FRACTAL DIMENSION AS MORPHOLOGY AND SIZE PARAMETERS OF FRACTURED PARTICLES OF ROCK [1]

Li, Gonbo Xu, Xiaohe
Northeast University of Technology, Shenyang 110006, China

ABSTRACT

Semi-automatic image analysis system was used for the surface fractal dimension measurement of 15 kinds of rock-fractured particles. The results indicate that the surfaces of the particles that have a size of about 200 mesh are fractal, and that the fractal dimension is in the range of 2.07～2.11. Therefore, some definitions of particle size such as surface diameter, equivalent volumetricly surface diameter are equivocal without giving the step length and the drag diameter, free-fall diameter, and stokes' diameter may be modified by the fractal dimension.

Key words: rock-fractured particle fractal dimension image analysis particle size particle shape

1 INTRODUCTION

Particle size is the most fundamental characteristic in the study of granular particles. The fact that a rock-fractured particle surface is fractal shakes this foundation: even though morphology is the principal characteristic of a particle, it is difficult to substantially quantify it. Nevertheless, fractal theory, using a single fraction-fractal dimension, can represent morphological characteristics. Fig. 1, which is an outline of a standard fractal curve (Koch island), can be regarded as a rugged boundary of a real particle. This figure has self-similarity and an infinite area. While the natural rugged outlines of the cross-section of rock-fractured particles may not meet all the requirements for a fractal curve, over a given specified range of resolution, any part of the outline, when magnified to a certain range, looks almost the same as the original one. That is to say, a rock-fractured particle may have a statistically self-similar surface. As Mandelbrot wrote[1]: "An actual stone's surface is surely not a standard surface, and it should be a fractal if it is scaling."

In 1969 and 1970, Meloy[2,3] presented papers in which fast Fourier transforms were used to process particle silhouettes as a signal, and this work was extended in 1977[4–5]. One of his main conclusions was that when the nth Fourier coefficient, A_n, was plotted versus lg n, a straight line was obtained. Meloy thus proved the particle surface to be fractal, even though he did not know the word "fractal" at that time. After fractal theory was introduced, Avinir and Pfeifer[6,7], Kaye[8], Flook[9] and others, have demonstrated that various of natural particles' surfaces are fractal.

The shape and size of a rock-fractal particle affect many mining procedures, such as blasting, free-falling ore, comminution and classification. Previous mathematical models were based on the supposition that a rock-fractured particle has a smooth and continuous surface. In this paper, a semi-automatic image analysis

[1] This paper was sponsored by China National Nature Science Foundation, and the manuscript was received in April 15, 1992.

system was used for the fractal dimension measurement of the outline of the cross-section of a ground particle, in order to see whether its surface is fractal. The effects of surface fractal dimension on morphology and size definition are discussed.

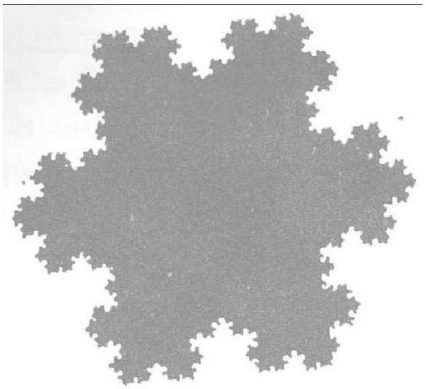

Fig. 1 A Koch island as a model for a cross-section of a natural particle

Fractal dimension = 1.261.8

2 EXPERIMENTAL METHOD

15 kinds of common rock or ore were used in the experimental research conducted in connection with this subject. After the rock crushing into fragments whose size was 5 cm or so, every kind of rock or ore was run in ball mill grinds for 10 min(70 r / min) and then sieved out. The particles with a size of about 200 mesh were prepared as samples. A particle sample was added to a mixture of heated epoxy resin and $(HOCH_2CH_2)_3$. This was poured into a plastic mould which was slowly rotated to ensure good mixing. The casting was cured at 65 ℃ for more than 5 h, then, cut as shown in Fig. 2. In order to get a polished cross-section of the rock-fractured particle which could be examined by the semi-automatic image analysis system.

The semi-automatic image analysis system includes an ASM68K computer, a digitizer, a microscope and a video camera, a monitor and a printer. A profile of a rock-fractured particle which was magnified 1,500 times was shown on the monitor screen. By tracing the outline of the profile with the mouse on a digitizer, the outline coordinates were transferred to the computer and the parameters such as perimeter, area and other geometric properties could be obtained automatically. For one sample, ten particle profiles were chosen at random and printed. For each profile outline, one edge point was chosen as a starting point, and a clockwise or counter clockwise convention and series of step lengths (filter diameters) from 0.4 to 10 mm were chosen. By tracing the particle outline, a polygon with the length of the chosen step length was formed. The perimeter was found by adding the length of the final step to the sum of all the equal step lengths (as shown in Fig. 3).

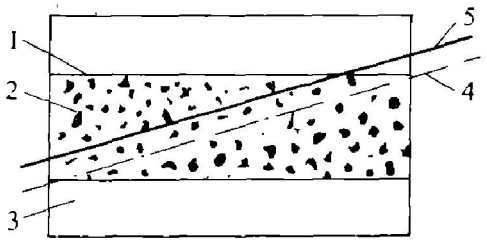

Fig. 2 Method of cutting mould

1, 3—epoxy resin; 2—particles in resin; 4—polished section; 5—cut section

The measured perimeter P was plotted against the step length δ on lg-lg coordinate paper, over a range of chosen step lengths from 0.27 to 6.7 μm. The fact that a straight line with a negative slope is found on the plot shows that the particle boundary has statistical self-similarity (as shown in Fig. 4).Therefore

$$P \propto \delta^m \tag{1}$$

The value for the fractal dimension of the profile is calculated from the slope m of the regression line for all coordinate points as

$$D_p = 1 - m \tag{2}$$

The average value of D_p for ten particle profiles is taken as the fractal dimension of a sample.

From Ref. [10], the suface fractal dimension D_s of rock-fractal particle is

$$D_s = 1 + D_p \tag{3}$$

As an example, results of D_P calculated for 3 profiles are presented in Table 1 and Fig. 4.

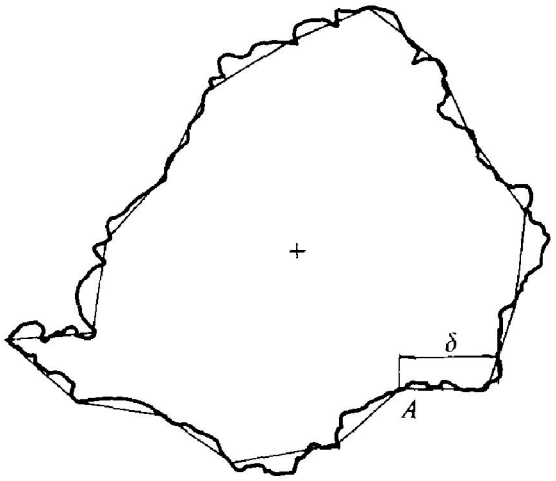

Fig. 3 The method of measurement of the fractal dimension of the particle profile

Dolerite: magnification: × 1.500; real perimeter—137.905 μm; real area—864.199 μm^2; *A*—starting point; *δ*—step length

Table 1 Calculation of D_p for a particle profile of Dolerite

material	δ mm								m	D_p
	0.4	0.8	1	2	4	6	8	10		
Dolerite	213.7	195.8	197.1	185.5	178.6	168.2	170.6	166.7	−0.077.02	1.077.02
Diorite	147.2	137.1	132.8	128.7	112.6	110.6	103.8	109.6	−0.104.76	1.104.76
Barite	242.4	226.4	218.7	209.6	203.2	196.5	181.3	175.0	0.090.52	1.090.52

3 RESULTS AND DISCUSSION

The experimental results of the fractal dimension measurement of 15 kinds of rock or ore particle are presented in Table 2. These D_P values range from 1.07～1.11(D_S from 2.07～2.11) with increasing D_P correlating with increasing № in the Table 2. The value of D_S, where $2 \leqslant D_S \leqslant 3$, labels a global level of particle surface irregularity, the bigger D_S is, the more rugged the particle surface[10]. It demonsttates that the surfaces of rock or ore particles are fractal, and it gives the order of irregularity. In addition, the degree of surface irregularity varies quite little when variation of D_S is limited to a small range.

3.1 D_S Signifies the Characteristic of the Particle Surface

It is valid for a particle surface to serve as the geometric parameter of particle shape, because the coordinates of its surface in 3-dimensional space form the shape. It is, however, not very important to describe exactly the actual shape by mathematics analysis. The values which can be compared with each other related to the physical and chemical properties of a particle.

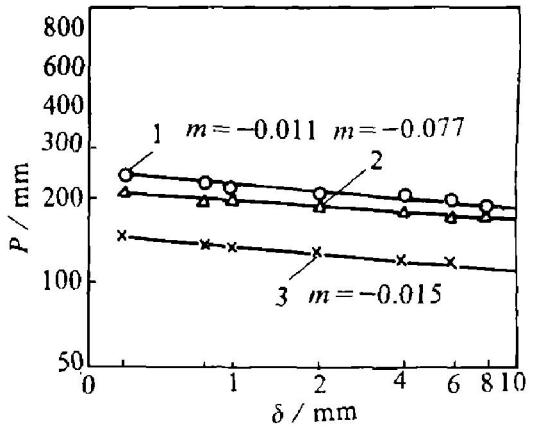

Fig. 4 Plot of lg *P* vs lg *δ*

Table 2 The average values of the fractal dimension D_P for 15 kinds of particles

№	material	D_P	№	material	D_P
1	Carborundum	1.070	9	Dark limestone	1.087
2	Iron ore 1	1.074	10	Galena	1.088
3	Dolerite	1.079	11	Barite	1.091
4	Gypsum	1.081	12	Calcite	1.093
5	Quartz	1.081	13	Light red limestone	1.094
6	Iron ore 2	1.083	14	Granite	1.098
7	Coal	1.083	15	Diorite	1.100
8	Syenite	1.085			

The morphology of a rock-fractured particle is related to two characteristics: general shape such as pyramid, sphere etc. and the microstructure of the general shape. Commonly, the general shape of a particle influences its physical behavior like fluidity; the microstructure influences its chemical behavior, like surface reactivity. The general shape can be learnt by sight or quantified by fractal harmonics methods[11], while the microstructure can be labeled by the fractal dimension with a given range of step lengths. In this paper, fractal dimension D_S, with a range of 0.27～6.7 μm of step length, reflects the microstructure of the rock-fractured particles.

3.2 *Influence of the Fractal Surface of the Particle on the Size Definition*

The outline of the cross-section and the surface of a rock-fractured particle are fractal. That means the profile perimeter and surface area depend on the step length. Therefore, for a given particle, the perimeter and surface area are not constant. When the smallest self-similar structure on a particle surface can not be known, the definitions of particle size based on two parameters, such as equivalent surface diameter, equivalent volumetricly surface diameter and perimeter diameter are equivocal. Only for a given step length, can the values of a diameter be scaled and compared to each other.

On the basis of the cross-section area and the volume of a particle are constant, volume diameter, projected area diameter, sieve diameter, Feret's diameter, Martin's diameter and unrolled diameter state precisely the meaning of a particle size.

3.3 *The Formula Relating the Length of a Fractal Outline or Area of a Fractal Surface Differ from the Previous One*

When the step length tends toward zero, theoretically, for any particle size, the outline length and surface area should be infinite. However, the particles of rock or ore in nature surely have the smallest self-similar structure so the perimeter P and surface area S can be defined.

Suppose an outline has a fractal dimension D_P, a surface D_S. The volume diameter d_V is chosen as the particle size, according to Ref. [12]. We have

$$P^{1/D_P} \propto V^{1/3} \tag{4}$$

where V is the volume of a particle.

$$S^{1/D_S} \propto V^{1/3} \tag{5}$$

Because $V = kd_V^3$ (k is constant),

$$P \propto d_V^{D_P} = \gamma_1 d_V^{D_P} \tag{6}$$

$$S \propto d_V^{D_P} = \gamma_2 (d_V^{D_S \cdot 2})^2 \tag{7}$$

where γ_1 and γ_2 are constants which depend on the smallest self-similar structure of a given particle.

Eqs. 6 and 7 are compared to the traditional equations for perimeter and area in a function of d_V: $P = \gamma_1 d_V^2$, $S = \gamma_2 d_V$. The particle size should be considered as $d_V^{D_P}$ and $d_V^{D_S \cdot 2}$.

The surface fractal dimension of rock-fractured particles, which is a characteristic value describing a particle's properties must be measured extensively. This paper serve as the beginning.

REFERENCES

1 Mandelbrot, B. B. Fractal Geometry of Nature. Freeman: San Francisco, 1982: 262.
2 Meloy, T. P., Screening. Washington D.C: AIME, 1969.
3 Meloy, T. P., In: Eng Found Conference. Deerfield, 1971.
4 Meloy, T. P. Powder Technol. 1977, 16(2): 233–254
5 Meloy, T. P., Powder Technol. 1977, 17(1): 27–36.
6 Avnir, D., Farin, D. *et al.* Nature (London). 1984, 308: 261.
7 Pfeifer, P., Avnir, D., J Chem Phy, 1983, 79: 3558.
8 Kaye, B. H., In: Proc Particle Size Analysis Conference, Salford: Chem Soc. Analyt. Div., 1977.
9 Flook, A. G., Powder Technol. 1978, 21: 295–298.
10 Mandlbrot, B. B. Nature, 1984, 308(19):721–722.
11 Clark, N. N. Nature, 1986, 319(20): 625.
12 Takayasu hazijn. (ed) Sheng, Buming (trans.). Fractal Dimension (In Chinese). Beijng: Earthquake Publishing House, 1989.

(*continued from page* № 5)

REFERENCES

1 Hakalehto, K. O., The behaviour of rock under impules loads. Doctor Thesis. Univ of Otaniemi-Helsinki. 1969.
2 Lundberg B., Int J Rock Mech Min Sci & Geomech Abstr. 1976, 13: 187–197.
3 Olsson W. A., Int J Rock Mech Min Sci & Geomech Abstr. 1991, 28: 115–118.
4 Li, Xibing, Lai, Haihui, J Cent-south Inst Min Metall, 1989, 20(6): 595–604
5 Fairhurst, C., Mine Quarry Engng, 1961, 27: 327–328.
6 Lundberg B., Some basic problems in percussive rock destruction. Dissertation. Chalmers Univ of Technology, Gothenburg. 1971.