### **BATH TEMPERATURE MODEL FOR**

## POINT—FEEDING ALUMINIUM REDUCTION CELLS<sup>®</sup>

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### **ABSTRACT**

To solve the problem of heat balance(HB) supervision & control of point-feeding aluminium reduction cells, an adaptive predicting estimation model (APEM) of dynamically balanced bath temperature (DBBT) is established, which co- operates with a temperature measuring device, the later employing the method of temperature dynamic measurement (TDM) and operating in intermittent way. Since it can realize quantitative description of the relationship between the output—DBBT and the inputs—heating power and equivalent feeding rate, of a HB system, APEM can be used in the design of a HB supervision & control system of a point-feeding cell. Its effectiveness has been tested by using the data measured on a 160 kA point-feeding prebaked anode aluminium reduction cell.

Key words: aluminium reduction cell bath temperature estimation model

### 1 INTRODUCTION

It is well known that good maintenance of heat balance (HB) of cells is a key to the attainment of good technological and economic targets, therefore, realizing on-line supervision and control of HB is a long-standing problem that aluminium metallurgists have hoped to solve. As the bath is a strongly corrosive melt, conventional contact-type sensors are unable to operate continiously for a long time, and so, they are only used in manual measurements of short-term tests. Contactless-type sensors are unavailable because the surface of a cell is covered by crust, and the crust thickness is variable. Nowadays, there is a kind of indirect

measurement in use, which measures celll shell temperature or side wall heat flux. However because the delay- time of its respose can reach one hour to several hours and its respose is disturbed by variations of ledge thickness, it can only be used for the qualitative analysis of medium-long term changing tendency of HB. At present, in some advanced control systems a kind of HB control model has been used which is based on energy balance calcultation, but obviously this is a kind of "open loop" control method of poor accuracy. To solve the difficult problem— HB supervision & control, our lab has carried on studies on the method temperature dynamic measurement

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(TDM) and has reaped first fruits in lab[1], The device of TDM can automatically insert a thermocouple into the bath, using the dynamic varying curve of the potential signal obtained in the transient contacting process to calculate the bath temperature corresponding to the steady state of the potential signal. In this way, thermocouple-bath contacting time can be reduced, and so, the service life (measuring times) can be extended. But the weakness of the method is that the service life of a thermocouple is still restricted by the material of its protective shell, and, therefore, the method may have industrial use value only when the measuring frequency is restricted in the range of once per several hours. To make up for this weakness and to realize real-time and accurate surpervision and control of the HB states of pointfeeding aluminium reduction cells, we have established a bath temperature model which co-operates with a temperature measuring device, the later employing TDM and operating in intermittent way.

## 2 ANALYSIS AND TESTING OF HB CHARACTERISTICS

Because crust thickness and ledge thickness vary with changing bath temperature, aluminium reduction cells possess quite strong self-balance ability. In addition, techniques of semicontinuous point-feeding and cell voltage autoregulation have been widely used, therefore, with average-time concept, the HB system of a cell can be regarded as " a dynamic balance process", the slow-time varying tendency (the smoothed curve) of a varying curve of bath temperature as " dynamically balanced bath temperature" (DBBT), and DBBT as the supervision & control parameter of HB.

The first input variable which influ-

ences HB is "heating power" (denoted by Q), which is calculated with:

$$Q = A_{\rm p} - A_{\rm r} - A_{\rm e} \tag{1}$$

where  $A_p$  denotes electric power inputted into the cell. It is calculated from on-line sampling values of cell voltage and line current:

$$A_{\rm p} = V_{\rm c} \cdot I \tag{2}$$

where  $V_c$  and I denote cell voltage (V) and line current(kA), respectively.

 $A_{\rm r}$  denotes electrochemical reaction power (kW) required by the electrolytic process. By using thermodynamic data ( at 1 223 K)<sup>[2]</sup>, an equation for approximate calculation of  $A_{\rm r}$  can be derived from the electrochemical reaction equation, that is:

$$A_{\rm r}=(0.441+1.131r)\cdot I$$
 (3) where  $I$  still denotes line current (kA);  $r$  denotes current efficiency, which is assumed to be a constant, but can be modified when data from aluminium tapping are available.

 $A_{\rm e}$  denotes the power(kW) dissipated in metered external resistances (outside the HB system). By giving the value of the external resistances  $R_{\rm e}(\mu \Omega)$ ,  $A_{\rm e}$  becomes a function of line current I(kA):

$$A_{\rm e} = R_{\rm e} \cdot I^2 / 1\,000 \tag{4}$$

The second input variable which influences HB is material feeding rate. In general, it is alumina feeding rate calculated and ordered by the process control system, but its accurate calculation becomes difficult when manual operations (anode setting, aluminium tapping) occur which can cause additional feedings and additional heat dissipation. Nevertheless, to maintain the continuity of HB supervision & control, we take the influence of a manual operation on HB as being equivalent to an increase of alumina feeding rate. This is realized on the basis of thermodynamic data<sup>[2]</sup> and our field tests, and the equivalency can be dealt with automatically by the process control system. As for electrolyte additions, due to long addition period and small addition amount, we also, on the basis of thermodynamic data<sup>[2]</sup>, take their influences on HB as being equivalent to increases of alumina feeding rate. As a result, the second input variable can be defined as equivalent feeding rate of alumina. Now, the HB system of a point-feeding cell has been simplified as a double input-single output (DISO) system, the inputs being heating power (denoted by Q) and equivalent feeding rate (denoted by  $T_{\rm eq}$ ), the output being DBBT (denoted by  $T_{\rm b}$ )

To test HB characteristics of point-feeding cells, we chose a 160 kA point-feeding prebaked anode aluminium reduction cell in normal operation as a test cell, and used a self-made data sampling system<sup>[3]</sup> to sample cell voltage, line current, bath temperature, etc. Raw bath temperature signal was obtained with a NiCr-NiSi thermocouple that was protected by a silicon nitride tube. To eliminate high frequency disturbances in the temperature smpling tunnel, lowpass filters were set in both hardware and software of the sampling system, and quite fast sampling rate  $(1 \sim 10 \, \text{s})$  was used to sample primary bath temperature signal.

In order to carry on system identification experiments, we designed the input variables according to pseudo-random binary sequence (PRBS), and this was realized by adjusting the anode-cathod distance (ACD) and the time interval of feeding.

It has been determined by our tests that, under normal conditions, the settling time and the time-constant of the HB system of the test cell are about 60 min and 15 min, respectively, hence, bit width of PRBS and sampling period for DBBT estimation were all set as 12 min, and the lowpass-filtered average of primary bath temperature signal in every sampling period (12 min) was taken as the measured value of DBBT in the corre-

sponding period.

## 3 ADAPTIVE PREDICTING ESTIMA-TION MODEL(APEM) OF DBBT

## 3.1 Identification of CAR Model

A controlled auto- regression (CAR) model has been used to describe the HB system of a cell. By using a linear difference equation, the CAR model is expressed by

$$y(k) = \sum_{i=1}^{n_0} a_i y(k-i) + \sum_{i=0}^{n_1} b_i u_1(k-i) + \sum_{i=0}^{n_2} c_i u_2(k-i) + d + e(k)$$
 (5)

where y(k) denotes the output variable, here representing DBBT;  $u_1(k)$  and  $u_2(k)$  denote the two input variables (i. e. the controlled terms); d denotes the model bias (here temporarily assumed as a constant); e(k) denotes a Gaussian white noise term with zero mean;  $n_0$ ,  $n_1$  and  $n_2$  are the orders of auto-regressive part, and of the two controlled terms, respectively. Suppose in a HB system the base output corresponding with the base inputs ( $Q = Q_0$ ,  $F_{eq} = F_0$ ) is equal to  $T_0$  (i. e.  $T_b = T_0$  ). In order to enhance model identification accuracy, the sampled sequences of input and output variables (denoted by Q(K),  $F_{eq}(k)$  and  $T_{b}(k)$ , respectively) should go through a disposal of subtracting their base values. Moreover, Q(k)should go through a disposal of reducing by suitable times. Hence, the input and output variables of the CAR model become:

$$u_1(k) = \lceil Q(k) - Q_0 \rceil / 100 \qquad (6)$$

$$u_2(k) = F_{eq}(k) - F_0$$
 (7)

$$y(k) = T_{b}(k) - T_{0} \tag{8}$$

where equivalent feeding rate  $F_{\rm eq}(k)$  is represented by the equivalent feeding times in the sampling period  $(k-1) \sim k$ ; Q(k) is calculated with eqs.  $(1) \sim (4)$ , but cell voltage  $V_{\rm c}$  and ling current I used in those equa-

tions should be replaced by their average values in the period  $(k-1) \sim k$  (denoted by  $V_c(k)$  and I(k)). Then, the following equation can be derived from eqs.  $(1) \sim (4)$ :

$$Q(k) = V_{c}(k) \cdot I(k) - (0.441 + 1.131r)$$

$$\times I(k) - R_{c} \cdot I(k)^{2} / 1000 (kW)$$
(9)

The base values  $Q_0$ ,  $F_0$  and  $T_0$  are approximately substituted by the assembly averages of Q(k),  $F_{eq}(k)$  and  $T_b(k)$ , respectively, With a CAR model auto-identification algorithm<sup>[4]</sup> based on recursive least-square method and the parsimony principle, the determination of suitable model order and time-delay, as well as the fitting of model parameters can be carried out automatically by computer. Finally, a parsimony-parameter CAR model can be obtained.

As for our test cell, DBBT sampling period( $T_{\rm s}$ ) has been set as 12 min . The base values:  $T_{\rm 0}=958$  C,  $Q_{\rm 0}=400$  kW,  $F_{\rm 0}=2$  times/12 minutes, have been given on the basis of statistical results. The CAR model identification results are:

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + b_0 u_1(k) + b_1 u_1(k-1) + b_2 u_1(k-2) + c_0 u_2(k) + c_1 u_1(k-1) + d + e(k)$$
(10)

where

$$[a_1,a_2, b_0, b_1, b_2, c_0, c_1, d]$$
= [1.075, -0.199, 3.694, -1.330, 0.276, -0.124, -0.204, -0.174]

# 3.2 Multistep Predicting Estimation (P. E.) of DBBT and P. E. Errors

Now that CAR model has been obtained, by setting the white noise term e(k) = 0, the model can be used to estimate DBBT in the way of multistep prediction. In order to keep generality, here we still use the general equation, (eq. (5)), in the following discussion, therefore, the following

P. E. equation can be acquired:

$$\hat{y}(k) = \sum_{i=1}^{n_0} a_i \hat{y}(k-i) + \sum_{i=0}^{n_1} b_i u_1(k-i) + \sum_{i=0}^{n_2} c_i u_2(k-i) + d$$
(11)

 $\hat{y}(k-i)$ , i=0,  $1\cdots n_0$ , denote where the P. E. values of y(k-i). Obviously, when  $\hat{y}(k-i)$ , i=1,  $2\cdots n_0$ , on the right side of eq. (11) are replaced with known y(k-i), eq. (11) becomes the case of one-step P. E., otherwise, it is the case of multistep P. E. When multistep P. E. is carried on , deviations between P. E. values and actual values will gradually increase due to measurement noises, environmental disturbances, timevarying characteristics of the process, and error accumulation caused by recursive calculation, therfore, available P. E. steps are limited. As to our test cell, by using eq. (10) for recursive P. E., it has been found that, under basically normal conditions, P. E. deviations are generally within  $\pm 2.5$  C as long as recursive P. E. steps do not go beyond 20. Hence, the average sampling interval of TDM (temperature dynamic measurement) can be set as 4h, i.e. every 4h or so, it is necessary to use observations of TDM to reset starting points of P. E. as well as to provide innovations for adaptive modification of model parameters.

As the sampling interval of TDM is by far larger than the sampling period of DBBT, observations of TDM are not enough to modify all the parameters in the CAR model. Our studies have shown that the majority of P. E. errors can be attributed to variations of model bias d. For example, when a variation of the overall heat transfer coefficient of the cell results in the base value of DBBT (corresponding to the base inputs  $Q_0$  and  $F_0$ ) varying from  $T_0$  to  $T_0$  +  $\triangle T_0$ , the model bias will vary from d to d

 $+\left[1-\sum\limits_{i=1}^{n_0}a_i\right]$  under the assumption that in the CAR model all parameters except d do not vary.

#### 3.3 Adaptive Modification of the CAR Model Bias d

With the assumptions that TDM sampling interval is m times DBBT sampling period and the estimate of d at point k is d(k), the estimate  $\hat{y}(k+m)$  of y(k+m) can be obtained with the CAR model (eq. (11) )calculating from k + 1 to k + m in the way of recursive prediction, that is:

$$\hat{y}(k+m) = \sum_{i=1}^{n_0} a_i \hat{y}(k+m-i) + \sum_{i=0}^{n_1} b_i u_1(k+m-i) + \sum_{i=0}^{n_2} c_i u_2(k+m-i) + d(k)$$
(12)

To derive a modification algorithm, we use several assumptions. First, it is assumed that TDM gives a measured value at point k + m, which, after being subtracted the base value  $T_0$ , is denoted by  $T_d(k+m)$  and used as the observation of y(k + m). Second, the observation noise term (denoted by  $\zeta(k+m)$ ) is assumed to be an independent Gaussian noise term with zero mean, i. e.

$$T_d(k+m) = y(k+m) + \zeta(k+m)$$
 (13)

Third, variations of model bias d are assumed to fit the generalized random-walk model, i.e.

$$d(k+m) = d(k) + w(k+m)$$
 (14)  
where  $w(k+m)$  is assumed to be an independent Gaussian white noise term with zero mean.

With these assumptions and steadystate Kalman filtering principle, an algorithm for adaptive modification of the model

bias d can be derived.

$$d(k+m) = \left[\frac{T_d(k+m) - \hat{y}(k+m)}{K_m}\right] \times d(k) + K_f$$
 (15)

where  $K_{\rm f}$  is a modification factor (  $K_{\rm f}$  < 1), which has the meaning of steady-state Kalman filter gain;  $T_d(k+m)$  is the observation given by TDM at point k + m;  $\hat{y}(k +$ m) is the P. E. value given by the CAR model at point k + m;  $K_m$  is calculated with the following recursive equation:

$$K_m = \sum_{i=1}^{n_0} a_i K_{m-i} + 1$$
, If  $m < i$ , then  $K_{m-i} = 0$  (16)

 $n_0$  is the sub-order of the auto -rewhere gressive part in the CAR model. ---

### Resetting of Starting Points of P. E.

After being used to modify the model bias d,  $T_d(k + m)$  is also used to reset starting points  $\hat{y}(k+m-i)$ ,  $i=0,1,\dots,n_0$ , which will be used in P. E. of DBBT during next interval  $(k + m + 1) \sim (k + 2m)$ . Because the estimate of d in the interval (k $+1) \sim (k+m)$  has been modified at point k + m from d(k) to dd(k + m), the first modified values (denoted by  $\hat{y}_1(k+m-i)$ of  $\hat{y}(k+m-i)$  can be obtained with eq. (11)

$$\hat{y}_1(k+m-i) = \hat{y}(k+m-i) + K_{m-i} \cdot |\hat{d}(k+m) - \hat{d}(k)|$$
 (17) where  $K_{m-i}$  is still calculated with the

same recursive algorithm as eq. (16).

Because at point k and point k + m the innovations provided by the observations of TDM are:

$$\varepsilon(k) = T_{d}(k) - \hat{y}_{1}(k)$$

$$\varepsilon(k+m) = T_{d}(k+m)$$

$$- \hat{y}(k+m)$$
(18)

the innovations at k + m - i may be approximately calculated with

$$\varepsilon(k+m-i) = \varepsilon(k) + (m-i)/m$$

$$\times |\varepsilon(k+m) - \varepsilon(k)|,$$

$$0 < i < m$$
(20)

Now, by imitating the steady-state Kalman filtering, the second modified values (denoted by  $\hat{y}_2(k+m-i)$ ) of  $\hat{y}(k+m-i)$  can be obtained:

$$\hat{y}_2(k+m-i) = \alpha \cdot \varepsilon(k+m-i)$$

$$\hat{y}_1(k+m-i) \tag{21}$$

where the modification factor a(a < 1) may be determind as a suitable constant by tests, but, when such manual operations as anode setting, manual feeding, etc, or anode effects take place during  $k \sim (k+m)$  due to poor reliability of  $\hat{y}(k+m)$ , a=1 should be set, let the resetting of the starting points of P. E. depend entirely on the observations of TDM. By replacing k,  $\hat{y}(k-i)$  and d in the CAR model (eq. (11)) with k+m-i,  $\hat{y}_2(k+m-i)$  and d(k+m), respectively, the resetting of the starting points of P. E. is realized.

### 3.5 General Constitution of APEM

From the above discussion, we can sum up the general constitution of APEM in Fig. 1.

### 4 VERIFICATION OF APEM

Since we have not carried on the testing of TDM together with that of APEM, we chose 4 h as the sampling interval and sampled the primary temperature values from those sampled by the data sampling system, so as to simulate observations of TDM, and then, to verify APEM.

By off-line testing, it has been shown that APEM can solve the problem of DBBT estimation as long as cell conditions are basically normal (i.e. except anode effects and about 1 h after an anode effect, as well as seriously abnormal cell conditions). Generally, estimation deviations are within  $\pm 3$  °C.

Fig. 2 shows a testing example.

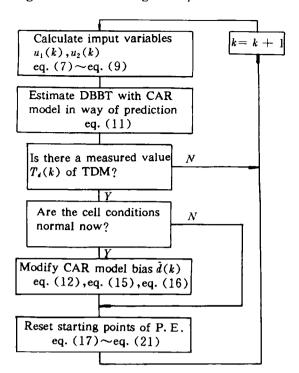


Fig. 1 General constituion of APEM

Finally we should point out that: (1) in actual use, the estimation accuracy of APEM will depend, to a great extent, on the measurement accuracy of TDM; (2) APEM does not demand that the sampling interval (m) of TDM is constant, hence, in order to keep observations of TDM being as close as possible to the actual values of DBBT, and then, to enhance the estimation accuracy of APEM, measurements of TDM can be arranged at the moments that the input variables are quite stable; (3) with average-time concept, it can be concluded that, as long as the mean of observation errors of TDM is equal to zero, the unbiassedness of DBBT estimation can be ensured.

### 5 CONCLUSIONS

(1) An adaptive predicting estimation model (APEM) of dynamically balanced

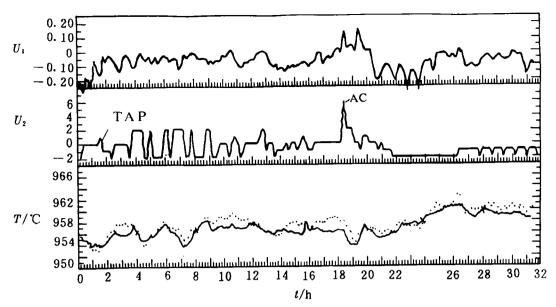


Fig. 2 Estimated curve and measured values of DBBT

 $u_1$ — the input variable depending on heating power Q(k);  $u_2$ — the input variable depending on equivalent feeding rate  $F_{eq}(k)$ ;  $\times$ —measured temperatures (°C) from simulated TDM; datted line—measured temperatures (°C) of DBBT; solid lines estimated curve of DBBT from APEM; TAP—aluminium tapping; AC—anode setting; Bath compisition;  $CR \approx 2.7$ ,  $\frac{9}{0}CaF_2 \approx 5$ ; Sampling period for primary signals = 1 s; Sampling period for DBBT estimation = 12 min.

bath temperature (DBBT) has been established, which co-operates with a temperature measuring device, the later employing the method of temperature dynamic measurement (TDM) and operating in intermittent way, APEM consists of a DISO CAR model, an algorithm for adaptive modification of the CAR model bias, and an algorithm for resetting of P. E. (predicting estimation) starting points.

(2) By using the data measured on a test cell, it has been verified that APEM possesses quite satisfactory estimation accuracy. The values of bath temperature measured intermittently (once per several hours) by TDM can be used to modify the slow-timevarying model bias, and therefore, APEM has the ability to be adaptive to the slow timevarying process of the characteristics of the heat balance (HB) of a cell.

(3) Having has realized quantitative de-

scription of the relationship between the output—DBBT and the inputs—heating power and equivalent feeding rate, of the HB system, APEM can be used in the design of a HB supervision & control system. The combination of the parameter estimation method with TDM can become an effective way to solve the problem of HB supervision & control of point-feeding aluminium reduction cells.

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