MODELLING OF P/M FORMING PROCESSES BY FEM①

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ABSTRACT A general, 2-dimensional finite element computer program, S-FORGE, has been developed to perform the simulation of full dense and porous metal preforms. The code allows the analysis of the forming operation of complex parts and gives the geometry evaluation and the distributions of the principal field-variables, including relative density. In addition, the yield criteria and constitutive equation proposed by Doraivelu was used to describe porous material plastic forming behavior, a FEM formulation to simulate the powder metallurgy (P/M) forming operation has been derived. In order to show the application of S-FORGE, simple compressions of porous preform with different friction factors were simulated.

Key words modelling P/M forming FEM yield criteria constitutive

1 INTRODUCTION

Over the last few years, the powder metallurgy industry has experienced great development. Powder forging has become a potentially economical method for the production of parts, and today is used widely in the automotive, electrical, aerospace and tool industry. Prerequisite to successful utilization of the process is understanding of the deformation behavior of the preform material. Due to the porosity of sintered powder preform, their deformation behavior is considerably different from that of conventional cast and wrought material. The finite element technique[1] has already proven to be an excellent analytical tool for the simulation of full dense metal forming operation. Therefore, the technique is used to predict the deformation behavior of porous material.

In the paper, the authors describe the basic theory and formulation including yield criteria, constitutive equation, and FEM formulation and introduce the system organization of S-FORGE. In the last part of this paper, the simulated results of simple compression are presented.

2 BASIC THEORY AND FEM FORMULATION

2.1 *Basic theory*

The constitutive equation for porous material differs from the Von-Mises equation for the full dense material due to the change produced in the volume of the porous workpieces during plastic deformation. In the past, various investigators have proposed several yield functions and the stress-strain relations for sintered powder materials. These are well reviewed in reference[2]. In present investigation, the one proposed by Doraivelu *et al*[2] is used to derive the FEM formulation. Here, a brief description of basic theory is given.

In the constitutive relation, it is assumed that the material is isotropic and that the deformation does not produce anisotropy. It is further assumed that the yield stress of the porous material can be represented by

$$F = AJ_2^1 + BJ_1^2 = Y_R^2 = \delta Y_0^2 \qquad (1)$$

where A, B and δ are functions of relative density, Y_R is the yield stress in uniaxial compression, and Y_0 is the yield stress of full dense material, J_2^1 and J_1 are the stress invariants defined by $J_2^1 = 1/2\sigma'_{ij}\sigma'_{ij}$ and $J_1 = \sigma_{kk}$. The

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condition that Y_R is the yield stress in uniaxial stress, requires $A = 3(1 - B)$. From the associated flow rule and the yield function given in equation (1), we have

$$\dot{\varepsilon}_{ij} = \frac{\dot{\bar{\varepsilon}}_R}{2\bar{\sigma}}[A\sigma'_{ij} + \frac{2}{3}(3 - A)J_1\delta_{ij}] \tag{2}$$

where $\dot{\varepsilon}_{ij} = \frac{1}{2}(V_{i,j} + V_{j,i})$ is a strain rate component, and $\dot{\bar{\varepsilon}}_R$ and $\bar{\sigma}$ are expressed as

$$\left.\begin{aligned} \bar{\sigma} &= \sqrt{3(AJ_2^1 + BJ_1^2)}; \\ \dot{\bar{\varepsilon}}_R &= \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^1\dot{\varepsilon}_{ij}^1 + \frac{1}{3(3 - A)}\dot{\varepsilon}_{kk}^2} \end{aligned}\right\} \tag{3}$$

The stress-strain rate relation can be rewritten as

$$\left.\begin{aligned} &\dot{\varepsilon}_{ij} = \frac{\dot{\bar{\varepsilon}}_R}{2\bar{\sigma}}[A\sigma_{ij} + (2 - A)J_1\delta_{ij}] \\ &\text{or} \\ &\sigma = \frac{\bar{\sigma}}{\dot{\bar{\varepsilon}}_R}[\frac{2}{A}\dot{\varepsilon}_{ij} + \frac{\delta_{ij}}{3(3 - A)}\dot{\varepsilon}_{kk}] \end{aligned}\right\} \tag{4}$$

The expressions as A, B and δ can be obtained by careful experiments. Doraivelu *et al* proposed that A, B and δ can be represented by

$$\left.\begin{aligned} A &= (2 + R^2) \\ B &= (1 - R^2)/3 \\ \delta &= (2R^2 - 1) \end{aligned}\right\} \tag{5}$$

where R is the relative density.

In addition to the stress-strain rate relation discussed above, P/M formulation requires some additional kinematic relations. The work hardening and straining rate hardening characteristics are, in general, available for full dense material, while the formulation is given in terms of the apparent strain. The relation between the strain of the porous material and that of the matrix material can be obtained by energy balance. That is

$$\dot{W} = Y_R\dot{\bar{\varepsilon}}_R V_R = Y_0\dot{\bar{\varepsilon}}_0 V_0 = RY_0\dot{\bar{\varepsilon}}_0 V_R \tag{6}$$

where $\dot{\bar{\varepsilon}}_R$ is the strain rate of the porous material and $\dot{\bar{\varepsilon}}_0$ is the average strain rate of the matrix material. By substituting $\sqrt{\delta}Y_0$ for Y_R, we have

$$\dot{\bar{\varepsilon}}_0 = \frac{\sqrt{\delta}}{R}\dot{\bar{\varepsilon}}_R \tag{7}$$

The effective strain of the matrix material can be expressed by

$$\bar{\varepsilon}_0 = \int_0^t \dot{\bar{\varepsilon}}_0 dt = \int_0^t \frac{\sqrt{\delta}}{R}\dot{\bar{\varepsilon}}_R dt \tag{8}$$

and the flow stress expression of the porous material is written by

$$\begin{aligned} Y_R &= \sqrt{\delta}\,Y_0(\bar{\varepsilon}_0, \dot{\bar{\varepsilon}}_0) \\ &= \sqrt{\delta}\,Y_0(\bar{\varepsilon}_0, \frac{\sqrt{\delta}}{R}\dot{\bar{\varepsilon}}_R) \end{aligned} \tag{9}$$

Also note that the change in relative density can be expressed in terms of volume strain as

$$\frac{R}{R_0} = \exp(-\Delta\varepsilon_V) = \exp(-\int_0^{\Delta t}\dot{\varepsilon}_V dt) \tag{10}$$

where R_0 is the initial relative density.

2.2 FEM *formulation*

The variational form of the equilibrium equation for porous medium is

$$\delta\pi = \int_V Y_R\delta\dot{\bar{\varepsilon}}_R dV - \int_S T_i\delta V_i dS = 0 \tag{11}$$

The discretization of this functional follows the standard finite element procedure, by introducing the shape funciton $[N]$ of the element under consideration. Consequently equation (11) at element level can be written as

$$\sum_m [\int_{V^m} \frac{Y_R}{\dot{\bar{\varepsilon}}_R}[K]\underset{\sim}{V}\,dV - \int_{S^m}[N]\underset{\sim}{F}\,dS] = 0 \tag{12}$$

where $\sum_m$ represents the assembly operation.

$$[K] = [B]^T[D][B] \tag{13-a}$$

$$\dot{\underset{\sim}{\varepsilon}} = [B]\dot{\underset{\sim}{V}}$$

$$\left.\begin{aligned} \dot{\bar{\varepsilon}}_R^2 &= \dot{\underset{\sim}{\varepsilon}}^T[D]\dot{\underset{\sim}{\varepsilon}} = \dot{\underset{\sim}{V}}^T[K]\dot{\underset{\sim}{V}} \\ \underset{\sim}{\sigma} &= (Y_R/\dot{\bar{\varepsilon}}_R)[D]\dot{\underset{\sim}{\varepsilon}} \end{aligned}\right\} \tag{13-b}$$

The nonlinear equation (12) is solved by the iterative Newton-Raphson procedure

$$\frac{\partial\pi}{\partial V_j}\Big|_{\underset{\sim}{V}=\underset{\sim}{V}_0} + \frac{\partial^2\pi}{\partial V_j\partial V_i}\Big|_{\underset{\sim}{V}=\underset{\sim}{V}_0}\Delta V_j = 0 \tag{14}$$

where

$$\frac{\partial\pi}{\partial V_j} = \int_V P_1[K]\underset{\sim}{V}\,dV - \int_S [N]\underset{\sim}{F}\,dS \tag{15-a}$$

$$\begin{aligned} \frac{\partial^2\pi}{\partial V_j\partial V_i} = \int_V \{&P_1([K] \\ &+ P_2[K]^T\underset{\sim}{V}^T[K]\underset{\sim}{V}) \\ &+ P_3[K]^T\underset{\sim}{V}^T[K]\underset{\sim}{V}\}dV \end{aligned} \tag{15-b}$$

and

$$\left.\begin{aligned} P_1 &= \frac{\sqrt{\delta}\,Y_0}{\dot{\bar{\varepsilon}}_R} \\ P_2 &= \frac{\sqrt{\delta}\,Y_0}{\dot{\bar{\varepsilon}}_R^3} \\ P_3 &= \frac{\delta}{R}\frac{1}{\dot{\bar{\varepsilon}}_R}\frac{\partial Y_0}{\partial \dot{\bar{\varepsilon}}_0} \end{aligned}\right\} \tag{16}$$

In order to achieve a good convergence character of iterative process, a good integration scheme must be adopted. Based on the work IM[3], the evaluation of the matrix $[K]$ is done in the following scheme: matrix $[D]$ was decomposed into two components, $[D]_1$ and $[D]_2$

$$\left.\begin{aligned} [D]_1 &= \frac{1}{3A}\begin{bmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \\ [D]_2 &= \frac{1}{3(3-A)}\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}\right\} \tag{17}$$

Consequently, matrix $[K]$ in equation (13) was also separated into two components $[K]_1$ and $[K]_2$. The first term, $[K]_1$ was integrated into regular Gauss points, while the second term, $[K]_2$ was evaluated at refuced integration points. This means four and one Gauss points respectively, for the four-node quadrilateral elements used in S-FORGE. The second, $[K]_2$, contains $1/[3(3-A)]$, which tends to infinity when the relative density approaches unity. A cut-off value of 0.9999 was employed to avoid numerical difficulties. This technique is equivalent to the penalty method used to enforce incompressibility in full dense materials. It must be mentioned that this last result is also the basis of the rigid-plastic formultaion based on the slight-compressible deformation of metals that has been proposed by Osakada *et al*[4].

3 GENERAL ORGANIZATION OF S-FORGE

The general organization of S-FORGE is illustraged in Fig. 1. The S-FORGE is designed and implemented as tightly integrated modules. The CAD system, PCDE, severs as the interface between users and S-FORGE. Preprocessor obtains the information about workpiece and dies form PCDE, generates a good FEM mesh and boundary condition. The result of preprocessor displays graphically in PCDE and is stored in text file, this enables the user to check the input data prior to FEM simulation.

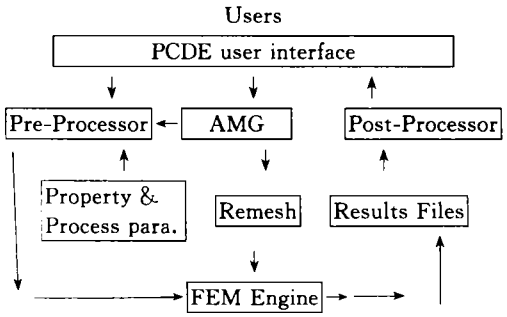

Fig. 1 General organization of S-FORGE

The interactive semi-automatic mesh generator allows a user to generate a good FEM mesh conveniently. The post-processor produces the simulation results at the given points in the deformation process. It displays load-stroke relationship and all process variables in a logical manner (e. g. x-y, vector, contour plots). The FEM Engine is the core of S-FORGE. It generate the initial velocity guess by direct iterative method and perform efficient FEM calculation. It can simulate: (1) Isothermal and non-isothermal forming process with arbitrarily shaped dies for full dense and sintered powder preforms; (2) Heat transfer process; (3) Coupled thermal mechanical analysis.

4 APPLICATION OF S-FORGE TO SINTERED METAL FORMING

In order to show the application of S-FORGE, simple compressions are simulated. A cylindrical sintered preform is compressed between flat dies. The sintered preform has a dimension of 20 mm in diameter and 20 mm in

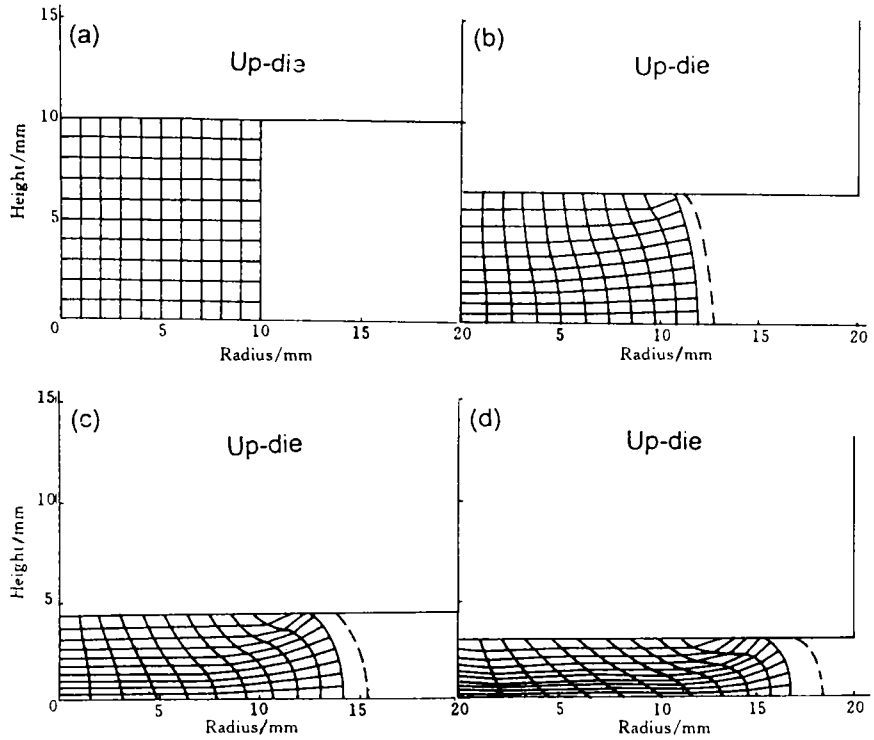

Fig. 2 Initial mesh layout and mesh distortions at 33.52%, 55.3% and 70% reduction in height
(a)—Initial mesh layout; (b)— $\Delta H/H_0 = 33.52\%$;
(c)— $\Delta H/H_0 = 55.3\%$; (d)— $\Delta H/H_0 = 70\%$

height. It is assumed that the preform has uniform relative density of 0.8. The simulation is carried out with three different friction factors, 0.0, 0.3 and 0.5. The flow stress of the matrix material is assumed as $\bar{\sigma} = 10.0 + 0.01\dot{\bar{\varepsilon}}$ (MPa). The updie velocity was 2 mm/s. Because of symmetry, it is sufficient to analyze the quarter section of the preform.

Fig. 2 shows the initial mesh layout and the mesh distortion predicted by S-FORGE at 33.52%, 55.3% and 70% reduction in height for the friction factor $m = 0.5$. For comparison, the boundary of the deformed workpiece with full dense preform predicted by S-FORGE is also showed in dashed lines. It is seen that the porous preform changed its volume.

The average density changes as a function of height reduction ($\Delta H/H_0$) are shown in Fig. 3. It is interesting to note that the average density varies very little with different friction case. Although the average density changes are not affected by the different

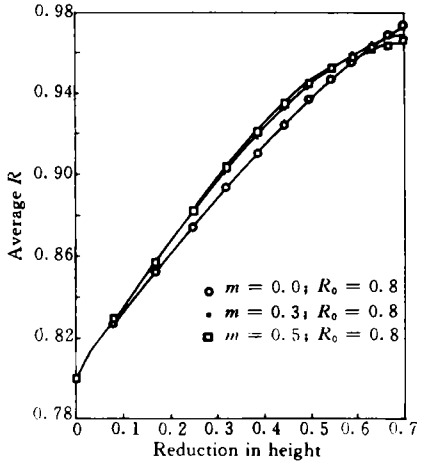

Fig. 3 The curve between average density and reduction in height

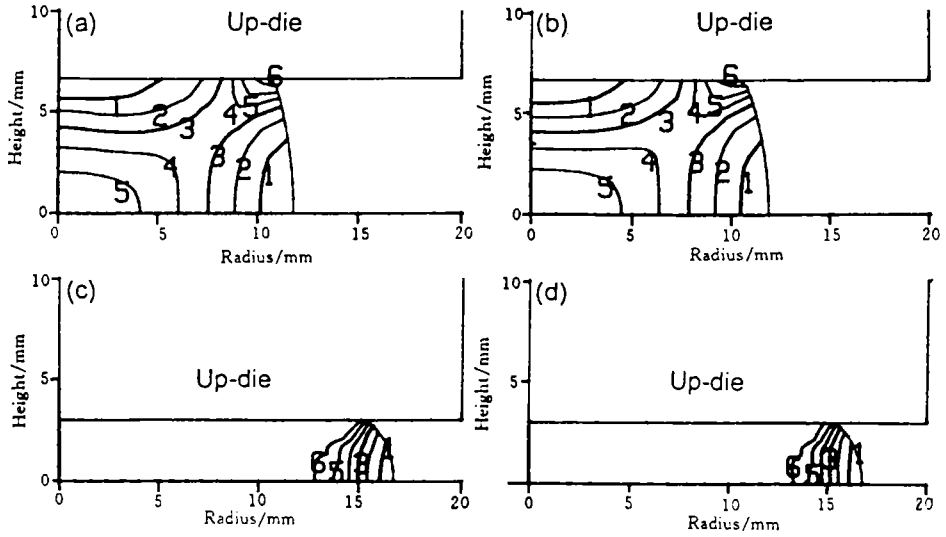

Fig. 4 The predicted relative density distributions

(a)— $m = 0.3$, $\Delta H/H_0 = 33.52\%$; (b)— $m = 0.5$, $\Delta H/H_0 = 33.52\%$;
(c)— $m = 0.3$, $\Delta H/H_0 = 70\%$; (d)— $m = 0.5$, $\Delta H/H_0 = 70\%$;

(a) Values of relative density		(b) Values of relative density		(c) Values of relative density		(d) Values of relative density	
$a_{max} = 0.97$	$a_{min} = 0.86$	$a_{max} = 0.98$	$a_{min} = 0.85$	$a_{max} = 1.00$	$a_{min} = 0.86$	$a_{max} = 1.00$	$a_{min} = 0.82$
1—0.88	2—0.89	1—0.87	2—0.89	1—0.88	2—0.90	1—0.85	2—0.87
3—0.91	2—0.92	3—0.91	2—0.93	3—0.92	2—0.94	3—0.90	2—0.92
5—0.94	6—0.95	5—0.95	6—0.97	5—0.96	6—0.98	5—0.95	6—0.97

frictions, the differences in the local density distributions are considerable. Fig. 4 shows the predicted relative density distributions at 33.52% and 70% reduction in height for the case of friction factor $m = 0.3$ and $m = 0.5$ respectively. It is seen from the Fig. 4(a), (b) that the density is the lowest at the top center where the deformation is restricted by die friction. The highest density takes place at the center of the workpiece. Fig. 4 (c), (d) show that the large portion of the workpiece has become full dense and the compression with higher friction reached high density throughout the most of the workpiece. However, the density near the free surface is lower when the friction higher due to more produced barreling.

5 CONCLUSION

The finite element formulation based on the yield criteria and constitutive equations proposed by Doraivelu *et al* has been derived. A general, 2-dimensional finite element code, S-FORGE, has been developed, which can be used to simulate the densitification and forming processes of porous material.

To demonstrate the validity of the FEM formulation and the usefulness of S-FORGE, simple compression results of sintered powder preform with different factors were presented.

REFERENCES

1 Kobayashi S *et al*. Metal Forming and the Finite Element Method, New York: Oxford University Press, 1989.
2 Doraivelu S M, Gegal H L *et al*. J Mech Sci, 1984, 26(9/10): 527—535.
3 IM Y T. Ph D Dissertation, University of California, Berkeley, 1985.
4 Mori K, Osakada K. J Mech Sci, 1987, 29(4): 229—238.