THE ESTABLISHMENT OF CALCULATION FORMULA FOR CASTING-ROLLING FORCE OF VISCOUS FLUID AND THE INFLUENCE OF TECHNOLOGICAL FACTORS①

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ABSTRACT Assuming the deformed body to be viscous fluid, a calculation formula of casting-rolling force for aluminium strip was established based on Navier-Stokes equation. The theoretical results were verified by experiments. And the influence of various technological factors on casting-rolling force was discussed.

Key words casting-rolling force viscous fluid flow function

1 INTRODUCTION

Casting-rolling force is a very important parameter in successive casting-rolling process, whereas hot-rolling formulas have constantly been used for the calculation of casting-rolling force, here the difference between casting-rolling and hot-rolling is neglected. In fact, the physical models in the forming processes of the two metals are not completely the same. though the contact surface of the metal has solidified, its inside is still in the state of liquid-solid viscous fluid. But metal is totally in solid state. The hot-rolling formula proposed normally is used to the condition that the deformed metal is considered as rigid-plastic and the solution to the formula requires the simultaneous equations of equilibrium differential in plastic mechanics and the criterion of plasticity. So, it is not reasonable to use these formulas in casting-rolling process. An urgent need in casting-rolling is to get a calculation formula to guide the production and online control. This parper is based on Navier-Stokes equation of hydromechanics. The calcultion formulas for casting-rolling force are established by using the flow function to set a velocity field and by simplifying the external force conditions in the contact boundary.

2 THE ESTABLISHMENT OF FORMULAS

The Navier-Stokes equation is as follows:

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = \rho F_i - \mathrm{grad}p + \eta\Delta^2 v_i \quad (i = x,\ y,\ z) \qquad (1)$$

where p —hydrostatic stress; F_i —volume force; ρ —denstity; v_i —velocity component.

The flow feature of metal forming in casting-rolling process is extremely similar to that of viscous fluid. Considering the deformed metal in casting-rolling are as viscous fluid, the ripe theory in viscosity hydromechanics can be used to discribe the flow feature of the deformed metal in casting-rolling area.

Formula (1) is the common form of N-S equation. It's not easy to get the solution of formula (1) analytically. But, if some necessary assumptions are given, it's also possible to solve formula (1).

These assumptions are as follows:

Considering the deformed body as linear viscous fluid, the imcompression condition

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$d_{iv}v_i = 0$ is satisfied; the casting-rolling area is considered as 2-D flow; the acceleration of casting-rolling can be neglected as the casting-rolling speed is very slow, that is, $dv_i/dt = 0$; the volume force can also be neglected since it is not as important as casting-rolling force in deformation; the velocity field can be assumed to be a potential field but not a curl field, $\tau = k$ at boundary(k: shear yield stress). Accepting the assumptionts above, formula (1) can be written as:

$$\left.\begin{aligned} \frac{\partial p}{\partial x} - \eta\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) = 0 \\ \frac{\partial p}{\partial y} - \eta\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right) = 0 \end{aligned}\right\} \quad (2)$$

2.1 *Velocity Field*

Replacing arc with chord at the contact boundary of the casting-rolling area (See Fig. 1), we have:

$$h_x = h_1 + \frac{\Delta h}{l}x \quad (3)$$

The flow function can be set up as:

$$\varphi = -\frac{h_1 l y v_1}{l h_1 + \Delta h x} \quad (4)$$

So the following velocity field can be derived:

$$\left.\begin{aligned} v_x &= -\frac{v_1 h_1 l}{h_1 l + \Delta h x} \\ v_y &= -\frac{\Delta h h_1 l v_1 y}{(h_1 l + \Delta h x)^2} \end{aligned}\right\} \quad (5)$$

This velocity field is kinematic permissible since we can prove that formula (5) satisfies the equation of continuity.

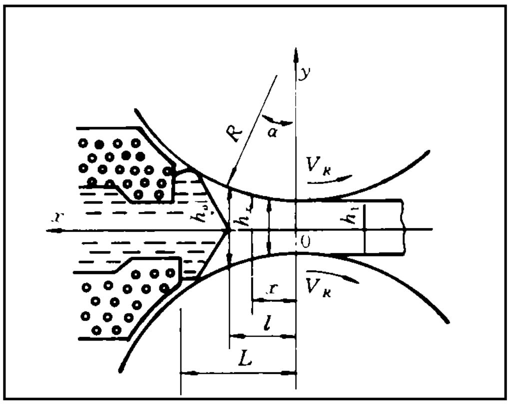

Fig. 1 The casting-rolling area give a hint with the eyes

2.2 *The Establishment of Calculation Formula of Casting-rolling Force*

Putting the known veloctity field into formula (2) and arranging in order:

$$\frac{\partial p}{\partial x} = -\frac{2v_1 B \Delta h^2 \eta}{(B + \Delta h x)^3} \quad (6)$$

where $B = h_1 l$

According to hydromechanics theory[1], we know that the shear stress is in direct proportion to the velocity gradient, so

$$\tau_{xy} = \frac{2v_1 B \Delta h^2 \eta y}{(B + \Delta h x)^3} \quad (7)$$

when $y = \frac{h_x}{2}$, $\tau_{xy} = \pm \tau_{xn}$ (τ_{xn}; shear stress on contact arc surface) and considering η to be only the function of x

$$\eta = \pm \frac{\tau_{xn}(B + \Delta h x)^2 l}{v_1 D \Delta h^2} \quad (8)$$

If the nip angle is very small and $\tau_{xn} = \tau_x$, (τ_x: frictional force on contact surface) approximately, formula (6) can be written as

$$\frac{\partial p}{\partial x} = \mp \frac{2\tau_x l}{B + \Delta h x} \quad (9)$$

Since $\frac{\partial p}{\partial y} \ll \frac{\partial p}{\partial x}$, taking $\frac{\partial p}{\partial y} = 0$, we have $\frac{\partial p}{\partial x} = \frac{dp}{dx}$; when $\tau_x = k$, formula (9) can be integrated

$$p = \mp \frac{2kl}{\Delta h}\ln(D + \Delta h x) + C \quad (10)$$

where C —arbitary constant, determined by boundary condition.

If the front tension is considered, the boundary condition in forward slip area will be

$$p = \sigma_q - k_1 \text{ when } x = 0 \quad (11)$$

where k_1 —resistance to deformation at exit end; σ_q —front tension stress.

The boundary condition in backward slip area is

$$p = 0 \text{ when } x = 1 \quad (12)$$

Using the boundary condition, the constans in forward and backward slip areas can be obtained respectively:

$$C_q = \sigma_q - k_1 + \frac{2kl}{\Delta h}\ln B \quad (13)$$

$$C_h = -\frac{2kl}{\Delta h}\ln(h_0 l) \quad (14)$$

Putting the constants into formula (10), the following distribution formulas of compression stress in forward and backward slip are as can also be obtasined:

$$p_q = \frac{2kl}{\Delta h}\ln(\frac{B}{B + \Delta hx}) - \sigma_q - k_1 \quad (15)$$

$$P_i = \frac{2kl}{\Delta h}\ln(\frac{B + \Delta hx}{h_0 l}) \quad (16)$$

The forward slip area's length is got by necessary mathematic derivation

$$l_q = \left\{\sqrt{(h_0 lB)/\exp[\frac{\Delta h}{2kl}(k_1 - \sigma_q)]} - B\right\} \frac{1}{\Delta h} \quad (17)$$

Integrating formula(15)、(16) along the length of deformation area, the average unit compression stress of contact surface can be obtained

$$\bar{p} = \frac{2k}{\Delta h^2}\{(B + \Delta hl_q)\ln(h_0 Bl) - 2(B + \Delta hl_q)\ln(B + \Delta hl_q) + \Delta h(2l_q - l)\} + \frac{l_q}{l}(\sigma_q - k_1) \quad (18)$$

If forward and backward slip areas are symmetric, the equation above can be written as

$$\bar{p} = 2kA\ln G + \frac{\sigma_q - k_1}{2} \quad (19)$$

where $A = \frac{\bar{h}l}{\Delta h^2}$; $G = \frac{h_0 h_1}{h^{-2}}$; $\bar{h}$ —mean thickness; k —when calculate, we can see the reference book[2].

3 DISCUSSION ON THE INFLUENCE OF CASTING-ROLLING TECHNOLOGICAL FACTORS ON $\bar{P}/2K$

To verify the correctness of theoretical formula and to determine the influence trend of technological factors on $\bar{p}/2k$, a casting-rolling force was done on a two-high horizontal experimental rolling mill and the data of casting-rolling force was obtained by measurement. In addition, the casting-rolling forces of d 650 mm and d 960mm type rolling mill were also obtained by measurement under the condition of industrial production(See Fig. 2—4).

$$\frac{\bar{p}}{2k} = \sqrt{\frac{R\bar{h}}{\Delta h^2}}\ln(\frac{h_0 h_1}{h^2}) + \frac{\sigma_q - k_1}{4k} \quad (20)$$

3.1 *The Influence of Δh on $P/2K$*

From Fig. 2 we see that as the absolute draft Δh increases, $\bar{p}/2k$ will also increase nonlineary, showing a conceave-down curve, in the beginning $\bar{p}/2k$ incerases quickly and then trends towards becoming slower. The reason is that with the increase of Δh, the volume of deformed body increases and leads to the increase of $\bar{p}/2k$. Besides, the increase of Δh, will certainly result in the lengthening of casting-rolling area. If the casting-rolling speed remains canstant, the deformed metal will stay longer, leading to the decrease of mean temperature of casting-rolling strip in deformation area and to the increase of resistance to deformation, resulting in the increase of $\bar{p}/2k$.

3.2 *The Influence of R on $\bar{p}/2k$*

The influence of radius R on $\bar{p}/2k$ is shown in Fig. 3. With the increase of R, $\bar{p}/2k$ has an approximate linear increase. This is becuse that when R increases the length of casting-rolling area will also increase, leading to the strengthening of heat exchange in casting-rolling area and to the increase of casting-rolling speed and resistance to deformation. As a result, $\bar{p}/2k$ increases.

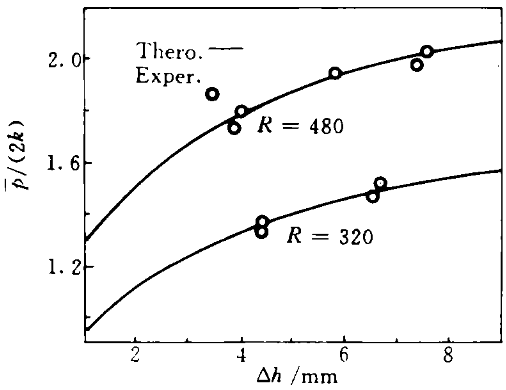

Fig. 2 The influence of Δh on $\bar{p}/2k$

3.3 *The influence of σ_q on $\overline{p}/2k$*

As shwon in Fig. 4, with increase of front tension σ_q, $\overline{p}/2k$ decreases linearly. The reason is that the function of front tension makes the stress state of 3-D campression in deformation area become the stress state of 2-D compression and tension in the other direction. The resistance to deformation is reduced, leading to the decrease of $\overline{p}/2k$.

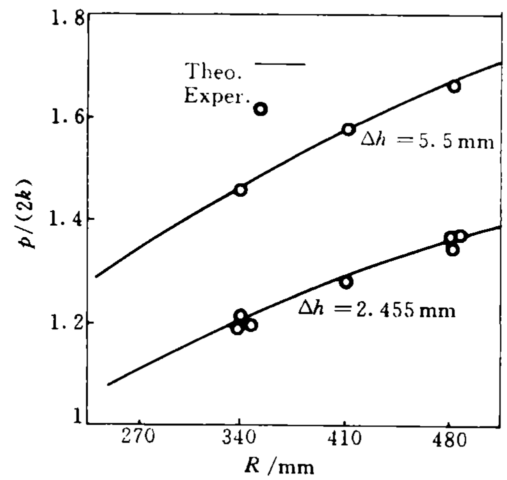

Fig. 3 The influence of R on $\overline{p}/2k$

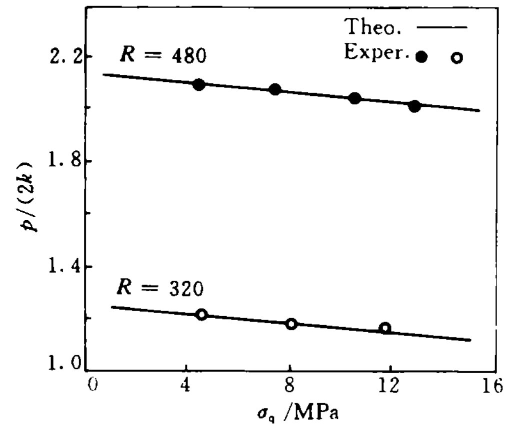

Fig. 4 The influence of on $\overline{p}/2k$

4 CONCLUSION

The 2-D velocity field that approximately satisfies the boundary condition of velocity was set up by means of flow function. And the calculation formula of casting-rolling foce was established on the basis of N-S equation. The formula is easy to use. Its computing mistake is controlled in 15%. So it can be used for engineering calculations. Comparing the theoretical value with the measured value, we see that to assume the deformed metal in casting-rolling area as viscous fluid is reasonable. the relation between Δh, R and $\overline{p}/2k$ is that with the increase of Δh and R, $\overline{p}/2k$ trends to increase, but it will decrease with the increase of σ_q.

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