DETERMINATION OF STATES OF STRESS AND STRAIN AT POLE OF HYDRAULIC BULGE ORTHOTROPIC SHEET SAMPLE①

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ABSTRACT The physical characteristics of deformation of the material at the pole of the hydraulic bulge specimen was analysed. Based on this analysis and Hill's hypothesis about the moving track of the material particle of bulge specimen, a method was established to determine the ratio of the two principal stresses at the pole of the orthotropic sheet sample directly from the experimental data, thus to determine the states of stress and strain of the pole. The theoretical value of the ratio stated relates to the yield function used. The application proves that the Hosford type yield function can give better results and is convenient for use.

Key words orthotropic sheet hydraulic bulge specimen pole stress state strain state

1 INTRODUCTION

As is well known, the hydraulic bulge test of sheets in a round groove die has become one of the most important means for measuring the basic properties of materials and examing the constitutive equations of anisoplasticity. So far, the hypothesis that the material at the pole of the specimen is deformed under equibiaxial tension state has been generally used. Obviously, for sheets with planar anisotropy, this hypothesis is not practical. However, no perfect solutions in theory and measurement have been found yet to determine the stress state at the pole.

In order to determine the ratio of two principal stresses at the pole of the orthotropic sheet sample, the author once put forward an approximate equation predicting indirectly based on the yield function used[1]. This paper tries further to remove the dependence on the specific yield function used and to establish a method to determine the stated ratio directly from the experimental data, consequently determine the states of stress and strain, then ensure the anisotropy to be expressed truly.

2 DETERMINATION OF RATIO OF TWO PRINCIPAL STRESSES AT POLE OF ORTHOTROPIC SHEET SAMPLE

2.1 *Theory*

2.1.1 Deformation Characteristics of Material at Pole of Bulge Specimen

The sketch of a bulge specimen is shown in Fig. 1, in which point *a* is the pole. Based on the orthotropy of the anisotropic properties of materials and the symmetry deformation of the bulge specimen, as for those material particles on rolling and transverse lines passing through point *a*, their two principal stresses are obviously just the normal stresses along the rolling direction (σ_0) and that along the transverse direction (σ_{90}). The yield surface

① Supported by the National Natural Science Foundation of China and Aviation Science Foundation;

Received Aug. 3, 1994; Accepted Sep. 29, 1994

profiles of these materials can be expressed by Fig. 2, in which point a expresses the state of stress at the pole of the bulge specimen and the left and right neighbouring areas of point a express, respectively, the stress states of the material particles on the transverse and rolling lines passing through point a. It is clear that the points at the left of point a, e. g. point b, express the cases that σ_{90} plays the main role, i. e. in the deformation work increment:

$$dW = \sigma_0 d\varepsilon_0 + \sigma_{90} d\varepsilon_{90}$$

the later term is larger than the former term; the points on the right of point a, e. g. point c, express the opposite cases. When point b and point c draw close point a, the effect of the other principal stress increases. Obviously, the deformation at the pole of the bulge specimen-results from the combination of the limits of the effects of the two principal stresses. Thus, the deformation at this point must occur under the condition

$$\sigma_0 d\varepsilon_0 = \sigma_{90} d\varepsilon_{90} \quad (1)$$

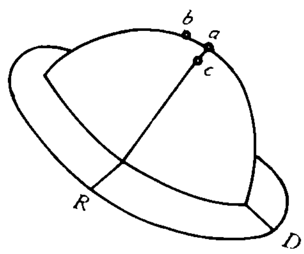

Fig. 1 Sketch of a hydraulic bulge specimen

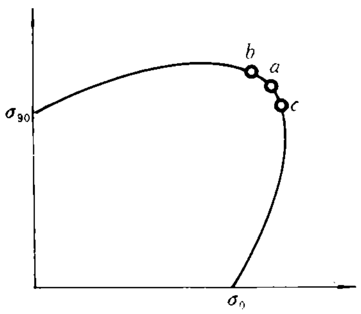

Fig. 2 Yield surface profile

When χ_p is used to express the ratio of the principal stresses, it can be expressed by

$$\chi_p = \sigma_{90}/\sigma_0 = d\varepsilon_0/d\varepsilon_{90} \quad (2)$$

The justification of the above mentioned ideas can be referred to as follows.

It is well known that when an element of material is deforming, the following yield criterion should be satisfied:

$$f = k\sigma_i^m \quad (3)$$

where f stands for an arbitrary orthotropic yield function (here expressed with σ_0 and σ_{90}); m is its power value; k is a constant; σ_i is equivalent stress, and it is generally assumed to be irrelative to the state of stress, namely for the state of planar stress, $\frac{\partial \sigma_i}{\partial \chi} = 0$ because the ratios of the principal stresses

$$\chi = \sigma_2/\sigma_1 \quad (\sigma_1 > \sigma_2) \quad (4)$$

are the expression of the different states of stress. Thus, the partial differential of eq. (3) to χ is:

$$\frac{\partial f}{\partial \sigma_0}\frac{\partial \sigma_0}{\partial \chi} + \frac{\partial f}{\partial \sigma_{90}}\frac{\partial \sigma_{90}}{\partial \chi} = 0$$

If the flow rule corresponding to the yield function

$$\left.\begin{aligned} d\varepsilon_0 &= d\lambda \frac{\partial f}{\partial \sigma_0} \\ d\varepsilon_{90} &= d\lambda \frac{\partial f}{\partial \sigma_{90}} \end{aligned}\right\} \quad (5)$$

and eq. (1) are introduced and the prerequisites $d\lambda \neq 0$, $d\varepsilon_0 \neq 0$, $d\varepsilon_{90} \neq 0$ are noted, then at the pole of the bulge specimen, there is

$$\frac{\partial}{\partial \chi}\ln\sigma_0 + \frac{\partial}{\partial \chi}\ln\sigma_{90} = \frac{\partial}{\partial \chi}\ln(\sigma_0\sigma_{90}) = 0$$

$$(\chi = \chi_p) \quad (6)$$

This implies that the product of σ_0 and σ_{90} is a constant or takes an extreme value at the pole. It is clear that the former case is impractical and the later case is reasonable in physical concept.

2. 1. 2 Relation Between Curvature Radius of Bulge Specimen and Strain Increment

Hill assumed that in the process of the hydraulic bulge forming, moving track of material particle coincides with the normal direction of the profile of the bulge specimen[2]. Due to the symmetry of the deformation, this assumption is no doubt practical for those materials at the pole of the bulge specimen. Moreover, in this case the directions of the principal curvatures in here are consistent with the rolling and transverse directions of the sheets. Therefore, if R_0 and R_{90} are used to express the instantaneous values of the curvatures radius, when there occurs further normal displacement dh at the pole, there will occur strain increments as follows:

$$\left.\begin{aligned} d\varepsilon_0 &= dh/R_0 \\ d\varepsilon_{90} &= dh/R_{90} \end{aligned}\right\} \quad (7)$$

thus $$\frac{d\varepsilon_0}{d\varepsilon_{90}} = \frac{R_{90}}{R_0} \quad (8)$$

2.2 *Experimental Determination of Ratio of Principal Stresses at Pole of Bulge Specimen*

If eq. (8) is substituted into eq. (2), the ratio of the principal stresses at the pole of the bulge specimen, i.e. χ_p, can be obtained:

$$\chi_p = \frac{\sigma_{90}}{\sigma_0} = \frac{d\varepsilon_0}{d\varepsilon_{90}} = \frac{R_{90}}{R_0} \quad (9)$$

thus it can be seen that the instantaneous χ_p value can be determined by measuring the instantaneous curvature radii, R_0 and R_{90}, at the pole of the bulge specimen.

2.3 *Theoretical Prediction of Ratio of Principal Stresses at Pole of Bulge Specimen*

Here, the theoretical prediction of the ratio of the principal stresses is based on selecting certain proper yield functions. If f is used to express the yield function selected, from eqs. (5) and (2), it can be found that the theoretical prediction equation for χ_p is

$$\chi_p = \frac{\partial f}{\partial\sigma_0} / \frac{\partial f}{\partial\sigma_{90}} \quad (10)$$

When different yield functions are selected, the specific predicting expressions and the calculated results are different.

(1) If Hosford type yield function[3, 4] is selected, then:

$$\chi_p = \frac{r_{90}}{r_0} \frac{1 + r_0(1-\chi_p)^{m-1}}{\chi_p^{m-1} - r_{90}(1-\chi_p)^{m-1}} \quad (11a)$$

this equation can also be rewritten as

$$\chi_p^m = \frac{r_{90}(1+r_0)}{r_0(1+r_{90})} + \frac{r_{90}}{1+r_{90}}[(1-\chi_p^2) \times (1-\chi_p)^{m-2} - (1-\chi_p^m)] \quad (11b)$$

where r_0 and r_{90} are two plastic strain ratio of materials. Specific calculation is in progress using the iterative method and taking the initial value of χ_p to be 1.0.

As described in Ref. [4], if $m = 2$, the Hosford type yield function reduces to the quadratic yield function proposed by Hill[5]. For this case, from eq. (11b) it can be seen that $\chi_p = \sqrt{\frac{r_{90}(1+r_0)}{r_0(1+r_{90})}}$.

(2) If Barlat's yiled fuction[6] is selected, then:

$$\chi_p = \frac{1}{h} \frac{a + c(1-h\chi_p)^{m-1}}{a(h\chi_p)^{m-1} - c(1-h\chi_p)^{m-1}} \quad (12a)$$

It can be clearly seen that the solution of eq. (12a) is $h\chi_p = 1$.

Therefore,

$$\left.\begin{aligned} \chi_p &= \frac{1}{h} \\ h &= \sqrt{\frac{r_0(1+r_{90})}{r_{90}(1+r_0)}}^{[6]} \end{aligned}\right\} \quad (12b)$$

From eq. (12b) it is clear that the value of χ_p does not relate to the power value of the yield function and is the same as the results obtained using Hill's quadratic yield function[5]. Which is right and which is wrong? This depends on the judgement of the experimental data.

When other yield functions are selected, the relevant equation predicting χ_p -values can be derived from eq. (10).

The comparisons between the theoretical χ_p values calculated using eq. (11) and the experimental ones of several materials are shown in Table 1. As mentioned above, if $m = 2$, the results are those obtained using Barlat's yield function or Hill's quadratic yield function. As for Hosford type yield function, according to the recommendations in Refs. [3, 4], it is appropriate to take $m = 6$ for *bcc* metals and $m = 8$ for *fcc* and *hcp* metals. It can be seen from Table 1 that except 1Cr18Ni9Ti, all the better results are obtained using the recommended m values. But it is interesting that similarly good results can also be obtained for 08Al steel sheets when taking $m = 2$.

Table 1 Comparisons between theoretical values calculated using eq. (11) and average experimental ones ($\overline{\chi}_E$)

Material	r_0	r_{90}	m				$\overline{\chi}_E$
			2	4	6	8	
1Cr18Ni9Ti	0.425	0.595	1.1184	1.0876	1.0577	1.0430	1.0169
08Al	1.983	2.014	1.0026	1.0039	1.0026	1.0019	1.0027
LF21M	0.488	0.466	0.9845	0.9885	0.9923	0.9943	1.0001
LF12M	0.744	0.842	1.0351	1.0314	1.0208	1.0156	1.0159
TA2M	2.052	2.929	1.0530	1.0921	1.0611	1.0455	1.0379

3 DETERMINATION OF STATES OF STRESS AND STRAIN

3.1 *Determination of Principal Stresses*

If eq. (9) is substituted into the well-known equilibrium equation, then:

$$p = \left(\frac{\sigma_0}{R_0} + \frac{\sigma_{90}}{R_{90}}\right)t = 2\frac{\sigma_0 t}{R_0} = 2\frac{\sigma_{90} t}{R_{90}} \qquad (13)$$

thus the two principal stresses at the pole of the bulge specimen are

$$\left.\begin{aligned} \sigma_0 &= \frac{PR_0}{2t} \\ \sigma_{90} &= \frac{PR_{90}}{2t} \end{aligned}\right\} \qquad (14)$$

where p is hydraulic pressure; t is the thickness at the pole of the bulge specimen.

3.2 *Determination of Principal Strain Increments*

If eq. (8) is substituted into the condition that the volume of the material is constant, then

$$\left.\begin{aligned} d\varepsilon_0 &= -\frac{R_0}{R_0 + R_{90}} d\varepsilon_t \\ d\varepsilon_{90} &= -\frac{R_{90}}{R_0 + R_{90}} d\varepsilon_t \end{aligned}\right\} \qquad (15)$$

where $d\varepsilon_t$ is the increment of the strain of thickness at the pole of the bulge specimen and it is obtained from the bulge test. Practical measurements indicate that the ratio of R_{90}/R_0 is not a constant in most cases. Therefore, the segmented integration method should be applied to calculate the strains ε_0 and ε_{90} so as to take into account the effect of this factor.

4 CONCLUSIONS

(1) The ratio of the two principal stresses at the pole of the hydraulic bulge orthotropic sheet sample equals the ratio of the two principal curvature radii at this point; and the ratio of the principal strain increments equals the reciprocal of the later ratio. Therefore, the effect of the anisotropy of the sheets can be taken into account by improving the present method for curvature radius measurement in bulge test.

(2) The theoretical ratio of the principal stresses at the pole of the bulge specimen depends on the yield function selected. The practical calculation shows that the Hosford type yield function can provide more accurate results in most cases, and is also convenient for use.

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