methods[3,4] are still used for the integration of $\int\Phi dx$ in the slip line field. An integral of inverse function of Φ will be presented and an analytical solution of $\int\Phi dx$ will be obtained in this paper.

2 PARAMETRIC EQUATION OF A SLIP LINE

For the compression of a rectangular section block between perfectly rough, rigid platens the slip line field is shown in Fig. 1. The frictional stress acting on the interface is $\tau = k$. The shear stress along the direction of thickness, which displays a linear distribution, is given by

$$\tau_{xy} = \pm 2ky / h = \pm \cos 2\Phi \quad (1)$$

From Mohr's circle of stress, the shear stress $\tau_{xy} = \pm k\cos 2\Phi$, the y and dy are given as

$$\left.\begin{aligned} y &= \pm h\cos 2\Phi / 2 \\ dy &= \mp h\sin 2\Phi d\Phi \end{aligned}\right\} \quad (2)$$

The differential equations for the α and β lines are

$$\left.\begin{aligned} dy / dx &= \mathrm{tg}\Phi \text{ on the } \alpha\text{-lineand} \\ dy / dx &= -\mathrm{ctg}\Phi \text{ on the } \beta\text{-line} \end{aligned}\right\} \quad (3)$$

Substituting Eq. (2) into Eq. (3) and taking the integral, then for the α line

$$\left.\begin{aligned} x &= \pm h(2\Phi + \sin 2\Phi) / 2 + C_1 \\ y &= \pm h\cos 2\Phi / 2 \end{aligned}\right\} \quad (4)$$

for the β line

$$\left.\begin{aligned} x &= \pm h(2\Phi - \sin 2\Phi) / 2 + C_2 \\ y &= \pm h\cos 2\Phi / 2 \end{aligned}\right\} \quad (4')$$

where Φ is the angle from the x axis to the tangent of α line. These parametric equations of the slip line show that the slip line field in the compression of a thin workpiece is a cycloid which is obtained from rolling a circle of

1 Manuscript recieved November 27.1992

radius $h/2$ along the straight line $y = \pm h/2$.

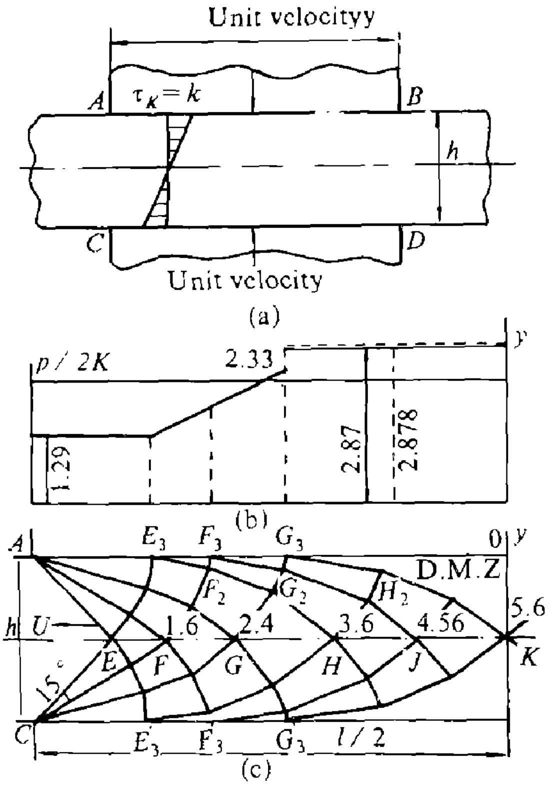

Fig. 1 Compress of a thin workpiece between rough parallel platens

(a)—linear distribution of shear stress;(b)—distribution of unit pressure;(c)—ship line field for $l/h = 5.6$

3 A SOLUTION WITH THE INVERSE FUNCTION OF Φ

As shown in Fig. 1, the slip line field for $l/h = 5.6$, which is from Ref.[3]~[5], is taken as an example. From Fig. 1, at any point on the left slip line AEC the horizontal principle stress is zero, so at the point E, $\sigma_x = 0$, $\Phi_E = -\pi/4$ and $p_E = k$[3,4]

Moving along the β line EE_3, $\Phi_{E_3} = 0$ and $\Phi_E = -\pi/4$; Using the Hencky stress equation, we obtain

$$p_{E_3} = p_E + 2k(\Phi_{E_3} - \Phi_E)$$
$$= k + 2k(0 + \pi/4) = 2.57k$$

Moving along the α line E_3F_2 and β line F_2F_3, $\Phi_{F_3} = -\pi/12$, $\Phi_{F_3} = 0$;

Using the Hencky stress equation, then

$p_{F_3} = 3.62k$

In a similar manner,

$p_{G_3} = 4.66k$

If the hydrostatic pressure at each point of E_3, F_3 and G_3 is divided by $2k$, the distribution of $p/2k$ at interface AG_3 is as shown in Fig. 1(b). In Fig.2, G_3K is an α line which bounded the dead metal zone. At any point on the slip line, the hydrostatic pressure is

$$p = p_G + 2k(\Phi_{G_3} - \Phi) \tag{5}$$

Take an element of length ds in α line G_3k. The shear stress k and hydrostatic pressure p act on it. The vertical component of the force on ds is $\delta F = (k\sin\Phi + p\cos\Phi)\mathrm{d}s$. Then, the total vertical force exerted on the line G_3K is

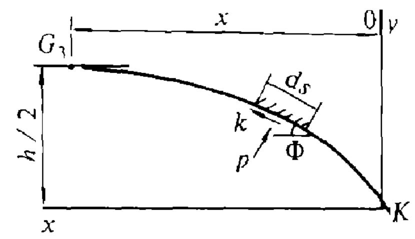

Fig. 2 Stresses acting on a line element ds of the α line G_3K

$$F = k\int\sin\Phi\mathrm{d}s + \int\cos\Phi\mathrm{d}s = k\int\mathrm{d}y + \int p\mathrm{d}x \tag{6}$$

Substituting Eq.(5) into Eq.(6) (note $\Phi_{G_3} = 0$, $\Phi_K = -\pi/4$), we get

$$F = k\int_0^{h/2}dy + \int_0^X(p_{G_3} - 2k\Phi)\mathrm{d}x$$
$$= hk/2 + p_{G_3}X - 2k\int_0^X\Phi\mathrm{d}x \tag{7}$$

According to traditional understanding, the last term of Eq.(7) is not a integrable function and still is solved by a graphical-numerical formula as follows.

$$F = hk/2 + p_{G_3}X + 2k\sum_{i=1}^{n}\Phi_i \cdot \Delta x_i \tag{8}$$

By formula (8), for $l/h = 5.6$, $h/2 = 2.5$, the numerical results are

$$F = 37.72k \tag{8'}$$

$$p/2k = 37.73k/2k \times 6.58 = 2.87 \tag{9}$$

In the above formulas[3],

$$\sum\Phi_i \cdot \Delta x_i = 2.28;\ \sum\Delta x_i = 6.50 \tag{10}$$

Since the above solution is not an analytical one, the author proposed an integration concerning the parameter Φ[5].However,recent work shows that the integral of the inverse function of Φ is simpler than that in Ref. 5. The procedures are as follows.

Dealing with the slip line G_3K, as shown in Fig. 2, and substituting $\Phi=\Phi_K=-\pi/4$, $x=0$ into *Eq.* (4) yields $C_l=(1+\pi/2)h/2$. Therefore, the equation of G_3K becomes

$$\left.\begin{array}{l} x = h(\sin2\Phi + 2\Phi + 1 + \pi/2)/2 \\ y = h\cos2\Phi/2 \end{array}\right\} \quad (11)$$

It is obviously that the integral of Φ in Eq. (7) is an implicit function of x, but from Eq. (11), we see that x is an explicit function of Φ. From Fig.3, the integral $\int_0^{\pi}\Phi dx$ is the area of a curvilinear trapezoid bounded by the curve $\Phi=\Phi(x)$, the x–axis, and the straight lines $x=0$ and $x=X$. That is

$$S=\int_0^X\Phi(x)dx \quad (12)$$

However, the area is equal to that of a curvilinear trapezoid bound by the same curve $x=x(\Phi)$[$x=x(\Phi)$ is the inverse function of $\Phi=\Phi(x)$], the Φ–*axis*, and the straight lines $\Phi=0$ and $\Phi=-\pi/4$. Since $0\geqslant\Phi(x)\geqslant-\pi/4$, the area is

$$S=\int_0^x\Phi(x)dx=\int_0^{-\pi/4}x(\Phi)d\Phi<0 \quad (12')$$

The function $x=x(\Phi)$ satisfies Eq. (11). Substituting Eq. (11) into Eq. (12′) yields

$$\begin{aligned}\int_0^x\Phi(x)dx&=\int_0^{-\pi/4}x(\Phi)d\Phi\\&=\int_0^{-\pi/4}[h/2(\sin2\Phi+2\Phi+1+\pi/2)]d\Phi\\&=h/4+(h/2)\times(\pi^2/16)\\&\quad-(h/2)(1+\pi/2)\times\pi/4\end{aligned}$$

Substituting $h/2=2.5$ into above the formula yields

$$\int_0^x\Phi dx=\int_0^{-\pi/4}xd\Phi=-2.26 \quad (13)$$

Substituting $\Phi_{G_3}=0$ into Eq. (11) yields

$$x=h/2(1+\pi/2)=6.427 \quad (14)$$

Substituting $x=6.427$, $\int_0^{\pi}\Phi dx=-2.26$ into Eq. (7) gives

$$F=2.5k+4.66K\times6.427+2k\times2.26+37k \quad (15)$$

$$n_\sigma=F/(2kx)=2.878 \quad (15')$$

Comparing Eqs. (15), (15′) with (8′) and , (9), it can be seen that the results from the integral of the inverse function of Φ are basically the same as those from the numerical method.

It should be pointed out that to the slip line field for $l/h=5.6$ and $h/2=2.5$, the exact value of $l/2$ is 14, but the numerical solution yield a value of 14.2[3,4]. The exact value of X should be 6.427, but the numerical solution yields $X=\sum\Phi_l\cdot\Delta x_l$, be 6.58.

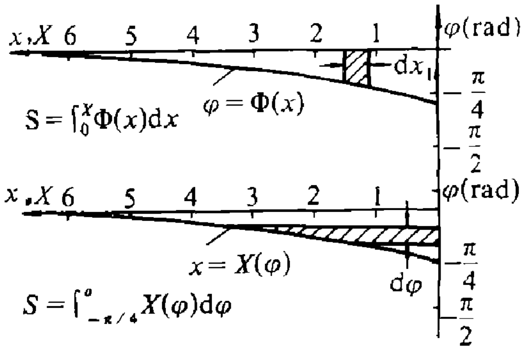

Fig. 3 $\int_0^x\Phi dx$ **and inverse function integral** $\int_0^{-\pi/4}xd\Phi$

4 CONCLUSIONS

(1) The integral of the inverse function of Φ can be carried out in the slip line field for the compression of a thin workpiece. An analytical solution of $\int_0^{\pi}\Phi dx$ can be obtained without using the conventional numerical method;

(2) For the slip line field of $l/h=5.6$ and $h/2=2.5$, the analytical solution with the integral of the inverse function of Φ is basically the same as that from a numerical one.

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