GREY DISPLACEMENT INPUT-OUTPUT MODELS OF MINERAL PROCESSING-METALLURGY SYSTEM①

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ABSTRACT According to the principle of input-output, a kind of input-output model of mineral processing-metallurgy system has been proposed. Taking uncertain elements in the input-output model as bounded grey numbers, the grey input-output models have been studied. In the light of linear programming, the grey displacement input-output models were formulated, and the solutions of the models were given. The models can be used for the research of the important proportional relations and economic structure in the economic activities of mineral processing and metallurgy system. The research method can be also used for the input-output analysis of other industries. A practical example was also given.

Key words: mineral processing-metallurgy system input-output grey theory economic model

1 INTRODUCTION

Modernization economy demands modernization management. As a kind of economic analysis and management techniques, input-output technique has been getting improvement in the process of studying economic problems and has been applied for the economic analysis of mineral industry in China[1]. However, the investigation on input-output technique is rather complicated. In the input-output analysis of mineral industry, there appear much more uncertain and inexact cases. Particularly, when science and technology are developed and the production structure is changed, it becomes a significant problem of input-output methodology that how to reflect exactly the change of economic system. So far there have appeared the methods of using Markov method to revise direct consumption coefficients[2], the input-output fuzzy mathematical models[3], the qualitative input-output analysis[4] and the optimal grey input-output models[5].

For effective analysis of mineral industry economy, we proposed an input-output model of mineral processing-metallurgy system in terms of the input-output principle, developed the grey input-output models and formulated grey displacement input-output models that used linear programming as the solution according to grey theory[6].

2 THE INPUT-OUTPUT MODELS OF MINERAL PROCESSING-METALLURGY SYSTEM

For mineral processing-metallurgy enterprise, there are multiproducts, multistages and connections of complicated productive technology.

In order to indicate its production and management activities, we drew up an input-output table according to the principle of determining product type by technological process. Considering that an overlarge model will bring difficulties for collecting data and and calculating the model, and that it is not necessary to divide the system in detail in every case, here we give an outline input-output model of mineral processing-metallurgy system shown as Table 1.

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Table 1 Input-output table of mineral processing-metallurgy system

Input	Output							
Intrinsic resources	Middle commodity				Final commodity			Total output
	1	2	…	n	Market	Store	Sum	
1	x_{11}	x_{12}	…	x_{1n}			y_1	x_1
2	x_{21}	x_{22}	…	x_{2n}			y_2	x_2
·	·	·	…	·			·	·
·	·	·	…	·			·	·
·	·	·	…	·			·	·
n	x_{n1}	x_{n2}	…	x_{nn}			y_n	x_n

The middle commodity in Table 1 refer to mineral processing commodity and smelting commodity. The equilibrium equation in the row of Table 1 is

$$\sum_{j=1}^{n} x_{ij} + y_i = x_i \quad i = 1, 2, \cdots, n \tag{1}$$

Let $X = (x_{ij})_{n\times n}$ be a discharge matrix, $y = (y_1, y_2, \cdots, y_n)^T$, $x = (x_1, x_2, \cdots, x_n)^T$, then equation (1) can be rewritten in matrix form:

$$Xe + y = x \tag{2}$$

where $e = (1, 1, \cdots, 1)^T$. Let $a_{ij} = x_{ij}/x_j$, $A = (a_{ij})_{n\times n}$, then equation (2) can be rewritten as follows:

$$Ax + y = x \tag{3}$$

The columns in Table 1 show the consumption of every commodity for the other commodity in the production process. Since the measure unit of each commodity in the input-output table of material object is different, the column data in Table 1 can not be summed up.

If the data are sufficient, Table 1 can also be expanded by adding some rows for the consumption of materials, electric power and so on.

3 GREY INPUT-OUTPUT MODELS

3.1 *Grey Numbers and Grey Matrices*

In grey theory, the system in which part of the information is certain and part uncertain is called grey system. Grey numbers are basic units in the system. $\otimes$ denotes general grey number; $\otimes(a)$ denotes the grey number which a is explicit value.

Definition 1 $\otimes$ is an interval grey number if $\otimes \in [\underline{a}, \overline{a}]$.

where $\underline{a}$ and $\overline{a}$ are called the lower bound and the upper bound of the grey number $\otimes$ respectively.

Definition 2 $\otimes(x)$ is a grey vector if there are some grey elements in the vector x. $\otimes(A)$ is a grey matrix if there are some grey elements in the matrix A.

3.2 *Grey Input-Output Models*

If there are grey numbers in the equilibrium equation (2), the grey input-output model is:

$$\otimes(X)e + \otimes(y) = \otimes(x) \tag{4}$$

It can be rewritten as follows:

$$\otimes(A)\otimes(x) + \otimes(y) = \otimes(x) \tag{5}$$

4 GREY DISPLACEMENT INPUT-OUTPUT MODELS

4.1 *Grey Displacement Models*

Definition 3 Let $\otimes(u)$ be a nonnegative grey vector consisting of the grey elements in model (1) or (2). $CS = \{(f, CL)\}$ is criterion set, where f is objective function, CL is constraint condition. For $J \in CS$, if $f(\underline{u}_J) = \min f(u)$, $f(\overline{u}_J) = \max f(u)$, then $\underline{u}_J$ and $\overline{u}_J$ are called the lower bound and upper bound of nonnegative grey vector $\otimes(u)$ respectively as to criterion J.

In definition 3, objective function f can be considered as a subjective demand for economic structure which is studied. Therefore, $\underline{u}_J$ and $\overline{u}_J$ are two explicit values about this subjective demand. The explicit values more precisely describing objective phenomenon can be structured as follows:

$$u_J = \underline{u}_J + t(\overline{u}_J - \underline{u}_J) \tag{6}$$

where $t \in [0, 1]$.

Theorem: If CL is a linear constraint about u, and $u \geqslant 0$, then, as for equation (6), there exists that $u_J \geqslant 0$, and u_J satisfies balance condition (2) or (3).

Proof: Since $u \geqslant 0$, we have $\underline{u}_J \geqslant 0$, $\overline{u}_J \geqslant 0$. But since $t \in [0, 1]$, hence $u_J \geqslant 0$.

Because balance condition is one of the constraint conditions, both $\underline{u}_J$ and $\overline{u}_J$ satisfy balance condition (2) or (3). But CL is a linear constraint, so the weighted combination of $\underline{u}_J$ and $\bar{u}_J$ still satisfy the linear constraint condition. The proof of theorem 1 is completed.

Since we can obtain different u_J from different t, we may get many reference values of the grey numbers in the grey input-output model. Equation (6) is called grey displacement model of the model (2) or (3), t is displacement factor. For $t \in [0,1]$, if the u_J obtained from (6) is still dissatisfied, we can select a new criterion $K \in CS$, according to the information of u_J, to compute $\underline{u}_K$ and $\bar{u}_K$; then using u_J, $\underline{u}_K$ or $\bar{u}_K$ to structure a new u_K according to equation(6).

4.2 *Solution of Displacement Model*

Let $\otimes(u) = (\otimes(u_1), \otimes(u_2), \cdots, \otimes(u_p))^T$ be a nonnegative vector consisting of the grey elements in model (2) and (3). A computing method for $\underline{u}_J$ is to solve the following equation:

$$\min \sum_{i=1}^{p} r_i u_i$$

$$\text{s. t.} \begin{cases} Xe + y = x \\ G(u) = 0 \\ u_i \geqslant 0 \quad (i = 1, 2, \cdots, p) \end{cases} \tag{7}$$

where $r_i \geqslant 0$ $(i = 1, 2, \cdots, p)$; $f = \sum_{i=1}^{p} r_i u_i$ is not a constant; $G(u) = 0$ is the additional constraint condition drew from the feature of the system. A computing method for $\bar{u}_J$ is to solve the following equation:

$$\max \sum_{i=1}^{p} r_i u_i$$

$$\text{s. t.} \begin{cases} Xe + y = x \\ G(u) = 0 \\ u_i \geqslant 0 \quad (i = 1, 2, \cdots, p) \end{cases} \tag{8}$$

The value of displacement factor t can be obtained one by one from interval $[0,1]$ according to equal steps, or be computed from actual demand u_0 so that u_J is the nearest to u_0.

The elements in input-output models are of clear economic meaning. In the economic management activities, people always hope to get the greatest profit with the lowest consumption. The equations (7) and (8) express a subjective desire.

5 A PRACTICAL EXAMPLE

According to the form of Table 1, using the production data of a certain metal mine in 1991, we can draw up an input-output table as Table 2.

Table 2 The input-output of a certain metal mine(t)

	Bismuth concentrate	Crude bismuth	Bismuth ingot	Final commodity	Total output
Bi concentrate		353		413	766
Crude Bi			98		98
Bi ingot				86	86

Now, in order to increase bismuth ingot output, the bismuth concentrate output is to be increased from 766 t to 1 089 t. The grey numbers, $\otimes(x_{12})$, $\otimes(y_1)$, $\otimes(x_{23})$, $\otimes(x_2)$, $\otimes(y_3)$, $\otimes(x_3)$, should be determined in the case of commodity structure not being changed.

Corresponding to (4), we have

$$\begin{pmatrix} 0 & \otimes(x_{12}) & 0 \\ 0 & 0 & \otimes(x_{23}) \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \otimes(y_1) \\ 0 \\ \otimes(y_3) \end{pmatrix} = \begin{pmatrix} \otimes(x_1) \\ \otimes(x_2) \\ \otimes(x_3) \end{pmatrix}$$

In order to increase bismuth ingot output, we take $f = y_3$ as an objective function.

First, displacement level of the initial year is considered. As for $x_1 = 766$, the uncertain elements make up $\otimes(u) = (\otimes(x_{12}), \otimes(y_1), \otimes(x_{23}), \otimes(x_2), \otimes(y_3), \otimes(x_3))^T$. From Table 2, the relation between input of bismuth concentrate and output of crude bismuth is $x_{12} = 3.062\,x_{23}$, and that between input of crude bismuth and output of bismuth ingot is $x_{23} = 1.1395\,y_3$. Hence, there are the following constraints:

$$\begin{cases} x_{12} + y_1 = 766 \\ x_{23} = x_2 \\ y_3 = x_3 \\ x_{12} = 3.602\,x_{23} \\ x_{23} = 1.1395\,y_3 \\ x_{12},\ y_1,\ x_{32},\ x_2,\ y_3,\ x_3 \geqslant 0 \end{cases}$$

The solution of $\min(y_3)$ is $\underline{u}_J = (0, 766, 0, 0, 0)^T$, and that of $\max(y_3)$ is $\bar{u}_J = (766, 0, 212.6481, 212.6481, 186.6153, 186.6153)^T$. For vector $u_0 = (353, 413, 98, 98, 86, 86)^T$ which is formed by the data of the initial year, the solution of $\min\|\underline{u}_J + t(\bar{u}_J - \underline{u}_J) - u_0\|_2$ is $t = 0.4608$.

Next, we will determine the output of the new plan. At this time, $x_1 = 1089$. Appropriately decrease the ratio of the input of bismuth concentrate to the output of crude bismuth, so that the ratio of the input of bismuth concentrate to the output of bismuth ingot is decreased from 4.1047 to 4. Then, the following constraints can be obtained:

$$\begin{cases} x_{12} + y_1 = 1089 \\ x_{23} = x_2 \\ y_3 = x_3 \\ x_{12} = 3.50103\, x_{23} \\ x_{23} = 1.1395\, y_3 \\ x_{12},\ y_1,\ x_{23},\ x_2,\ y_3,\ x_3 \geqslant 0 \end{cases}$$

The solution of $\min(y_3)$ is $\underline{u}_J = (0,\ 1089,\ 0,\ 0,\ 0,\ 0)^T$. The solution of $\max(y_3)$ is $\bar{u}_J = (1\,089,\ 0,\ 310.2289,\ 310.2289,\ 272.25,\ 272.25)^T$. According to the displacement level of the initial year, $t = 0.4608$, so

$$u_J = \underline{u}_J + t(\bar{u}_J - \underline{u}_J) = (501.8079,\ 587.1888,\ 142.9535,\ 142.9535,\ 125.4528,\ 125.4528)^T$$

If the levels of production and management are raised, $t = 0.48$, then

$$u_J = (522.7183,\ 566.28,\ 148.9099,\ 148.9099,\ 130.68,\ 130.68)^T$$

In this way, under the given conditions, the decision and predicting values of bismuth concentrate quantity and bismuth smelting quantity are obtained.

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