# INFLUENCE OF STRESS STATE ON EARING BEHAVIOR①

## (Ⅰ) CRYSTALLOGRAPHIC ANALYSIS OF *FCC* METALS

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**ABSTRACT** The influence of stress state on earing behavior in cup drawing of *FCC* metal sheets was analyzed using the crystallographic method. The results showed that, under the partial-slip(PS) assumption, both the type and extent of earing change with the stress ratio significantly; under the total-slip(TS) assumption, the earing extent varies with the stress ratio while the earing type changes only slightly. Simple method was adopted to predict the cup heights of an aluminum single crystal sheet.

**Key words** earing texture deep-drawing

## 1 INTRODUCTION

Since earing behavior in cup drawing is initially caused by and closely related with crystallographic textures, many researchers have attempted to predict the earing behavior using the crystallographic methods. The Turker method of earing has been widely used in both single crystals and polycrystals with the advantage of explicit mathematic form[1-5]. The ear profiles predicted with this method are generally in good agreement with those observed. However, it is interesting that all these successful predictions have been carried out based on the assumption for stress state in the flange, another important factor which may influence the earing behavior, is a plane stress state with the ratio of radial to circumferential stress simply to be unit. Recently, Inagaki[5] has studied the influence of stress state on earing and concluded that, with increasing stress ratio, anisotropy in the total radial strain or cup height increases significantly, while positions of ears alter slightly. However, the total radial strain he has concluded from is related with the indeterminable stress level [cf. Eqn. (6) in Ref. [5] )]. In his calculation, different stress levels are arbitrarily adopted for different stress ratios, so that the variations of total radial strain with different stress ratio involve the interference of stress levels. Therefore, the validity of his conclusion is extremely questionable.

In this paper, the Tucker method of earing is modified by introducing a more realistic representation of the anisotropy of radial strain in cup drawing, and then the influence of stress state on earing is analyzed in detail.

## 2 THEORY

### 2.1 Basic assumptions

In the calculation of radial strain induced by each slip system with Tucker method, the main assumptions used may be summarized as follows:

(1) The flange of deep drawn blank is under a plane stress which consists of a tensile stress radially, $\sigma_r$, and a compressive stress circumferentially, $\sigma_\theta$, while the stress normal to flange is neglected.

(2) The effects of bending, ironing, and stretching in deep drawing operation on earing

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can be ignored.

(3) The possible texture change induced by deformation can be ignored in the calculation of resolved shear stress of each slip system.

(4) The deformation modes of *FCC* metals are the twelve {111}< 110> slip systems. The relationship between shear strain $\gamma$ and shear stress, $\tau$, in each slip system can be expressed by

$$\gamma = k\tau^2 \qquad (1)$$

where $k$ is a constant.

## 2.2 Radial strain induced by single slip system

Let $\rho = -\sigma_r/\sigma_\theta$ be the stress ratio, according to the Schmid law[6] the resolved shear stress of slip system $i$ at $\theta$ degree from the rolling direction (RD) in the flange, $\tau_i$, can be calculated by

$$\begin{aligned}\tau_i(\theta) &= [(\boldsymbol{T}\cdot\boldsymbol{n}_i)(\boldsymbol{T}\cdot\boldsymbol{b}_i) - \\ &\quad (\boldsymbol{R}\cdot\boldsymbol{n}_i)(\boldsymbol{R}\cdot\boldsymbol{b}_i)\rho]\sigma_\theta \\ &= S_i(\theta)\sigma_\theta \end{aligned} \qquad (2)$$

where $S_i(\theta)$ is the Schmid factor, $\boldsymbol{n}_i$, $\boldsymbol{b}_i$, $\boldsymbol{R}$ and $\boldsymbol{T}$ are unit vectors of the slip plane normal, slip direction, radial direction and circumferential direction, respectively.

Since the deformation is large in cup drawing, the change of orientation must be considered in the calculation of radial strain. The radial strain induced by the shear strain of some active slip systems, therefore, is given by the following equation

$$\varepsilon_i = |(\boldsymbol{R}\cdot\boldsymbol{n}_i)|\gamma_i \qquad (3)$$

with Eqns. (1) to (3), the radial strain contributed by slip system $i$ at $\theta$ degree from RD can then be calculated with

$$\begin{aligned}\varepsilon_i(\theta) &= |\boldsymbol{T}\cdot\boldsymbol{n}_i|[(\boldsymbol{T}\cdot\boldsymbol{n}_i)(\boldsymbol{T}\cdot\boldsymbol{b}_i) - \\ &\quad (\boldsymbol{R}\cdot\boldsymbol{n}_i)(\boldsymbol{R}\cdot\boldsymbol{b}_i)\rho]k\sigma_\theta^2\end{aligned} \qquad (4)$$

## 2.3 Total radial strain and its normalization

In Tucker's investigation[1], it is assumed that only slip systems that sustain the maximum resolved shear stress can contribute to the radial strain, and that the total radial strain is the average of radial strains induced on these systems. This will be designated as partial-slip (PS) assumption. The total radial strain calculated under this assumption can be expressed as

$$\varepsilon(\theta) = \frac{1}{N_{max}}\sum_{i=1}^{N_{max}}\varepsilon_i(\theta) \qquad (5a)$$

where $N_{max}$ is the number of slip systems that sustain the maximum resolved shear stress.

Another assumption proposed recently by Inagaki states that[5], the total radial strain is the sum of radial strains induced on all slip systems. This will be designated as total-slip (TS) assumption. Under this assumption, the total radial strain then must be calculated by

$$\varepsilon(\theta) = \sum_{i=1}^{N_{all}}\varepsilon_i(\theta) \qquad (5b)$$

where $N_{all}$ is the number of all the slip systems and equal to 12 for {111}< 110> slip mode in *FCC* metals.

It can be seen from Eqn. (4) that, the radial strain can not be calculated directly even if the stress ratio, $\rho$, has been given because the stress level $\sigma_\theta$ is indeterminable. In the investigations carried out by Tucker[1] and Inagaki[5], the radial strain $\varepsilon(\theta)$ is calculated by setting $k\sigma^2$ as unit. In order to express the anisotropy of radial strain or cup height quantitatively, in the present study, the total radial strain is normalized as

$$E(\theta) = \varepsilon(\theta)/\bar{\varepsilon} \qquad (6)$$

where $\bar{\varepsilon}$ is the average total radial strain along the circumference. The normalized radial strain, $E(\theta)$, thus defined does not contain the unknown factors $\sigma_\theta$ and $k$, therefore, it can be calculated and used to indicate the anisotropy of the radial strain directly.

For quantitative representation of the earing extent, the earing extent index (EEI) should be calculated on the basis of the $E(\theta)$ by the following equation:

$$\begin{aligned}EEI &= \frac{\varepsilon_{max} - \varepsilon_{min}}{(\varepsilon_{max} + \varepsilon_{min})/2} \\ &= \frac{E_{max} - E_{min}}{(E_{max} + E_{min})/2}\end{aligned} \qquad (7)$$

where $E_{max}$ and $E_{min}$ are the maximum and minimum of $E(\theta)$, respectively.

# 3 RESULTS AND DISCUSSION

## 3.1 Influence of stress state on earing profile

It has been well established that the

assumption of plane stress in the flange of deep drawn blank is acceptable for general operations and very realistic for the case of no blank holder. For a typical drawing ratio of 2, Chung and Swift[7] have demonstrated that the stress ratio $\rho$ as stated above varied from zero at the rim of the blank to 2.3 at just before the rim of the die, while the normal stress in thickness direction can be neglected. In this study, therefore, the influence of stress state on earing behavior is analyzed under plane stress state within this range of $\rho$. The normalized radial strain for three importantly ideal orientations in *FCC* metals are calculated using the PS (Eqn. (5a)) and TS (Eqn. (5b)) assumptions, respectively. Since the $E(\theta)$ results are symmetrical to both the rolling and transverse directions, only those for $\theta$ in the range of 0° to 90° at $\rho = 0$, 1 and 2.3 were illustrated in Figs. 1 and 2 for the PS and TS assumptions, respectively. For comparison, the earing extent indexes calculated by using Eqn. (7) for both assumptions were plotted in Fig. 3, as a function of stress ratio.

As has been concluded by Inagaki[5], the ear profiles predicted under the TS assumption are more continuous than those under the PS assumption. This is more explicitly demonstrated in the present study by using the normalized radial strain (compare Figs. 1 and 2).

In addition, another important difference concerning the influence of stress state on earing may be found. It can be seen from Figs. 1 and 2 that, the ear profiles predicted under the PS assumption change obviously with the stress ratio, while those predicted under the TS assumption change only slightly, and the latter has not been demonstrated by Inagaki using the radial strain calculated under the assumption of $\sigma_\theta = -1$[5]. With the change of stress ratio, the ear profiles predicted under the PS assumption change significantly in the (100)[001] and (110)[001] orientations, and slightly in the (112)[11$\bar{1}$] orientation (Fig. 1). In the (100)[001] orientation (Fig. 1(a)), the $E(\theta)$ results show prominent ears at 0° and 90° directions for both $\rho = 0$ and 1[1,8], while indicate abnormally small ears at 0°, 25°, 65° and 90° directions for $\rho = 2.3$. In the (110)[001] orientation (Fig. 1(b)), the widely observed 90° ear is satisfactorily predicted for $\rho = 1$ and 2.3[1,8], while not indicated for $\rho = 0$. On the other hand, the ear profiles predicted by the TS assumption change only slightly for the three orientations of interest (Fig. 2).

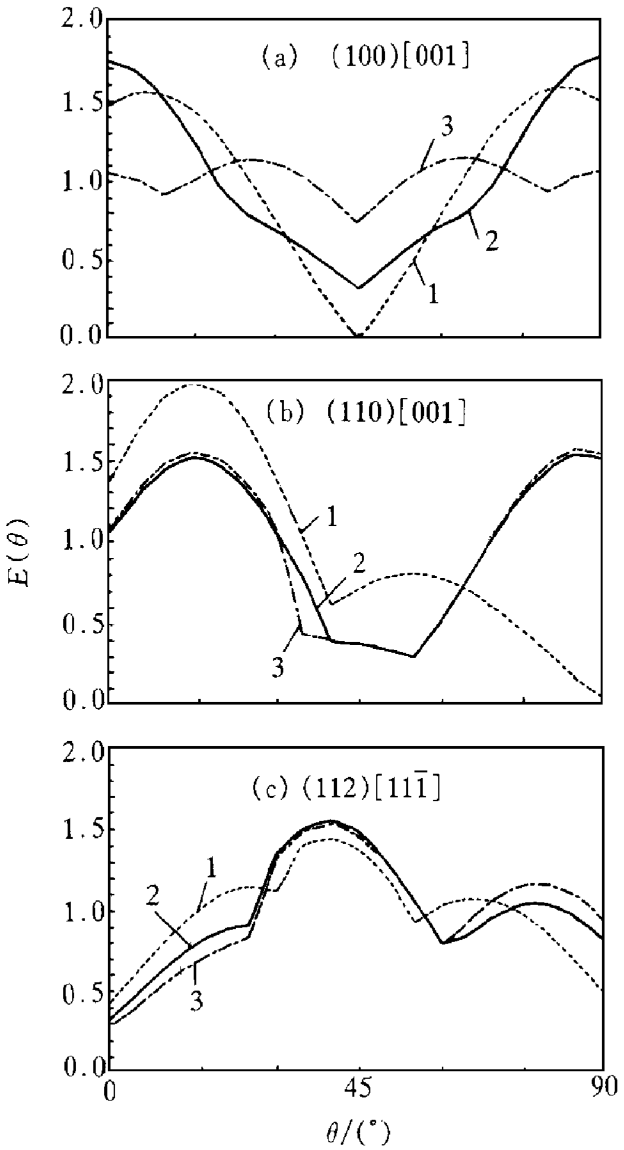


**Fig. 1 Ear profiles calculated under PS assumption at different stress ratio**

1 — $\rho = 0$; 2 — $\rho = 1$; 3 — $\rho = 2.3$

### 3.2 Influence of stress state on earing extent

Fig. 3 shows that in both predictions the change of earing extent with stress ratio is obvious, and is different for different orientations. In the TS predictions, with the increasing of stress ratio, the EEI increases obviously in the (110)[001] and (112)[11$\bar{1}$] orientations and decreases significantly in the (100)[001] orientation. In the PS predictions, the changes of

EEI with stress ratio are similar to that in the TS predictions, though less steadily they are in the (100) [001] (Fig. 3(a)) and (110) [001] (Fig. 3(b)) orientations.

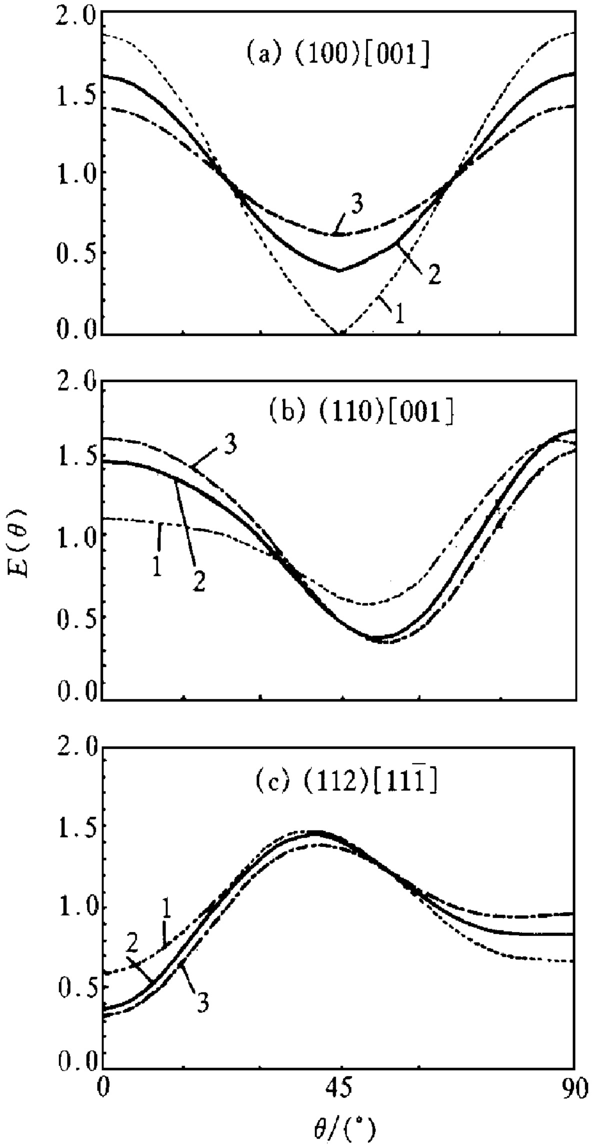


**Fig. 2 Ear profiles calculated under TS assumption at different stress ratio**

1 — $\rho$ = 0; 2 — $\rho$ = 1; 3 — $\rho$ = 2.3

Though obvious change of earing with stress state has been demonstrated as stated above, it can also be inspected that the $E(\theta)$ results calculated at about $\rho$ = 1 can represent the average earing behavior in a satisfactory degree, as shown in Figs. 1 to 3. This is identical to the CMTP analysis carried out by the authors[9, 10], and confirms the reasonability of the simplified method adopted in the Tucker analyses of earing by several investigations[1-4].

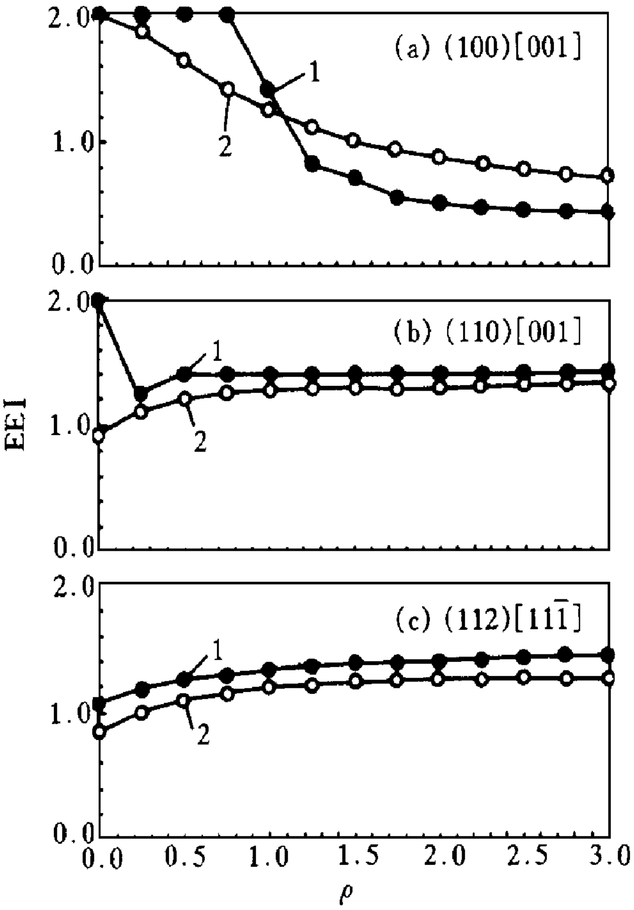


**Fig. 3 Earing extent indexes calculated for different stress ratio under PS and TS assumptions**

1 —PS; 2 —TS

## 3.3 Prediction of cup profile

As it is proportional to the radial strain, the cup height can be calculated by the use of radial strain and geometric conditions in cup drawing. In a series of papers, Kanetake *et al*[2-4] have reported the method of calculating the cup height from the radial strain, and the cup heights they obtain agree very well with those observed. However, the values of $n$ and $\tau_0$ used in their calculations (see Eqn. (10) in Ref. [3]) are not taken from experimental data. They are used as adjustable parameters to obtain the best fit between the calculated and measured results.

In this paper, a more realistic method is used for the calculation of cup height by using the normalized radial strain, which has been proposed and successfully used in the CMTP analysis of earing behavior[9, 10]. Moreover, both the average extent and type of earing can be well represented by the normalized radial strain calculated at $\rho$ = 1 as stated above. For simplification, therefore, the stress state of $\rho = 1$ was adopted in this study. Accordingly, the cup height at $\theta$ degree from the rolling direction, $h(\theta)$, can be calculated by the following equation,

$$h(\theta) = h_0 + (h_f - h_0)E(\theta) \tag{8}$$

where $E(\theta)$ is the normalized radial strain calculated under plane stress state of $\rho = 1$. $h_0$ and $h_f$ are the cup height without and with radial strain, respectively, the details of which can be seen in Ref[9].

In an aluminum single crystal sheets, the cup heights predicted in this way are compared with those observed by Tucker[1], as shown in Fig. 4. It is clear that, the predicted cup heights for both assumptions are in reasonable agreement with those observed, though compared with the experimental cup profile more discontinuity is indicated in the PS predictions.

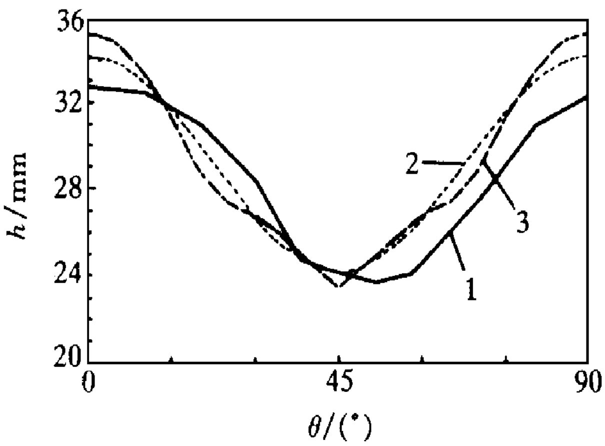


**Fig. 4 Comparison of the predicted cup heights ( $\rho = 1$ ) with those measured, for an Al single crystal sheet with (100)[001] orientation**
1—Exp; 2—TS; 3—PS

## 4 CONCLUSIONS

Using the normalized radial strain derived from Tucker theory, the influence of stress state in the flange of blank on earing behavior has been analyzed quantitatively under plane stress state with different ratios of radial stress to circumferential one. The results could be summarized as follows.

(1) The stress state has obvious influence on earing behavior. Using the total-slip assumption, the earing extent changes obviously and steadily with the stress ratio, while the ear profiles change only slightly using the partial-slip assumption; both the extent and profile of earing change obviously with the stress ratio.

(2) The change of the earing extent with the stress ratio can be different for different orientations. With increasing stress ratio, the earing extent increases in the (110)[001] orientation and (112)[$\bar{1}$11] orientation, and decreases in the (100)[001] orientation.

(3) The average earing behavior for typical deep drawing ratio can be well predicted under plane stress state with $\rho = 1$.

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