ANALYSIS OF DRAWING UNDER LUBRICATION OF NON-NEWTONIAN LUBRICATING MATERIALS①

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ABSTRACT Based on the rheologic features of non-Newtonian materials, an equation of the drawing under the lubrication of non-Newtonian lubricating material through upper-bound analysis was presented. The equation includes the theoretical results of Avitzur B under the lubrication conditions of invaricant fricition factor hydrodynamic lubricating, and an optimum thickness-diameter ratio is proposed, which is important for practical application.

Key words drawing force rheology upper-bound method non-Newtonian material tribology

1 INTRODUCTION

In drawing thread, wire, and bar, ox-oil lime, soap, grease are usually used as lubricating materials, a layer of desiccated lubrication is thus formed on the surface of drawing blank. Most of the available calculating equations of the drawing force are derived assuming that the friction between the drawing blank and the die coincides with Coulomb's law, or assuming that the invaricant tangential stress factor is far from steel wire production process, so they could not fit the practical production. In recent years, Avitzur has achieved many results through analysing metal forming process using hydrodynamic lubrication[1]. The analysis of drawing force taking into consideration the effect of the friction feature of surface lubrication coating is still open to be fulfiled. This paper will give some results on the research of this problem using the method of rheology and tribology.

2 KINEMATICALLY ADMISSIBLE VELOCITY FIELD

As shown in Fig. 1, the deformation zone

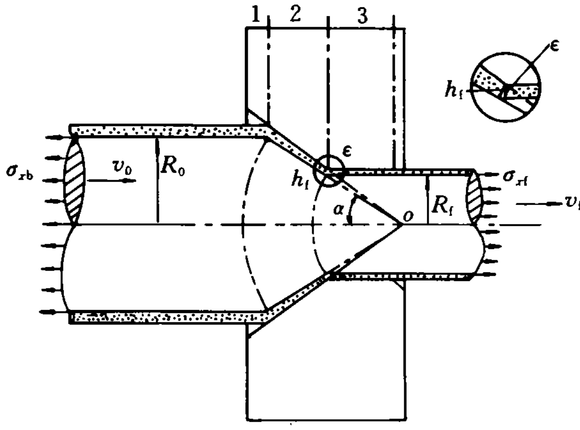

Fig. 1 Illustration of deformation zone

(zone 2) is shaped like an arch; the center of the bar enters the deformation zone earlier than outer. The fricitional resistance to outer metal is larger than that of inner, therefore the inner metal flows fast, and outer metal flows slow. Because the cylindrical bar drawn in the conical die is deformed axial-symmetrically and not epicyclic-twisting, metal mass points on the symmetric-axial flow along the axial direction, and other metal mass points

① Received Oct. 17, 1994; accepted Jun. 17, 1995

flow along conical generating line of deformation cone to the vertex of cone. As shown in Fig. 2 assuming at metal mass point flowing velocity in any point of deforming cone is along the diameter-direction and joins at the vertex of cone. Zone 1 and zone 3 are rigid zones, they move rigidly along axial direction with velocities of v_0 and v_f respectively. According to the conservation law of metal volume, we have

$$v_0 = v_f(R_f/R_0)^2 \quad (1)$$

In order to simplify mathematical analysis, Γ_1 and Γ_2 are treated as surfaces of concentric spheres whose centers are o and radii are r_0 and r_f. After one point of rigid zone one moves along axial-direction on to the spherical surface Γ_2 with the velocity v_0 and enters deformation zone 2, it flows along diameter-direction to the spherical surface Γ_1. And then

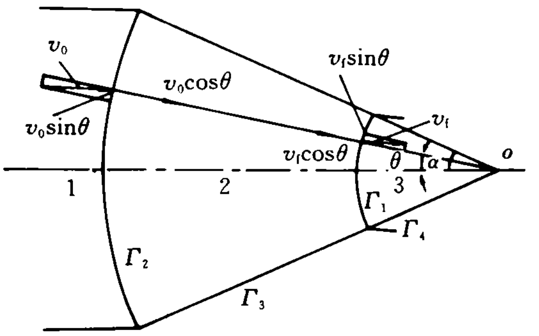

Fig. 2 Velocity field

the metal mass point leaves zone 2, enters rigid zone 3, moves along axial-direction with velocity v_f.

Therefore, Γ_1 and Γ_2 are velocity discontinuous surfaces. The female die is fixed on the carrier and keeps stationary, the tangential velocity is discontinuous on the conical surface Γ_3 and Γ_4, the normal velocity equals zero constantly. As shown in Fig. 2, using Avitzur B spherical centripetal velocity field, the velocity discontinuity in spherical coordinates(r, φ, θ) is[1, 2]:

Velocity field

$$\left.\begin{aligned} &\text{Zone 1: } v_0 \\ &\text{Zone 2: } U_r = v = - v_f r_f^2 \cos\theta / r^2 \\ &\qquad U_\theta = U_\varphi = 0 \\ &\text{Zone 3: } v_f \end{aligned}\right\} \quad (2)$$

Velocity discontinuity

$$\left.\begin{aligned} &\text{Surface } \Gamma_1: \quad \Delta v_1 = v_f \sin\theta \\ &\text{Surface } \Gamma_2: \quad \Delta v_2 = v_0 \sin\theta \\ &\text{Surface } \Gamma_3: \quad \Delta v_3 = v_f r_f^2 \cos\theta / r^2 \\ &\text{Surface } \Gamma_4 \quad \Delta v_4 = v_f \end{aligned}\right\} \quad (3)$$

3 DEFORMATION POWER

The deformation powers in zone one and two are zero, because they are all rigid zones. Zone two is symmetrical about φ axis, and the strain velocity is

$$\left.\begin{aligned} &\dot{\varepsilon}_{rr} = \frac{\partial U_r}{\partial r} = - 2\dot{\varepsilon}_{\theta\theta} = \\ &\quad - 2\dot{\varepsilon}_{\varphi\theta} = 2 v_f r_f^2 \cos\theta / r_3 \\ &\dot{\varepsilon}_{r\theta} = (1/2) v_f r_f^2 \sin\theta / r_3 \\ &\dot{\varepsilon}_{\theta\varphi} = \dot{\varepsilon}_{r\varphi} = 0 \end{aligned}\right\} \quad (4)$$

Any particle in zone two moves along radial direction and has a constant angle θ, then eq. (4) becomes $\dot{\varepsilon}_{rr}:\dot{\varepsilon}_{\theta\theta}:\dot{\varepsilon}_{\varphi\varphi}:\dot{\varepsilon}_{r\varphi}:\dot{\varepsilon}_{\varphi\theta}:\dot{\varepsilon}_{\theta r} = 1: -2: -2:1/4:0:0$, which indicates proportional straining. The lubricating layer's thickness at the exit is ε, the radius of the cylinder part of the die is R_f, and when the product's radius is R_f-ε, substituting (5) by eq. (4) gives[3]

$$\dot{W} = \frac{2}{\sqrt{3}} \sigma_s \int_v \sqrt{\frac{1}{2} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}}\, dv \quad (5)$$

The deformation power in zone 2 is

$$\dot{W} = 2\pi \sigma_s v_f (R_f - \varepsilon)^2 f(\alpha) \ln\left(\frac{R_0}{R_f - \varepsilon}\right) \quad (6)$$

where

$$f(\alpha) = \frac{1}{\sin^2\alpha}\left\{1 - \cos\alpha\sqrt{1 - \frac{11}{12}\sin^2\alpha} + \frac{1}{\sqrt{11 \times 12}} \ln\left[\left(1 + \sqrt{\frac{11}{12}}\right) \div \left(\sqrt{\frac{11}{12}}\cos\alpha + \sqrt{1 - \frac{11}{12}\sin^2\alpha}\right)\right]\right\}$$

4 EXHAUST POWER

(1) Powers exhausted to velocity discontinuity surface Γ_1 and Γ_2:

$$\dot{W}_{s1,2} = \int_{\Gamma_1+\Gamma_2} \tau \Delta v \mathrm{d}s = (2/\sqrt{3})\sigma_s \pi v_f (R_f - \varepsilon)^2 \times (\alpha/\sin^2\alpha - \mathrm{ctg}\alpha) \qquad (7)$$

(2) Power exhaused by fricition:

The lubricating grease is non-Newtonian material, the rheology's features of which can be represented by [5, 6]

$$\tau = \tau_s + \Phi(\dot{\gamma})^n \qquad (8)$$

where τ_s —yield stress; $\dot{\gamma}$ —shearing strain rate; τ —shearing stress; Φ, n —constants. As shown in Fig. 3, when $\tau_s = 0$, eq. (8) represents pseudoplasticity and expansible fluid; when $n = 1$, eq. (8) is Newtonian fluid, Φ is kinetic viscosity; $\tau = \tau_s + \Phi(\dot{\gamma})^n$ represents plastic body, or Bingham body. When sheering stress exceeded τ_s, flowing occurred. If $\tau = \tau_s = m\sigma_s/\sqrt{3}$ is assumed, that is the assumption of invaricant fricition factor, m is fricition factor($0 \leqslant m \leqslant 1$). Therefore, it is concluded that eq. (8) is of universal significance.

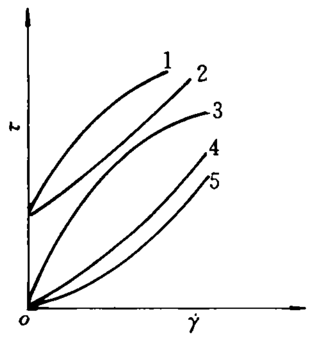

Fig. 3 Shearing strain rate ($\dot{\gamma}$) versus shearing stress (τ)

1—Grease; 2—Pseudo-plasticity; 3—Expansible; 4—Newtonian body; 5—Plasticity

The velocity distribution rule of lubricating layer is that the velocity of lubrication adhering to the surface of the female die is zero, and that the flowing velocity of lubrication adhering to the surface of the bar blank equals the velocity of the surface mass point of the bar:

$$v = v_f r_f^2 \cos\alpha / r^2$$

If the lubricating layer's velocity is linearly distributed along the lubricating layer's thickness, the average velocity of the lubricating layer will be

$$\bar{v} = (1/2) v_f r_f^2 \cos\alpha / r^2 \qquad (9)$$

When flowing through a ring region, with a radius of $R = r\sin\alpha$ and a gap of h, the lubricating layer's flowing rate is

$$\dot{V}_l = 2\pi r h \bar{v} \sin\alpha \qquad (10)$$

The conical part of the female die is $r = r_f$, the lubricating layer's thickness h_f is

$$h_f = \varepsilon/\cos\alpha \qquad (11)$$

Because of the constancy of the lubricating layer's volume, we can get the function of the lubricating layer's thickness r from eqs. (10) and (11)

$$h = h_f r / r_f = \varepsilon r / (r_f \cos\alpha) \qquad (12)$$

The total consumed power on the lubricating layer's volume is

$$\dot{W}_s = \int_s \tau |\Delta v| \mathrm{d}s \qquad (13)$$

where

$$\left.\begin{aligned}
&\tau = \tau_s + \Phi(\dot{\gamma})^n \\
&\Delta v = v = v_f r_f^2 \cos\alpha / r^2 = v_f (R_f/R)^2 \cos\alpha, \\
&h = (r/r_f)\ (\varepsilon/\cos\alpha) = (R/R_f)\ (\varepsilon/\cos\alpha), \\
&\mathrm{d}s = 2\pi R\ \mathrm{d}R/\sin\alpha,\ \dot{\gamma} = \Delta v/h, \\
&\dot{W}_{s3} = \int_{R_f}^{R_0} [\tau_s + \Phi(\dot{\gamma})^n] \Delta v 2\pi R \mathrm{d}R \div \\
&\quad \sin\alpha = 2\pi \mathrm{ctg}\alpha\ v_f R_f^2 \{\tau_s \ln(R_0/R_f) + \\
&\quad \Phi/(3n)\ v_f^n \cos^{2n}\alpha/\varepsilon^n [1 - (R_f/R_0)^{3n}]\}
\end{aligned}\right\} \qquad (14)$$

The fricition consumed power of cylindrical surface is

$$\dot{W}_{s4} = \tau_s v_f 2\pi R_f L + \Phi(v_f^{n+1}/\varepsilon^n) 2\pi R_f L = 2\pi R_f L v_f [\tau_s + (v_f/\varepsilon)^n \Phi] \qquad (15)$$

5 DRAWING POWER

(1) Drawing force power

$$J^* = \pi v_f (R_f - \varepsilon)^2 \sigma_{xf} \qquad (16)$$

(2) Post-drawing tensile force power

$$\dot{W}_b = -\int_{S_t} T_i v_i \mathrm{d}s = \pi v_0 R_0^2 \sigma_{xb} = \pi v_f R_f^2 \sigma_{xb} \qquad (17)$$

6 FORMULA INFERENCE

According to the upper-bound law[1, 2],

$$J^* = \frac{2}{\sqrt{3}}\sigma_s\int_v \sqrt{\frac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}\,dv + \int_{s_\Gamma} \tau|\Delta v|ds - \int_{s_t} T_i v_i ds \qquad (18)$$

Substituting R_f in eqs. (14), (15) and (17) to R_f-ε, and substituting (18) by eqs. (16), (6), (7) gives

$$\frac{\sigma_{xf}}{\sigma_s} = \frac{\sigma_{xb}}{\sigma_s} + 2f(\alpha)\ln(\frac{R_0}{R_f-\varepsilon}) + \frac{2}{\sqrt{3}}(\frac{\alpha}{\sin^2\alpha} - \text{ctg}\alpha) + \frac{2}{\sigma_s}(\frac{v_f}{\varepsilon})^n\Phi \times \{\frac{\text{ctg}\alpha\cos^{2n}\alpha}{3n}[1-(\frac{R_f-\varepsilon}{R_0})^{3n}] + \frac{L}{R_f-\varepsilon}\} + \frac{2\tau_s}{\sigma_s}[\frac{L}{R_f-\varepsilon} + \text{ctg}\alpha\ln\frac{R_0}{R_f-\varepsilon}] \qquad (19)$$

From eq. (19) we can see that σ_{xf} rises, when ε reduces, the inner deformation power rises when ε raises. Taking σ_{xf} as a function of ε seek optimum ε_0.

Let $A = R_0/R_f$, $B = L/R_f$, $X = \varepsilon/R_f$, then eq. (19) can be written as

$$\frac{\sigma_{xf}}{\sigma_s} = \frac{\sigma_{xb}}{\sigma_s} + 2f(\alpha)\ln\frac{A}{1-X} + \frac{2}{\sqrt{3}} \times (\frac{\alpha}{\sin^2\alpha} - \text{ctg}\alpha) + \frac{2\Phi}{\sigma_s}(\frac{v_f}{R_f})^n\frac{1}{X^n} \times \{\frac{\text{ctg}\alpha\cos^{2n}\alpha}{3n}[1-(\frac{1-X}{A})^{3n}] + \frac{B}{1-X}\} + \frac{2\tau_s}{\sigma_s}[\frac{B}{1-X} + \text{ctg}\alpha\ln\frac{A}{1-X}] \qquad (20)$$

Derivation of X in eq. (20) and let it equal zero, considering that X is very small, $1 - X \approx 1$, optimum X_0 is obtained

$$X_0 = \left[(\frac{v_f}{R_f})^n\frac{\Phi n}{\sigma_s}\frac{\frac{\text{ctg}\alpha\cos^{2n}\alpha}{3n}(1-\frac{1}{A^{3n}}) + B}{f(\alpha) + \frac{\tau_s}{\sigma_s}(B + \text{ctg}\alpha)}\right]^{\frac{1}{n+1}} \qquad (21)$$

$$\varepsilon_0 = R_f X_0 \qquad (22)$$

Substituting eq. (20) by eq. (21), we can get the minimum stretch stress. Wistreich J G[4] proved that. The metal wire's radius at exit is smaller than the female die's radius, and the difference is lubricating layer's thickness ε.

7 CONCLUSIONS

(1) When $\tau_s = 0$, $\Phi = \eta$, $n = 1$, eqs. (20) and (21) change to Avitzur B result under the hydraulic lubrication condition.

(2) When $\tau_s = m\sigma_s/\sqrt{3}$, $\Phi = 0$, eq. (20) changes to Avitzur B conclusion under the lubrication condition of the invaricant fricition factor.

(3) According to eq. (20), we can get the drawing force under the lubrication of grease and soap. From eq. (21), we can get the optimum ratio between lubricating layer's thickness and female die's radius at exit. Eq. (20) is more general, it includes Avitzur B results as special cases.

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(Edited by Peng Chaoqun)