

# EQUIVALENT CIRCUIT OF NONLINEAR EFFECT OF SPECTRAL IP<sup>①</sup>

He, Jishan Li, Daqing Tang, Jingtian

*Department of Geology, Central South University of Technology, Changsha 410083*

**ABSTRACT** A new nonlinear equivalent circuit was proposed to describe the mineral-electrolyte interface when the nonlinear system being neared to equilibrium state, i. e., the induced current density is not very high. The Volterra/Winner nonlinear system theory was employed to analyse the equivalent circuit. A mathematical formula to calculate overvoltage response of the circuit to arbitrarily induced current has been obtained, and cathode/anode polarization curves of pyrite and graphite were calculated theoretically with the formula. Comparison of these curves with experiment results showed that the proposed equivalent circuit is suitable to describe the nonlinear system composed of mineral-electrolyte, it can be used to study the nonlinear effect of spectral IP.

**Key words** spectral IP nonlinear effect equivalent circuit admittance anode/cathode polarization

## 1 INTRODUCTION

The evaluation of IP anomalies is an important but difficult problem in practice. The methods used can be divided into two groups, the first uses the charging/discharging curves or the spectral IP curves; the second measures the nonlinear IP response.

In this paper, a new equivalent circuit to describe the mineral-electrolyte interface when the induced current density is not very high will be proposed. Furthermore, a mathematical formula to calculate the overvoltage response of the circuit to arbitrarily induced current will be given. At the last, the cathode/anode polarization curves theoretically calculated will be compared with the experimental results.

## 2 ELECTROCHEMISTRY MECHANISM OF IP NONLINEAR EFFECT

The system composed of mineral-electrolyte is a very comprehensive physico-

chemistry system, also is the electrode reactions happened on the mineral-electrolyte interface, those are related not only to the mineral component and the interface property but also to the electrolyte component and its concentration, etc., and play an important role in the consideration of nonlinear IP effect.

It was found that the cathode/anode polarization curves of various minerals were almost the same in shape, as shown in Fig. 1,

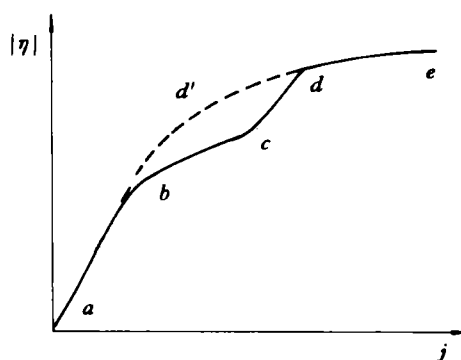


Fig. 1 Typical curve of cathode/anode polarization

① Received Nov. 12, 1994

where the horizontal axis indicated the induced current density ( $j$ ), while the vertical axis indicated the overvoltage response ( $\eta$ ). This typical polarization curve can be explained as follows according to the electrochemistry theory:

(1) Section  $ab$  A double layer structure is formed around the mineral-electrolyte interface due to various electrochemistry reactions. The electrode reactions around the interface are weak because little current flows through the interface. The overvoltage response has a linear relation with the current induced density approximately.

(2) Section  $bc$  The polarization curve begins to bend with increasing current density. the electrode reactions becomes obvious when the overvoltage increases to the value at point  $b$ . Charges from the induced current are mainly consumed in the electrode reactions, little of it are concentrated on the interface. Thus, the overvoltage doesn't increase anymore or increases very slowly with the increase of induced current density at this section  $bc$ . Most of the induced current would flow through the interface. At this stage, ions would be separated out on cathode or dissolved on anode. the relationship of the overvoltage response with the induced current density would become to nonlinear.

(3) Section  $cd$  The remained charges are accumulated on double layer structure, so overvoltage increases with the increase of induced current density. Electrode reactions would be dominated by diffusion process.

(4) Section  $de$  Hydrogen/oxygen begins to be separated out when the overvoltage increases to the value at point  $d$ . The overvoltage increases little with the increase of induced current density.

(5) As for the non-dissolvable minerals such as graphite, the polarization curve would moves along the dashed line from  $b$  to  $d'$ .

### 3 EQUIVALENT CIRCUIT OF NON-LINEAR EFFECT OF IP

According to the above analysis, one part

of induced current would charge to the double layer structure, which was named as non-faradaic current on electrochemistry theory. Another part of the current would flow through the mineral-electrolyte interfaces, which was called faradaic current. Thus, the mineral-electrolyte system could be describe as two parallel path, one is called faradaic path, the other is non-faradaic path. Considered the electrolyte resistance, the system could be equivalent to a circuit shown in Fig. 2.

In linear condition,  $G_1$  can be expressed as  $(i\omega x)^c$  in frequency domain, while  $G_2$  is pure

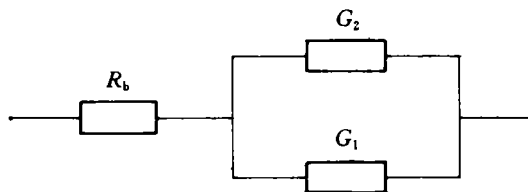


Fig. 2 The equivalent circuit of mineral-electrolyte system

$R_b$  — the electrolyte resistance;  $G_1$  — the equivalent admittance of non-faradaic path;  $G_2$  — the equivalent admittance of faradaic path

conductance with  $G_2 = 1/R_c$ . Then Fig. 2 can be redrawn as Fig. 3, which is the cole-cole equivalent circuit in frequency domain. The transfer impedance of cole-cole equivalent circuit is

$$Z(\omega) = Z(o) \left\{ 1 - m \left( 1 - \frac{1}{1 + (i\omega\tau)^c} \right) \right\} \quad (1)$$

$$\begin{aligned} \text{where } Z(o) &= R_b + R_c; \\ m &= R_c / (R_b + R_c); \\ \tau &= \chi(R_c)^c \end{aligned}$$

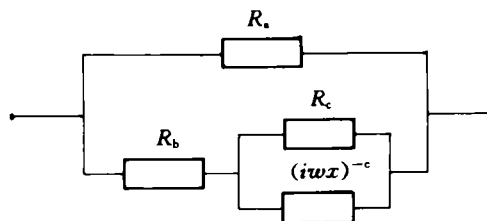


Fig. 3 Cole-cole equivalent circuit of IP

$Z(\omega)$  is the impedance when angle frequency is equal to zero.  $m$  is called chargeability, and  $\tau$  is the time-constant.  $c$  is named as frequency related coefficient, which indicates the relationship between the impedance and the angle frequency.

The linear relation between the overvoltage and the induced current density would be broken when the current density increases to a threshold value, and the Cole-Cole equivalent circuit was not suitable for a nonlinear system anywise.

It could be supposed that the equivalent admittance of the non-faradaic path,  $G_1$ , which relates to the double layers structure, is still  $(i\omega\chi)^c$  when the induced current density is not very high. The equivalent admittance of faradaic path,  $G_2$ , is a complex response of various electrode reactions related to the nonlinear effect. By analysing the equivalent admittance of various electrode reactions, it can be assumed that:

$$G_2 = G_0 + a \frac{G_0^2}{S} V_2 + b \frac{G_0^3}{S^2} V_2^2 \quad (2)$$

where  $G_0$  is a pure conductance with  $G_0 = 1/R_c$ , which indicates the linear effect of faradaic path;  $a$  and  $b$  are parameters related to nonlinear effect of faradaic path, which have the dimensional relation with the induced current density:  $[a] = [j]^{-1}$ ,  $[b] = [j]^{-2}$ ;  $S$  indicates the effective section area of various electrode reactions;  $V_2$  is the overvoltage response of faradaic path.

Substituting  $G_1/G_2$  in Fig. 2 with above equivalent admittance, the equivalent circuit to describe the mineral-electrolyte system is established as shown in Fig. 4.

Because  $G_2$  is a function of  $V_2$ , the relationship between  $V_2$  and the current density  $j$  is nonlinear. Analysis methods and conclusions in linear system are not suitable here. It should deal the circuit with nonlinear system in next paragraph.

Supposing the nonlinear system composed of mineral-electrolyte satisfies the time-invariant and causality. According to the Weierstrass theorem, the response of any such a nonlinear system to arbitrary input can be ex-

pressed as a convolution sequence:

$$\begin{aligned} y(t) = & \int_{-\infty}^{\infty} h_1(\tau) u(t - \tau) d\tau \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2) \\ & \times u(t - \tau_1) u(t - \tau_2) d\tau_1 d\tau_2 \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_3(\tau_1, \tau_2, \tau_3) \\ & \times u(t - \tau_1) u(t - \tau_2) u(t - \tau_3) \\ & \times d\tau_1 d\tau_2 d\tau_3 + \dots \end{aligned} \quad (3)$$

where  $y(t)$  indicates the response of a nonlinear system to the input  $u(t)$ . Equation (3) is a series presented by the Italy mathematician, Vito-Volterra in 1880. Wiener first used it to analyse nonlinear system in 1942. In the series,  $h_1(\tau)$ ,  $h_2(\tau_1, \tau_2)$ ,  $h_3(\tau_1, \tau_2, \tau_3)$ ... are named as Volterra Kernels, which define the nonlinear system exactly.

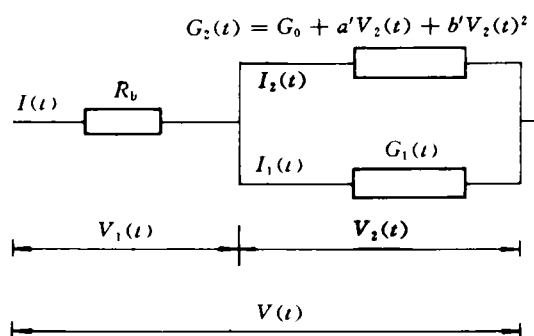


Fig. 4 Equivalent circuit of nonlinear electrolyte-mineral system

In Fig. 4, the response of the equivalent circuit,  $V(t)$  can be written as

$$V(t) = V_1(t) + V_2(t) \quad (4)$$

where  $V_2(t)$  is the overvoltage response of faradaic path, which can be expressed as a Volterra series with the input  $I(t)$ :

$$\begin{aligned} V_2(t) = & \int_{-\infty}^{\infty} Z_1(\tau) \\ & \times I(t - \tau) d\tau \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_2(\tau_1, \tau_2) I(t - \tau_1) \\ & \times I(t - \tau_2) d\tau_1 d\tau_2 \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_3(\tau_1, \tau_2, \tau_3) \\ & \times I(t - \tau_1) I(t - \tau_2) I(t - \tau_3) \end{aligned}$$

$$\times d\tau_1 d\tau_2 d\tau_3 + \dots \quad (5)$$

where  $Z_1(\tau)$ ,  $Z_2(\tau_1, \tau_2)$ ,  $Z_3(\tau_1, \tau_2, \tau_3) \dots$  are Volterra Kernels of the nonlinear system

Assuming the current  $I(t)$  is arbitrarily, it can be expanded in the power series, then:

$$I(t) = \sum_{k=-\infty}^{\infty} I_k e^{ik\omega t} \quad (6)$$

Substituting  $I(t)$  in equation (5) with expression (6), then get

$$\begin{aligned} V_2(t) = & \sum_{k_1=-\infty}^{\infty} Z_1(k_1) I_{k_1} e^{ik_1 \omega t} \\ & + \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} Z_2(k_1, k_2) \\ & \times I_{k_1} I_{k_2} e^{i(k_1+k_2)\omega t} \\ & + \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} \sum_{k_3=-\infty}^{\infty} Z_3(k_1, k_2, \\ & k_3) I_{k_1} I_{k_2} I_{k_3} e^{i(k_1+k_2+k_3)\omega t} \end{aligned} \quad (7)$$

where  $Z_1(k_1)$ ,  $Z_2(k_1, k_2)$ ,  $Z_3(k_1, k_2, k_3) \dots$  are Volterra Kernels in frequency domain

$$\left. \begin{aligned} Z_1(k_1) &= \int_{-\infty}^{\infty} Z_1(\tau) e^{-ik_1 \omega \tau} d\tau \\ Z_2(k_1, k_2) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_2(\tau_1, \tau_2) \\ &\times e^{-ik_1 \omega \tau_1} e^{-ik_2 \omega \tau_2} \\ &\times d\tau_1 d\tau_2 \\ Z_3(k_1, k_2, k_3) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_3 \\ &\times (\tau_1, \tau_2) e^{-ik_1 \omega \tau_1} \\ &\times e^{-ik_2 \omega \tau_2} e^{-ik_3 \omega \tau_3} \\ &\times d\tau_1 d\tau_2 d\tau_3 \end{aligned} \right\} \quad (8)$$

Thus the response of a linear system can be gotten by letting the Volterra Kernels with high-order ( $Z_2, Z_3 \dots$ ) be equal to zero in equations (5) or (7), and the response of any system can be calculated with equation (5) or (7) as long as the Volterra Kernels are solved.

In Fig. 4, the relation between currents is

$$I(t) = I_1(t) + I_2(t) \quad (9)$$

where  $I_1(t)$  is the current through non-faradaic path, which is defined by a convolution with  $V_2$  and  $G_1$ :

$$I_1(t) = \int_{-\infty}^{\infty} G_1(\tau) V_2(t - \tau) d\tau \quad (10)$$

where  $I_2(t)$  indicates the current in nonlinear faradaic path, i. e.

$$\begin{aligned} I_2(t) &= G_2(t) V_2(t) \\ &= G_0 V_2(t) + a' V_2^2(t) \\ &\quad + b' V_2^3(t) \end{aligned} \quad (11)$$

In equation (11),  $a' = aG_0^2/S^2$ ,  $b' = b \cdot G_0^3/S^2$ . Because of the formula:

$$f(t) = \int_{-\infty}^{\infty} \delta(\tau) (t - \tau) d\tau$$

where  $\delta(t)$  is a Diract function. Equation (11) can be rewritten as

$$\begin{aligned} I_2(t) = & \int_{-\infty}^{\infty} G_0 \delta(\tau) V_2(t - \tau) d\tau \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a' \delta(\tau_1) \delta(\tau_2) \\ & \times V_2(t - \tau_1) V_2(t - \tau_2) d\tau_1 d\tau_2 \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b' \delta(\tau_1) \\ & \times \delta(\tau_2) \delta(\tau_3) V_2(t - \tau_1) \\ & \times V_2(t - \tau_2) V_2(t - \tau_3) \\ & \times d\tau_1 d\tau_2 d\tau_3 \end{aligned} \quad (12)$$

put equations (6), (10) and (12) into equation (9), then get:

$$\begin{aligned} \sum_{k=-\infty}^{\infty} I_k e^{ik\omega t} = & \int_{-\infty}^{\infty} [G_0 \delta(\tau) + G_1(\tau)] \\ & \times V_2(t - \tau) d\tau \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a' \delta(\tau_1) \delta(\tau_2) \\ & \times V_2(t - \tau_1) V_2(t - \tau_2) d\tau_1 d\tau_2 \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b' \delta(\tau_1) \\ & \times \delta(\tau_2) \delta(\tau_3) \\ & \times V_2(t - \tau_1) V_2(t - \tau_2) \\ & \times V_2(t - \tau_3) d\tau_1 d\tau_2 d\tau_3 \end{aligned} \quad (13)$$

substituting  $V_2(t)$  in equation (13) with expression (7), after hard work and comparing the right side with the left side of the equation, then have:

$$\left. \begin{aligned} Z_1(k_1) [G_0 + (ik_1 \omega \chi)^c] &= 1 \\ Z_2(k_1, k_2) [G_0 + (i(k_1 + k_2) \omega \chi)^c] \\ &\quad + a' Z_1(k_1) Z_2(k_2) = 0 \\ Z_3(k_1, k_2, k_3) [G_0 + i(k_1 + k_2 \\ &\quad + k_3) \omega \chi)^c] + a' Z_2(k_1, k_2) Z_3(k_3) \\ &\quad + a' Z_2(k_2, k_3) Z_1(k_1) \\ &\quad + b' Z_1(k_1) Z_2(k_2) Z_3(k_3) = 0 \end{aligned} \right\} \quad (14)$$

from equations (14), the Volterra Kernels are solved as follows

$$\left. \begin{aligned} Z_1(k_1) &= 1/[G_0 + (ik_1w\chi)^c] \\ Z_2(k_1, k_2) &= -a'G_0/\{[G_0 + (ik_1w\chi)^c] \\ &\quad [G_0 + (ik_2w\chi)^c][G_0 + (i(k_1 + k_2)w\chi)^c]\} \\ Z_3(K_1, K_2, K_3) &= \\ &\quad (1/\{[G_0 + (ik_1w\chi)^c][G_0 + (ik_2w\chi)^c] \\ &\quad [G_0 + (ik_3w\chi)^c] \\ &\quad \times [G_0 + (i(k_1 + k_2 + k_3)w\chi)^c]\}) \\ &\quad \times (a'^2G_0^4/[G_0 + (i(k_1 + k_2)w\chi)^c] \\ &\quad + a'^2G_0^4/[G_0 + (i(k_2 + k_3)w\chi)^c] \\ &\quad - b'G_0^3) \end{aligned} \right\} \quad (15)$$

Since  $V_1(t) = \sum_{k=-\infty}^{\infty} R_b I_k e^{ik_1 \omega t}$ , put it and the equation (15) into equation (4) and (7), and note that:

$a' = aG_0^2/S$ ,  $b' = b \cdot G_0^3/S^2$ ,  $G_0 = 1/R_c$ , then get:

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) \\ &= \sum_{k_1=-\infty}^{\infty} [R_b + \frac{R_c}{1 + (ik_1w\tau)^c}] I_{R1} e^{ik_1 \omega t} \\ &\quad + \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} (-aR_c/\{[1 + (ik_1w\tau)^c] \\ &\quad \cdot [1 + (ik_2w\tau)^c][1 + (i(k_1 + k_2)w\tau)^c]\}) \\ &\quad \cdot \frac{I_{k1} I_{k2}}{S} e^{i(k_1 + k_2) \omega t} \\ &\quad + \sum_{k_1} \sum_{k_2} \sum_{k_3=-\infty}^{\infty} (R_c/\{[1 + (ik_1w\tau)^c] \\ &\quad \cdot [1 + (ik_2w\tau)^c][1 + (ik_3w\tau)^c] \\ &\quad \cdot [1 + (i(k_1 + k_2 + k_3)w\tau)^c]\}) \\ &\quad \cdot (a^2/\{[1 + (i(k_1 + k_2)w\tau)^c] \\ &\quad + a^2/[1 + (i(k_2 + k_3)w\tau)^c] - b\} \\ &\quad \cdot \frac{I_{k1} I_{k2} I_{k3}}{S^2} e^{i(k_1 + k_2 + k_3) \omega t}) \end{aligned} \quad (16)$$

where  $\tau = \chi(R_c)^{1/c}$ . Equation (16) is the formula to calculate the overvoltage response of the nonlinear equivalent circuit to arbitrarily induced current  $I(t)$ . There is only the first item in the formula if  $a = b = 0$ , its corresponding complex impedance in frequency domain is

$$\begin{aligned} Z(\omega) &= V(\omega)/I \\ &= R_b + \frac{R_c}{1 + (i\omega\tau)^c} \end{aligned} \quad (17)$$

It is the same as the impedance of the Cole-Cole equivalent circuit, which is a special case of nonlinear equivalent circuit at a relatively small induced current density.

#### 4 THEORETICAL CALCULATION AND EXPERIMENT RESULTS OF CATHODE/ANODE POLARIZATION CURVES

Supposing the induced current in Fig. 4 is  $I(t) = I_0 + I_1 \cos \omega t$  (18)

There are only three exponential items after expanded in power series, i. e.  $k = -1, 0, 1$ , the corresponding amplitude is  $I_k = -I_1/2, I_0, I_1/2$ , respectively. Put them into formula (16), resolve the overvoltage  $v(t)$  in time domain, then the overvoltage  $v(\omega')$  in frequency domain with Fourier transform can be gotten. In the expression of  $v(\omega')$ , there is not only the items containing angle frequency  $\omega' = \omega$ , but also that items containing  $\omega' = 0, 2\omega, 3\omega$ , etc. It is an important characteristic appeared in overvoltage response, which has been once measured by Mayb юб *et al.* to evaluate the nonlinear effect of mineral polarization.

For the frequency  $\omega' = \omega$ , the complex impedance is defined still as  $Z(\omega) = V(\omega)/I$ , but  $Z(\omega)$  is the function of the induced current here.

For the actual mineral-electrolyte, the nonlinear parameters  $a$  and  $b$  are usually small when the induced current density is not very high. If the direct current  $I_0$  is much greater than the alternate current  $I_1$  in the induced current, then the approximate expression of the impedance can be gotten as:

$$\begin{aligned} Z(\omega) &= R_b + \frac{R_c}{1 + (i\omega\tau)^c} \\ &= Z'(0) [1 - m' (1 - \frac{1}{(1 + i\omega\tau)^c})] \end{aligned} \quad (19)$$

where

$$\left. \begin{aligned} Z'(0) &= R_b + R_c \\ m' &= \frac{R'}{R_b + R_c} \\ \tau &= \chi(R_c)^c \end{aligned} \right\} \quad (20)$$

and

$$\left. \begin{aligned} R'_c &= \frac{R_c}{1 + 2aj_0 + 3bj_0^2} \\ j &= \frac{I_0}{S} \end{aligned} \right\} \quad (21)$$

Thus, the impedance  $Z(\omega)$  of a nonlinear system is of the same form as the impedance of a linear system. The only difference of them is that the parameters ( $Z'(0)$ ,  $m'$ ,  $\tau'$ ,  $R'_c$ ) are not only related to nonlinear parameters  $a$  and  $b$ , but also related to the induced current density for a nonlinear system.

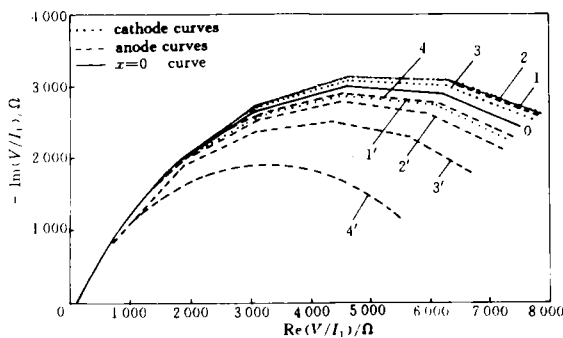
For dissolvable minerals (such as metal sulfates), the nonlinear effect of IP is determined mainly by the electrochemistry reactions on the mineral-electrolyte interface when the induced current density is not very high. Fig. 5 shows the electrode polarization curves theoretically calculated with equations (19)~(21) for the dissolvable minerals. The most important characteristic of these curves is the "cathodic polarization dominant", which becomes more obvious with the increase of induced current density. That means the IP response on the cathode is greater than on the anode for the dissolvable minerals.

As for the non-dissolvable minerals such as graphite, the nonlinear effect IP is dominated mainly by the reaction in which the hy-

drogen and oxygen are separated out. Their theoretical impedance curves are shown in Fig. 6. The most obvious characteristic in Fig. 6 is the "anode polarization dominant", Which means that the IP response on anode is greater than on cathode for the nondissolvable minerals.

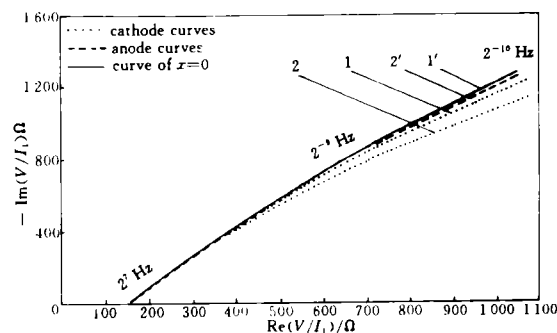
Fig. 7 and Fig. 8 show the experimental cathode/anode polarization curves of pyrite and graphite, respectively. Comparing these figures with Fig. 5 and Fig. 6 respectively, it is clear that the curves shown in Fig. 7 and Fig. 8 are almost the same in shape as those in Fig. 5 and Fig. 6. This fact indicates that the equivalent circuit and the formulae proposed above are suitable to analyse the mineral-electrolyte system.

It is found from the comparison that the nonlinear characteristics on cathode/anode polarization curves for the metal sulfate and graphite are different. (1) The polarization curves for the dissolvable minerals such as metal sulfate are "cathode polarization dominant", while those for the non-dissolvable minerals such as graphite are "anode polarization dominant". This is because of that the electrochemistry mechanism of nonlinear IP effect for different minerals are different. (2) The nonlinear effect of IP for the sulfate is much greater than that for the graphite be-



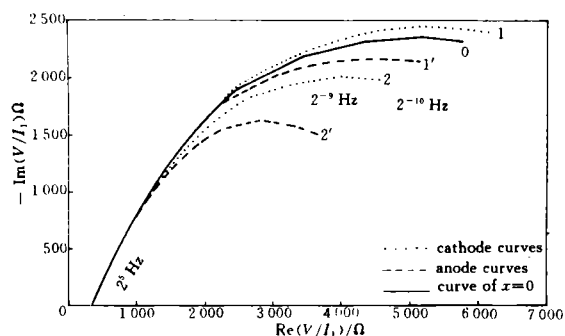
**Fig. 5 Cathode/anode polarization curves theoretically calculated for dissolvable minerals, in calculation:**

$Z(0) = 10\,000\Omega$ ,  $m = 0.985$ ,  $\tau = 50\text{ s}$ ,  
 $c = 0.7$ ,  $a = 0.004\text{ cm}^2/\mu\text{A}$ ,  $b = 0.0001\text{ cm}^4/\mu\text{A}^2$ .  
 For curve 0,  $j_0 = 0\mu\text{A}/\text{cm}^2$ ; 1 and 1',  $j_0 = 4\mu\text{A}/\text{cm}^2$ ; 2 and 2',  $j_0 = 8\mu\text{A}/\text{cm}^2$ ; for curves 3 and 3',  $j_0 = 16\mu\text{A}/\text{cm}^2$ ; for curves 4 and 4',  $j_0 = 32\mu\text{A}/\text{cm}^2$



**Fig. 6 Theoretical cathode/anode polarization curves for non-dissolvable minerals**

(The parameters are the same with Fig. 5 except  $\tau = 2\,000\text{ s}$ )



**Fig. 7 Experimental cathode/anode polarization curves of pyrite**

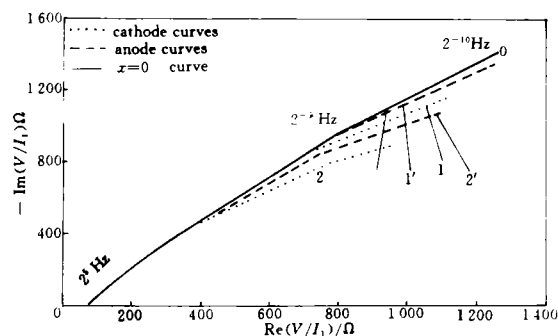
$$S = 23.40 \text{ cm}^2, L = 4.2 \text{ cm},$$

$$j_1 = I_1/S = 0.53 \mu\text{A}/\text{cm}^2$$

For curve 0,  $j_0 = 0 \mu\text{A}/\text{cm}^2$ , for curves 1

and 1',  $j_0 = 4.24 \mu\text{A}/\text{cm}^2$ ;

for curves 2 and 2',  $j_0 = 16.96 \mu\text{A}/\text{cm}^2$



**Fig. 8 Experimental cathode/anode polarization curves of graphite**

$$S = 28.50 \text{ cm}^2, L = 3.2 \text{ cm},$$

$$j_1 = I_1/S = 0.85 \mu\text{A}/\text{cm}^2$$

For curve 0,  $j_0 = 0 \mu\text{A}/\text{cm}^2$ ,

for curves 1 and 1',  $j_0 = 6.8 \mu\text{A}/\text{cm}^2$ ;

for curves 2 and 2',  $j_0 = 27.2 \mu\text{A}/\text{cm}^2$

cause the time-constant for graphite is usually much greater than those for the sulfates. This means that the value  $x$  for graphite is much greater than those for the sulfates when  $\text{Re}$  and  $c$  are approximately equal for the two kinds of minerals. Thus,  $G_1 = (i\omega x)^c$  would increase with the increase of  $x$ , and the role played by the nonlinear equivalent admittance  $G_2$  becomes weaker relatively. That is why the

nonlinear effect of IP for graphite is weaker than that for sulfates. Because  $x$  determines the compactive reactance of the double layer structure, which is related closely to the mineral property, The nonlinear effect of IP for different minerals is different.

## 5 CONCLUSIONS

A new equivalent circuit to describe the mineral-electrolyte system has been proposed and the overvoltage response of the circuit induced by any arbitrary current is calculated. The circuit proposed is proved to be reasonable and computing reliable by comparison of theoretically calculated cathode/anode polarization curves of sulfates and graphite with those curves of experiment results. It has been found that the nonlinear effect of IP is different for different minerals. In general, electrode polarization curves for dissolvable minerals are "the cathode polarization dominant", while those for non-dissolvable minerals are "the anode polarization dominant". The nonlinear effect of IP for dissolvable minerals is much greater than that for the nondissolvable minerals usually. Thus, it can be used to evaluate IP anomalies.

## REFERENCES

- 1 Pelton W H *et al.* Geophysics, 1978, 43(3): 588~603.
- 2 Chua LO, Ng C Y. IEEE J Electronic and syst. 1979, (3): 165-185.
- 3 Chua LO, Ng CY. IEEE J Electronic and syst. 1979, 3: 257~269.
- 4 Schetzen M. The volterra/winner theories of nonlinear systems, New York: Wiley, 1980.
- 5 Rugh, WJ. Nonlinear system theory—the volterra/winner approach. Baltimore: Johns Hopkins, 1981.
- 6 Sumner, JS. Principles of induced polarization for geophysical exploration. Amsterdam-Oxford-New York: Elsevier scientific publishing company, 1976.
- 7 Комаров В А, Злекеро Азвеля КА. Методом Визанполяризации Пзание Второею Лерераотанное и Дополненное Ленинград «Недбра», Ленинградское Отделение, 1980.