CHARACTERIZATION OF VELOCITY SINGULARITY AND SUPPRESSION OF NOISE①

Song, Shougen Tang, Jintian
Department of Geology, Central South University of Technology, Changsha 410083

ABSTRACT

Based on the imaging formulas that reported by the authors, the characterization of velocity singularity has been studied across the wavelet scale. Furthermore, the imaging method can suppress the noise in the observed data. Finally, the examples are given.

Key words: imaging characterization of singularity noise multiscale

1 INTRODUCTION

This paper is the second part of the study on the imaging of reflector in the earth with wavelet transform, of which the first part has been reported in the last issue of the Transactions of NFsoc, i. e. Ref[1]. Based on the imaging formulas in Ref. [1], the authors obtained some results as follows.

2 CHARACTERIZATION OF VELOCITY SINGULARITY

To map more precisely the location of the reflector surface, it is useful to analyze the velocity singularity, for the band- limited input data. In mathematics, singularities are generally characterized by their lipschitz exponents.

Definition 1. Let $0 \leqslant \alpha \leqslant 1$, a function $f(x)$ is lipschitz γ at x_0 if and only if there exists a constant k such that for all x in a neighborhood of x_0, we have

$$|f(x) - f(x_0)| \leqslant k|x - x_0|^\gamma \qquad (1)$$

The function $f(x)$ is uniformly lipschitz γ over an open interval (a, b) if and only if there exists a constant k such that inequality (1) holds for any $x, x_0 \in (a, b)$.

According to the Definition 1, if $f(x)$ is continuously differentiable at x_0, then it is Lipschitz $\gamma = 1$. If $f(x)$ is discontinuous but bounded at x_0, for example, $f(x)$ is a step function at x_0, then its Lipschitz exponent $\gamma = 0$.

In two dimensions, Lipschitz exponent are defined with a simple extension of Definition 1. Let $0 \leqslant \gamma \leqslant 1$ and $x \in R^2$. A function $f(x)$ is said to be Lipschitz γ at a given point $x_0 \in R^2$, if and only if, there exists $\delta > 0$ as well as $k > 0$ such that for any $x \in U(x_0, \delta) = \{x | x \in R^2, |x - x_0| < \delta\}$

$$|f(x) - f(x_0)| \leqslant k\delta^\gamma \qquad (2)$$

If there exists a constant k such that inequality (2) is satisfied for any points x_0 and x within a open set of R^2, the function $f(x)$ is uniformly Lipschita γ over this open set.

The modulus of the wavelet operator at the scales can be difined by

$$|W_s\alpha(x)| = \sqrt{|W_s^1\alpha(x)|^2 + |W_s^3\alpha(x)|^2} \qquad (3)$$

Based on Eq. (3), the Jaffard's result in Ref. [4] can be extend as follow. Suppose that the wavelet $\varphi^j(x)$, $j = 1, 3$, are continuously differentiable and $|\varphi^j(x)| = 0(\frac{1}{1 + |x|^2})$, as $|x| \to \infty$, we has the following lemma.

Lemma 1. Let $0 < \gamma < 1$ and $f(x) \in L^2(R^2)$. A function $f(x)$ is uniformly Lipschitz γ over an open set A of R^2 if and only if there exists a constant $B > 0$ such that for all $x \in A$, the modulus of the wavelet operator satisfies

$$|W_s f(x)| \leqslant Bs^\gamma \qquad (4)$$

① Received Dec. 17, 1993; accepted in revised form Aug. 3, 1994

The proof of this lemma is a simple extension of the proof of the Jaffard's result in Ref. [4]. Lemma 1 characterizes uniform Lipschitz exponents over an open set but not pointwise Lipschitz exponents. To study isolated singularities, however, according to the Mallat's view in Ref. [3], lemma 1 is sufficient. We shall say that a function has an isolated singularity at x_0 if there exists a neighborhood $U(x_0, \delta)$ of x_0, where the the worst singularity is at x_0. In other words, the uniform Lipschitz regularity of the function over $U(x_0, \delta)$ is equal to the pointwise Lipschitz regularity at x_0.

Based on Eqs. (29), (36) in Refs. [1] and [4] in lemma 1, we obtain the following theorem.

Theorem 1. For the band-limited input data $u_s(x_g, x_r, \omega)$, on the surface S in the earth, the Lipschitz exponent γ of $\alpha(x)$ is not less than zero.

particularly, for the full band input data $u_s(x_g, x_r, \omega)$, by Eqs. (30) and (36) in Ref. [1], and Eq. (4) in lemma 1, we know that the Lipschitz exponent γ of $\alpha(x)$ is equal to zero on the surface S in the earth. Hence, we have the following corollary.

Corollary 1. For the full band input data $u_s(x_g, x_r, \omega)$ the function $\alpha(x)$ is step function and its discontinuty is on the surface S.

Theorem 1 and Corollary 1 show clearly the characterization of velocity singularity, respectively for the band-limited and the full band input data. Using this information about the velocity singularity, we can easily suppress the effect of the noise in the real world data.

Because there always exists noise in the obaserved data, it is important to find the velocity inversion method which can suppress the noise.

First, let us consider the effects of noise in the data and suppose that $u_s(x_g, x_r, \omega)$ is affected by $\hat{n}(\omega)$ and the observed data becomes $u_s(x_g, x_r, \omega) + \hat{n}(\omega)$, here $\hat{n}(\omega)$ is the Fourier transform of the noise $n(t)$. Then, the function $\alpha(x)$ in equation (5) in Ref. [1] correspondingly becomes $\bar{\alpha}(x)$, that is

$$\bar{\alpha}(x) = \alpha(x) + n_1(x) \tag{5}$$

In this equation,

$$n_1(x) = \frac{8c_0^3}{i\pi^2}\int dk_1 dk_3 d\zeta_1 dt \times \frac{k_3}{\omega^2} t n(t) e^{2i[k_1(x_1-\zeta_1)-k_3x_3]+i\omega t} \tag{6}$$

Eq. (6) implies that the noise in the observed data should infects the function $\alpha(x)$ in Eq. (5) in Ref. [1]. And then, if $n(t)$ is real wide sense stationary white noise, using the methods provided by Mallat in Ref. [3] to calculate the lipschitz exponent γ of $n_1(x)$ in Eq. (6), for a few special cases (for example, for the zero offset constant background case), we have known that the Lipschitz exponent γ of $n_1(x)$ in Eq. (6) is less than zero. That is to say, there exists a constant B_1 such that

$$|W_s n_1(x)| \leqslant B_1 s^{\gamma} \tag{7}$$

In this equation, γ is less than zero.

Therefore, if we apply the wavelet operator W to Eq. (5), we obtain

$$W_s\bar{\alpha}(x) = W_s\alpha(x) + W_s n_1(x) \tag{8}$$

Inequality (7) implies that $|W_s n_1(x)|$ should decrease when the scale s increases. On other hand, by Theorm 1 and Lemma 1, the amplitude of $|W_s\alpha(x)|$ should increase or remain constant when the scale s incrcase. Hence, we can take the advantage of the spatial coherence of the image of velocity singular structure to suppress the effects of the noise in observed data when the scale s increase.

3 EXAMPLE AND CONCLUSIONS

For the band-limited inverse problem, the inverse operator $\beta(x)$, introduced by Bleistein in Ref. [2], is a band-limited Delta function on the surface in the earth. On the other hand because the inversion operator $\beta(x)$ is a generalization of the derivative operator $\partial\alpha(z)/\partial z$, $\beta(x)$ is highly sensitive to the noise in the real world data. But, the method in this paper can suppress the effects of the noise.

Let us consider a single inclined planar reflector. The angle of inclination will be 30° with respect to a horizontal ox_1 axis, and the planar reflector is papallel to ox_2 axis, above the plane with speed c_0 = 4 500 m/s, and below the plane with speed c_1 = 5 500 m/s. In this paper, the coordinate system $x = (x_1, x_2, x_3)$ is a right-hand system with x_3 being positive in the downward direction into the earth. The inclined plane is assumed to be at depth 2 000 m below the original point(0, 0, 0).

For the theory model, we add the real white

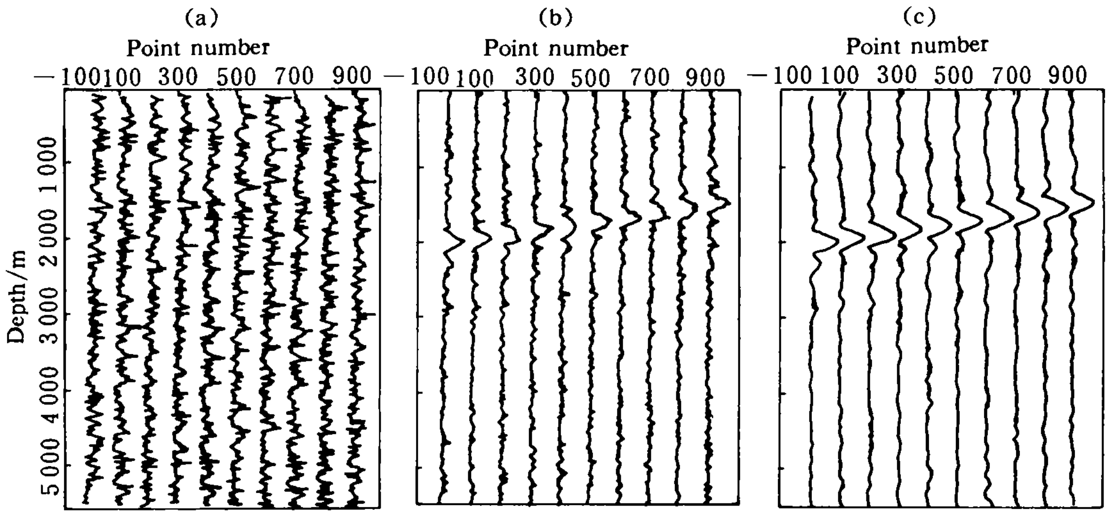

Fig. 1 (a) ~ (c) Adding real white noise to input data
(a)—A 6~50 Hz bandwidth depth section determined by $\partial \alpha / \partial n$
(b)~(c)—6~50 Hz bandwidth depth section determined by $|W_s \bar{\alpha}(x)|$
under scale s taking 2^1 and 2^2, respectively.

noise to the data $u_s(x_g, x_r, t_j)$. The image produced by $\partial \alpha / \partial n$ is shown in Fig. 1 (a). In Fig. 1 (a), the effects of the noise are very serious, and we can not know where the location of the planar reflector and how many there exist the reflectors.

But, in Fig. 1 (b) ~ (c), the image produced by the method in this paper are far more desirable. As the wavelet scale increasing, the image of the reflector becomes more and more clear.

Acknowledgments. This paper is motivated by Bleistein's lecture (1992) in Shandong university of P. R. China. The authors wish to thank Professof Bleistein for his help.

REFERENCES

1 Song, Shougen *et al.* Trans of NFsoc, 1994, 4(3): 1.
2 Bleistein, N *et al.* Geophysics, 1987, 52: 26.
3 Mallat, S *et al.*, IEEE Trans Inform Theory, 1992, 38: 617.
4 Jaffard, S. Publications Math, 1991, 35.