DAMAGE MECHANICS MODEL PREDICTING METALLIC MINE SUBSIDENCE [1]

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ABSTRACT

The mine subsidence and ground movement is a complicated mechanical phenomenon. Based on the rock mass being a part of the geologic body suffering deformation and failure, whereas mine subsidence is a phenomenon of rock mass redeformation and refailure, the damage mechanics model for predicting metallic mine subsidence and a planar finite element program considering the damage are proposed; The organizations and rules of metallic mine subsidence are studied. The numerical results agree nicely with the observed data.

Key words: mine subsidence damage mechanics Elastic Finite Element (FEM)

1 INTRODUCTION

As early as the late 19th century, the subsidence and failure of overlying rock mass caused by mining had drawn attention.

Many theories and methods for predicting the ground movement at coal mines, especially for predicting the subsidence caused by coal mining under buildings, water and railways have been developed and widely used at home and abroad[1, 2]. However, the research on metallic mine subsidence has not progressed yet. In particular the study of the methods for theoretical prediction has remained negligible[3−5]. This is because: (1) the geologic mineralization of metallic mines is more complicated; (2) the rock is very hard, with sophisticated geologic structures and the abundant joints; (3) various shapes of ore bodies make the goal very complicated, and it is very difficult to get accurate results from surveying stations on the ground surface. All those difficulties resulted in the failure of anybody to summerize any general rules or set up any mathematical models as have already been done for coal mines[4, 5]. So it is important to find a theoretical method to predict metallic mine subsidence. Such a method must consider the shape of the goal, the structures of rock mass, the residual stress and the properties of rock, especially the influence of the joint sets. The FEM based on a consecutive medium is an ideal method to meet some of these requirements, but the traditional elastic or elasto-plastic FEM is unable to simulate the influence of the joints and get satisfactory answers. If we adopt the Goodman join unit element and use a computer to simulate the joints separately, it not only takes a long time but also is impossible based on the limited storage capacity of the computer. However, damage mechanics can deal with this kind of medium well.

Although damage mechanics has been applied to rock mechanics for only about ten years, it has been used successfully to solve many

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engineering problems in rock mechanics[6, 7]. This mechanics considers the discontinuous joints in the rock mass as a macroscopic damage, the rock mass with unpenetrating joints as consecutive medium with micro-field-damage, the initiation and propagation and penetration of the joint as the process of damage evolution.

Therefore the authors thought this mechanics could be used to predict the subsidence of metallic mines. In addition we considered the rock mass as a part of the geologic body suffering deformation and failure, and the subsidence as the process of redeformation and refailure of the rock mass for mining. The planar computation program considering damage was also compiled and used to simulate the subsidence process at the metallic mine with hard rock and abundant joints. Although this method was the first attempt, it seems to be a new available and effective method.

2 MODEL FOR PREDICTING SUBSIDENCE

2.1 *The Damage Tensor of the Rock Mass and Its Physical Modification*

The damage concept was originally introduced in treating damage in 1859 [6]. It was defined as the ratio of the total area of the joints and defects of a cut surface in a representative volume element to the total surface, that is $\Omega = A_W / A$. The effective stress δ^* was defined as the ratio of the force acting upon the surface to its actual effective load-bearing area A_{ef} ($A = A_w + A_{ef}$), that is $\delta^* = P / A_{ef}$. So we can get the relation between the effective stress and Cauchy stress:

$$\sigma^* = \sigma(I-\Omega)^{-1} \tag{1}$$

In order to take its direction into account, Murakami and Ohno suggested a tensor to describe the damage variable:

$$\Omega = \Omega(n \otimes n) \tag{2}$$

where n is the unit normal and $\otimes$ denotes the tensor product.

Obviously, the damage tensor Ω expresses the direction and density of the joints.

We consider the rock mass composed of fundamental units and joints which are distributed on the surface and can extend along the surface of the unit. Suppose the fundamental unit has a representative volume v which corresponds to the unit cube cut by joints with average spacing l, i.e. $v = l^3$, and the damage tensor of the rock mass can be described as

$$\Omega = l \sum_{k=1}^{n} s^k (n^k \otimes n^k) / V \tag{3}$$

where V denotes the volume of the rock mass containing n joints, n^k and s^k denote the area and the normal unit vector of the k th joint.

It is impossible to measure all s and n of the joints separately, but we can divide the joints into some groups by practical investigation, and then calculate the damage tensor of each group by means of the average values:

$$\Omega = \bar{l} / V \cdot \overline{SN}(\bar{n} \otimes \bar{n}) \tag{4}$$

The damage tensor of the rock mass is the sum of damage tensors of all joint groups:

$$\Omega = \sum^{i} \Omega_i \tag{5}$$

where i denotes the damage tensor of the ith joint group.

The damage tensor described above only considers the geometrical properties of the joints and does not consider the physical properties. Equation (1) is deduced under the supposition that all the joints are fully open and cannot transmit any stress. In fact, the reduction of the effective area is different due to the difference in physical properties and closing and sliding of the joints from the effects of different kinds of stresses.

It is generally considered that the joint-section cannot bear tensile stress but can bear compression and shearing stresses. So we think,

Ω plays the whole role in the case of the damage resulting from tensile stress and part of role in the case of damage resulting from compression stress and shearing stress for the attenuation of the effective area. Let c_n and $c_t(0 \leqslant c_c, c_t \leqslant 1)$ to characterize the rates of transmission of compression and shearing. The effective areas are $(I-\Omega)$, $(I-c_n\Omega)$ and $(I-C_t\Omega)$ under the three stresses state respectively. Thus, the equation for the net stress must be revised to:

$$\begin{aligned}\sigma^* = T'\{&\sigma_t'(I-c_t\Omega')^{-1} \\ &+\sigma_n'[H<\sigma_n'>(I-\Omega')^{-1} \\ &+H<-\sigma_n'>(I-c_n\Omega')^{-1}]\}T\end{aligned} \tag{6}$$

where T is the orthogonal diagonalizer of Ω(.)t, that is $\Omega' = T\Omega T^t$, means the transpose, and $\sigma' = T\sigma T^t$, $\sigma_n' = \sigma' \cdot I$, $\sigma'_t = \sigma' - \sigma_n'$.

H<.> is the switching operator such as

$$H < x > = \begin{cases} 1 & x > 0 \\ 0 & x \leqslant 0 \end{cases}$$

2.2 *The FEM Model Considering Damage*

According to the supposition of equivalent strain in damage theory, we can find the constitutive relation of the rock mass with joints from that of rock mass considering no direct damage, just by equating the net stress σ' with the Cauchy stress σ:

$$\sigma^* = D \cdot \varepsilon \tag{7}$$

In classical mechanics the equation of equilibrium using the Cauchy stress σ is written by a virtual work form as:

$$\int \sigma \delta\varepsilon \mathrm{d}v = \int t^{\circ} \delta u \mathrm{d}s_t + \int f \delta u \mathrm{d}v \tag{8}$$

Substituting the net stress σ' defined by equation (6), the Cauchy stress σ is represented as

$$\sigma = \sigma^* - \Psi \tag{9}$$

Where Ψ denotes the mechanical effect of demage.

Then we can get the virtual work equation of the damaged body.

$$\int \sigma^* \delta\varepsilon \mathrm{d}v = \int t^{\circ} \delta u \mathrm{d}s_t + \int f \cdot \delta u \mathrm{d}v + \int \Psi \delta\varepsilon \mathrm{d}v \tag{10}$$

Where

$$\left.\begin{aligned}\Psi = T'\{&\sigma_t'(\Phi_t - I) \\ &+\sigma_n'[H<-\sigma_n'>\Phi \\ &+H<-\sigma_n'>(\Phi_n - I)]\}T \\ \Phi = (I-&\Omega')^{-1}, \\ \Phi_t = (I-&c_t\Omega')^{-1}, \\ \Phi_n = (I-&c_n\Omega')^{-1}\end{aligned}\right\} \tag{11}$$

and it describes the mechanical effects of the damage.

Now, we can get the standard finite element discretization from equation (10).

$$[k]\{u\} = \{F_e\} + \{F_e^*\} \tag{12}$$

Where $[k] = \int [B]^t [D][B] \mathrm{d}v$

$$\{F_e\} = \int [N]^t \{t^{\circ}\} \mathrm{d}S_t + \int [N]^t \{f\} \mathrm{d}v \tag{13}$$

$$\{F_e^*\} = \int [B]^t \{\Psi\} \mathrm{d}v \tag{14}$$

The additional force vector $\{F_e^*\}$ describes the mechanical effect of the damage.

The excavation analysis for damage theory is carried out as the following programs.

(1) Calculate $\{U_0\}$ in the original stress state and calculate the Cauchy stress $[\sigma_0]$, which is the primary stress of the first excavation step. Then we can simulate the excavation process.

(2) Calculate the equivalent joint force vector $\{F_e\}$ in accordance with the primary Cauchy stress or the find excavating Cauchy stress of the last step.

$$\{F_e\} = \sum_{i=1}^{M} \left(\int [B]^t [\sigma_i] \mathrm{d}v - \int [N]^t \{f\} \mathrm{d}v\right) \tag{15}$$

where M denotes the number of excavated units.

(3) Solve the simultaneous equations and get the displacement $\{u'\}$ of the undamaged state.

$$[k]\{u'\} = \{F_e\} \tag{16}$$

(4) Calculate the Cauchy stress with the aid of the constitutive relation from the displacement $\{u'\}$.

$$[\sigma] = [B][D]\{u'\} \tag{17}$$

where $[\sigma]$ will be the primary stress of the next step's excavation.

(5) Substitute $[\sigma]$ and $[\Omega]$ into Equation (6) to get the net stress $[\sigma^*]$, then calculate the additional force vector $\{F_e^*\}$ from Eq. (11).

(6) Solve the equation: $[k]\{u''\}=\{Fe'\}$ to get the additional displacement $\{u''\}$ caused by the damage. The subsidence caused by the excavation will be

$$\{u\}=\{u'\}+\{u''\} \tag{18}$$

Repeating the second through the sixth steps, we can finish the simulation of the excavation process.

2.3 *Analysis of Subsidence Affected by Mining*

In fact, the main subsidence and ground movement caused by underground mining is a movement from an unbalanced stress state to a new balanced state by the movement and deformation of the rock mass. There is a difference between mine subsidence and rock mechanics whose final object is to judge whether the object studied will be stable or not. Mine subsidence cares not only whether the rock mass will cave in or not and the progression of the caving, but also the dynamic state of the whole overlying rock mass, that is the rules of the movement and the deformation of the rockmass. Mine subsidence prediction must predict the height and the range of caving, the cause of failure of each rock layer, and the values and the ranges of the movement and deformation of the rock mass.

2.3.1 The Criterion for Instability

To choose the criterion of instability, you must consider the stress state in the rock mass and the failure forms of the rock mass. When a part of an ore body was worked out, the overlying rock mass always displays the same stress distribution. The vertical stress σ_y near the working surface would be tensile stress, and as the inner area of the rock mass was entered, the stress in it would become compression stress. The overlying rock mass in such a stress state may also experience failure besides tensile failure. The tensile failure criterion may be

$$F=\sigma_i-\sigma_t \geqslant 0 \quad i=1,2,3 \tag{19}$$

where σ_i denotes the main stress componerits and σ_t, the indivadual tensile strength.

Adopt the following criterion for shear failure:

$$\tau>[\tau_0]\sigma_n \mathrm{tg}\varphi \tag{20}$$

where τ and σ_n are the tangential stress and normal stress and τ_0 and φ are the cohesive force and angle of internal friction of the rock mass.

If we use σ_x and σ_y to express τ and σ_n, it can be written as

$$\left.\begin{aligned} \tau &= (\sigma_x \cdot \cos\beta - \sigma_y)/2 \cdot \sin 2\alpha \\ \sigma_n &= \sigma_x \cdot \cos\beta \cdot \sin^2\alpha + \sigma_y \cdot \cos^2\alpha \end{aligned}\right\} \tag{21}$$

where β denotes the azimuth strike angle of the joint group deviating from the ore body strike line, and α is the angle between the shear section and direction of the maximum compressive stress vector.

Calculating the shear stress and maintenance stress of all the joint groups according to this criterion, we can confirm along which joint group the rock mass will experienced shear failure with the greatest probability[8].

2.3.2 Primary Research of Caving Criterion

The rock mass is likely to be unstable when the stresses at some spots reach their peak values, but the criterion of stability can not judge whether the rock mass will cave in due to its own deed weight yet.

The following is a simple discussion of the condition for natural caving of the rock mass.

At first, the rock body must get rid of the effect of residual stress fields. The factors which control wether the overlying rock body will cave in or not are the sliding force and frictional resistance caused by the dead weight of the rock body. If the former is larger than the latter, the rock body will slide till caving occurs.

The rock body which experienced shear failure is still affected by residual stress fields. For the overlying rock body, the sliding force is the vector sum of the components in the direction of failure of the dead weight of the rock body and residual stress, the sliding resistance is the vector sum of the components of friction caused by the normal force of the dead weight of the rock body and residual stress. If the sliding force is larger than the sliding resistance, the unit will cave in.

2. 3. 3 Simulation of the Caving Process

Considering the caving unit as an excavated unit and transferring the stress by means of "equivalent releasing load", we can simulate the process of caving well. We can do as follows.

(1) Get the net stress σ^* in a certain excavation state;

(2) Use σ^* to confirm whether the unit will be stable or not;

(3) Confirm whether the unstable unit will cave in or not;

(4) Consider the caving unit as an excavated unit. Transfer the equivalent-nodal-load of these units to the surface of the new worked-out section, and then calculate the displacement of the nodes and the stress of the units after this caving step.

Repeat the above steps till the unit no longer caves, i.e., the stress field reaches a new balances state, so we have determined the progression of caving at each excavating stage.

According to the criterion of instability and caving, we can determine the "three zones of movement" from the stress distributions: the last caving area is the caving zone, the area with movement but still stable is the global bending zone.

3 TDED-1(TWO-DIMENSION ELASTIC DAMAGE) PROGRAM

According to the model mentioned before, we program a two dimensional elastic finite element program TDED-1, which can consider the effect of the damage in Fortran-77. The program adopts a one-dimensional dynamic method to store the global stiffness matrix and adopts the Gauss-Integration method to solve the equations directly. The program composition and all computations are carried out on a Siemens 7570-c. The following are its practical functions.

(1) Divide the units, form the information of the units and the joints and calculate the coordinates of the joints automatically;

(2) Calculate the equivalent loads of the joints automatically;

(3) Simulate the caving process and effects of the excavation process on mine subsidence;

(4) Confirm the edge of the caving area, calculate the caving height, define the range of the fissure zone, calculate the value of the displacement and deformation of the global subsidence zone, and get the limit angle and the boundary angle;

(5) All results are given in a diagram.

4 ENGINEERING EXAMPLES

A simulation computation was made using the rules for the subsidence of block caving in Tonkuangyu Copper Mine[9], which has seven large ore bodies. The main ore bodies are metamorphic porphyry copper ores. № 5 ore body is a lenticular one, whose direction is essentially parallel to its wall rock. For this body, the lengths of control strike and control height are 980 m and 120 ~ 1,061 m, the mean thickness is 115 m. The joints of the rock mass are very abundant. The ore reserves are rich but the grades of the ores are very low. The undercut engineering was arranged at 810 m level and the height of one phase is 120 m.

Before under-cuting, the precisely polygonometric net-work at two levels 870 m and 930 m in the upper rock mass had been positioned. The height was measured by precision levelling using a Zeiss Ni004 gradienter and a two-meter-long Invar rod. Valuable observational data at the level lane of 870 m were obtained and used to compare with the computed results[10].

In computation, consider no damage propagation and an inability of the joint-section to transfer tensile stress and shearing stress but an ability to transfer all the compression stress, that is $c_t = 1$ and $c_n = 0$.

In computation, the length along x direction is 950 m, which is about 8 times the range of undercut. In z–direction, we consider the whole rock mass from the level of 670 m to the ground surface. The nets total of finite elements includes 2,400 units and 2,511 joints. There are two groups of computation schemes. One considers no damage; the other considers damage. Each scheme simulates the excavation of six steps which represents the circument of undercut being 20, 40, 60, 80, 100 and 120 m respectively. The computation results, which ware in fair agreement with the observed data and illustrated in detail in Ref.[9], were based on the CAD station of Siemens 7570–c directly.

5 CONCLUSION

(1) The values of subsidence considering damage are about five times those computed by traditional FEM. It is proved for mine subsidence that the effect of the joints is very obvious and it is one of the characters of the metallic mine subsidence;

(2) Joints make the subsidence curve different to normal distribution. The model from the damage FEM can consider well the anisotropy affected by joints and it has displayed the advantages of the damage FEM;

(3) The computation results of the model FEM considering damage nicely agree with the observed data, and the feasibility and effectiveness has thus been proven.

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