WAVE FIELD EXTRAPOLATION IN VISCOELASTIC MEDIUM AND VARIABLE FOCUS METHOD TO SEPARATE SEISMIC COMPOUND WAVE[①]

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ABSTRACT In viscoelastic medium, a new method for wave field extrapolation, which was very different from the three classical methods of wave field extrapolation in elastic medium was introduced. In addition, a method for separating the compound wave response of two (or more) nearby reflectors through wave field extrapolation was presented. It meaned that not only seismic migration to enhance lateral resolution, but also separate seismic compound wave to enhance vertical resolution could be done. This paper in which the Born's approximation was not needed, was motivated by Bleistein's lecture, but it was very different from his linearization method.

Key words seismic wave viscoelastic medium wave field extrapolation separating compound wave

1 INTRODUCTION

Seismic waves propagate in an imperfect elastic medium, thus modeling this propagation by a wave equation for a perfect elastic medium cannot describe the practical physical processes sufficiently and accurately for some applications. For example, it may be difficult to obtain sufficient high-resolution images from migration or tomography for models based on the acoustic or perfect elastic wave equation when there are two nearby reflectors. More specifically, in travel-time reflection tomography combined with waveform inversion, it is first necessary to identify arrival times of reflected waves on the seismic trace. Then, the reflected waves are modeled, based on the modeling wave equation in a background medium. The problem we will address here is that the reflected wave produced by two (or more) nearby reflectors forms a compound reflected wave which may appear as a single distorted spike on the trace, we cannot discriminate the two reflected waves from the trace.

On the other hand, trying to match the apparent single arrival time observed for this compound wave leads to an unsatisfactory result when it is interpreted as a single reflection event. To remedy this problem, one must separate the compound wave. At present the primary method of choice for doing this is deconvolution. However, because of its imperfect basic theory, there are two serious drawbacks to use deconvolution for this purpose. The first is that the necessary conditions for deconvolution are not satisfied, for example, the seismic wavelet in deconvolution is time invariant; this leads to lack of fidelity and makes it difficult to distinguish true reflectors from false reflectors. The second problem is that deconvolution is sensitive to noise. Moreover, the viscosity of the earth medium causes changes in the waveform and leads to distortions of the full waveform inversion based on a nonviscose wave equation, and little is known about the inversion for viscoelastic waves.

Here, based on the viscoelastic wave equation, we present a new method for wave field extrapolation, which was very different with the three classical methods of wave field extrapola-

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tion. By this method, we can not only do seismic migration to enhance lateral resolution, but also separate seismic compound wave to ehnhance vertical resolution.

2 WAVE FIELD EXTRAPOLATION IN VISCOELASTIC MEDIUM

We use Stokes equation, which was used by Ricker to investigate the wave propagation in viscoelastic medium.

The Stokes equation is

$$\nabla^2\left[u + \frac{\eta_1 + 4/3\,\eta_2}{\rho c^2}\frac{\partial u}{\partial t}\right] = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \delta(x - x_s) \qquad (1)$$

The wavespeed is represented as a perturbation on a known reference speed, $c_0(x)$, expressed mathematically as follows:

$$\frac{1}{c^2(x)} = \frac{1}{c_0^2(x)}[1 + \alpha(x)] \qquad (2)$$

And we set

$$\frac{1}{\omega_0} = \frac{\eta_1 + 4/3\,\eta_2}{\rho c_0^2(x)}, \quad a = 1 - \frac{\omega}{\omega_0} i.$$

Then, in frequence domain, Enq. (1) is reduced to

$$a\,\nabla^2 u(x, x_s, \omega) - (1 - a)\,\nabla^2[\alpha(x) u(x, x_s, \omega)] + \frac{\omega^2}{c_0^2} u(x, x_s, \omega) + \frac{\omega^2}{c_0^2}\alpha(x) u(x, x_s, \omega) = -\delta(x - x_s) \qquad (3)$$

The total field is decomposed into an incident and scattered field

$$u(x, x_s, \omega) = u_I(x, x_s, \omega) + u_s(x, x_s, \omega) \qquad (4)$$

In which $u_I(x, x_s, \omega)$ is the solufion of the following equation:

$$a\,\nabla^2 u_I(x, x_s, \omega) + \frac{\omega^2}{c_0^2} u_I(x, x_s, \omega) = -\delta(x - x_s)$$

Futhermore, let $g(x, x_g, \omega)$ satisfy the following equation:

$$a\,\nabla^2 g(x, x_g, \omega) + \frac{\omega^2}{c_0^2} g(x, x_g, \omega) = -\delta(x - x_g) \qquad (5)$$

Similarly as Bleistein *et al*[1], by using Green function, from Eqn. (3), one obtain

$$u_s(x_g, x_s, \omega) = \int_D \left[(1 - a)\,\nabla^2(\alpha(x) u(x, x_s, \omega)) - \frac{\omega^2}{c_0^2}\alpha(x) u(x, x_s, \omega)\right] g(x_g, x, \omega)\,\mathrm{d}x \qquad (6)$$

In which, x_g —receiver position, x_s —source position.

Eqn. (6) is a integral differential equation. In Eqn. (6), $\alpha(x)$ that we seek and total field $u(x, x_s, \omega)$ are all unknown function, according to this, the Eqn. (6) is a nonlinear inverse problem. Motivated by Bleistein's lecture, but different with his linearization method, we take $\left[(1 - a)\,\nabla^2(\alpha u) - \frac{\omega^2}{c_0^2}(\alpha u)\right]$ as one term. Namely, set

$$O(x, x_s, \omega) = (1 - a)\,\nabla^2(\alpha u) - \frac{\omega^2}{c_0^2}(\alpha u) \qquad (7)$$

and Eqn. (6) is reduced to

$$u_s(x_g, x_s, \omega) = \int_D O(x, x_s, \omega) \cdot g(x, x_g, \omega)\,\mathrm{d}x \qquad (8)$$

We view here the integral Eqn. (8) as representing a source image $O(x, x_s, \omega)$ degraded by a filter function, $g(x, x_g, \omega)$, into a new image $u_s(0, 0, \omega)$. The physical motivation for our analysis is the reconstruction source image from the degraded image. First, let's inverse $O(x, x_s, \omega)$ from Eqn. (8). For one dimension case, suppose $\frac{\omega}{\omega_0} \ll 1$ and $c_0(x) = c_0 =$ constant, then

$$g(x, x_g, \omega) = -c_0 e^{-\frac{\omega_0}{2c_0}\left(\frac{\omega}{\omega_0}\right)^2 \cdot |x - x_g|} e^{i\omega|x - x_g|/c_0} / 2i\omega.$$

If geophone is on the surface of earth and $x_g = x_s = 0$, then from Eqn. (8) we obtain

$$u_s(0, 0, \omega) = -\frac{c_0}{2i\omega}\int_0^\infty O(x, 0, \omega) \cdot e^{-\frac{\omega^2}{2\omega_0 c_0}x} e^{i\omega x/c_0}\,\mathrm{d}x \qquad (9)$$

Similarly as Yang *et al*[2], set

$$s^2 = x/c_0\omega_0, \text{ namely, } x = c_0\omega_0 s^2 \qquad (10)$$

$$\tilde{O}(s, 0, \omega) = O(x, 0, \omega) e^{i\omega x/c_0} \qquad (11)$$

and

$$\hat{\psi}(s\omega) = (s\omega)^2 e^{-\frac{1}{2}(s\omega)^2} \quad (12)$$

in which, overhat "^" means Fourier transform respect with t .

From Eqns. (9) ~ (12), one obtain

$$-\frac{i\omega^3 u_s(0, 0, \omega)}{c_\psi \omega_0 c_0^2} = \frac{1}{c_\psi}\int_0^\infty WO(s, 0, \omega) \cdot \hat{\psi}(s\omega)\frac{ds}{s} \quad (13)$$

where

$$c_\psi = \int_0^\infty \frac{(\omega^2 e^{-\frac{1}{2}\omega^2})^2}{\omega} d\omega$$

By wavelet theory, from Eqn. (13) we know that

$$O(s, x_s, \omega) = -\frac{i\omega^3 u_s(0, x_s, \omega)}{c_0^2 c_\psi \omega_0} \quad (14)$$

Due to

$$WO(s, 0, \omega) = O(s, 0, \omega)\hat{\psi}(s\omega)$$

We get

$$O(x, 0, \omega) = -\frac{i\omega^3 u_s(0, 0, \omega)}{c_0^2 c_\psi \omega_0} \cdot \hat{\psi}(s\omega) e^{-i\omega x/c_0} \quad (15)$$

We can easily check that $O(x, 0, \omega)$ in Eqn. (15) satisfies integral Eqn. (9). In fact, substituting $O(x, 0, \omega)$ in Eqn. (15) into the right side of Eqn. (9), one obtains that the right side of Eqn. (9) equals to

$$\frac{\omega^2}{c_\psi} u_s(0, 0, \omega)\int_0^\infty (s\omega)^2 e^{-(s\omega)^2} s ds$$

$$= \text{left side of Eqn. (9)} .$$

Note that the term $O(x, x_s, \omega)$ in Eqn. (8) relates only to source position x_s and does not relate to receiver position x_g .

Put Eqn. (15) into Eqn. (8), for one dimension case and $x_s = 0$, one obtain

$$u_s(x_g, 0, \omega) = -\frac{i\omega^3 u_s(0, 0, \omega)}{c_\psi \omega_0 c_0^2} \cdot \int_0^\infty \hat{\psi}(s\omega) e^{-i\omega x/c_0} g(x, x_g, \omega) dx \quad (16)$$

in which, $x_g > 0$.

Eqn. (16) means that one can calculate the scatter wave field value under the surface, $u_s(x_g, 0, \omega)$, $x_g > 0$, from the scatter wave field value on the surface, $u_s(0, 0, \omega)$. Eqn. (16) is the wave field extrapolation formula, which has a clearer physical meanings than the classical three methods for wave field extrapolation in elastic medium. The data were migrated using the developed scheme based on Eqn. (16) for wave extrapolation with compensation for absorption. Using this result will further improve the resolution of the output.

3 RESOLUTION ENHANCEMENT

In this section, we describe a method for resolution enhancement. This work is an outgrowth of the wave extrapolation Eqn. (16) described in previous section. We explain the method with the aid of an example, shown in Fig. 1. Fig. 1(a) and (b) show two reflection events that have been transformed by the low-pass filter effects of the underground medium. Fig. 1(c) depicts the compound wave representing the sum of these two events. We then perform focus analysis of this compound wave (that means input different ω_0 in Eqn. (9)) and seek the value of the scaling variable that maximize the peak amplitudes of the output. Fig. 1(d) shows a non-maximized output. Fig. 1(e) shows the maximized output. It is easy to see that the peaks of the original signal are far better resolved in Fig. 1(e) than in Fig. 1(d) and are also better resolved than in the input signal, in Fig. 1(c).

The second example arises from a field data set. Fig. 2 shows the migration output in elastic

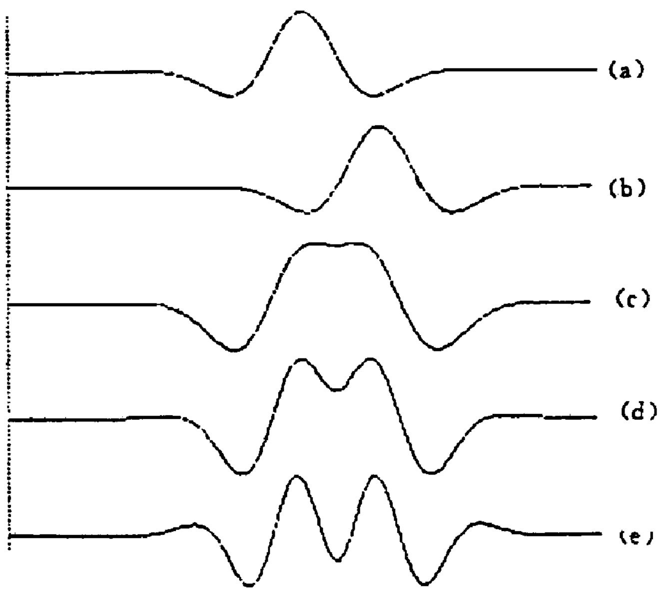

Fig. 1 An example of peak separation using focus method

medium. The data comes from a survey in northwest China. Fig. 3 is the output processed by the method described here. The resolution can be seen to be much higher than in Fig. 2. In particular, note the region between 2875 ms and 3040 ms. The two reflectors of Fig. 2 have been sharpened and a third weak reflector has emerged between them in Fig. 3. The structure depicted in the latter figure was later confirmed by drilling. Another feature of the multiscale processed data of Fig. 3 is a general improvement in the signal-to-noise ratio.

4 CONCLUSIONS

We have demonstrated here a method for wave field extrapolation in viscoelastic medium. Based on this method, we have further proposed a method to separate seismic compound wave for overcoming the attenuation of waves due to viscoelastic effects. This method has been checked by numerical example and a field data, through wave extrapolation. This has shown that we can not only to do seismic migration to enhance lateral resolution, but also separate compound wave to enhance vertical resolution. The method for wave extrapolation in this paper is very different with the three classical methods for wave extrapolation. Our method is motivated by Bleistein's

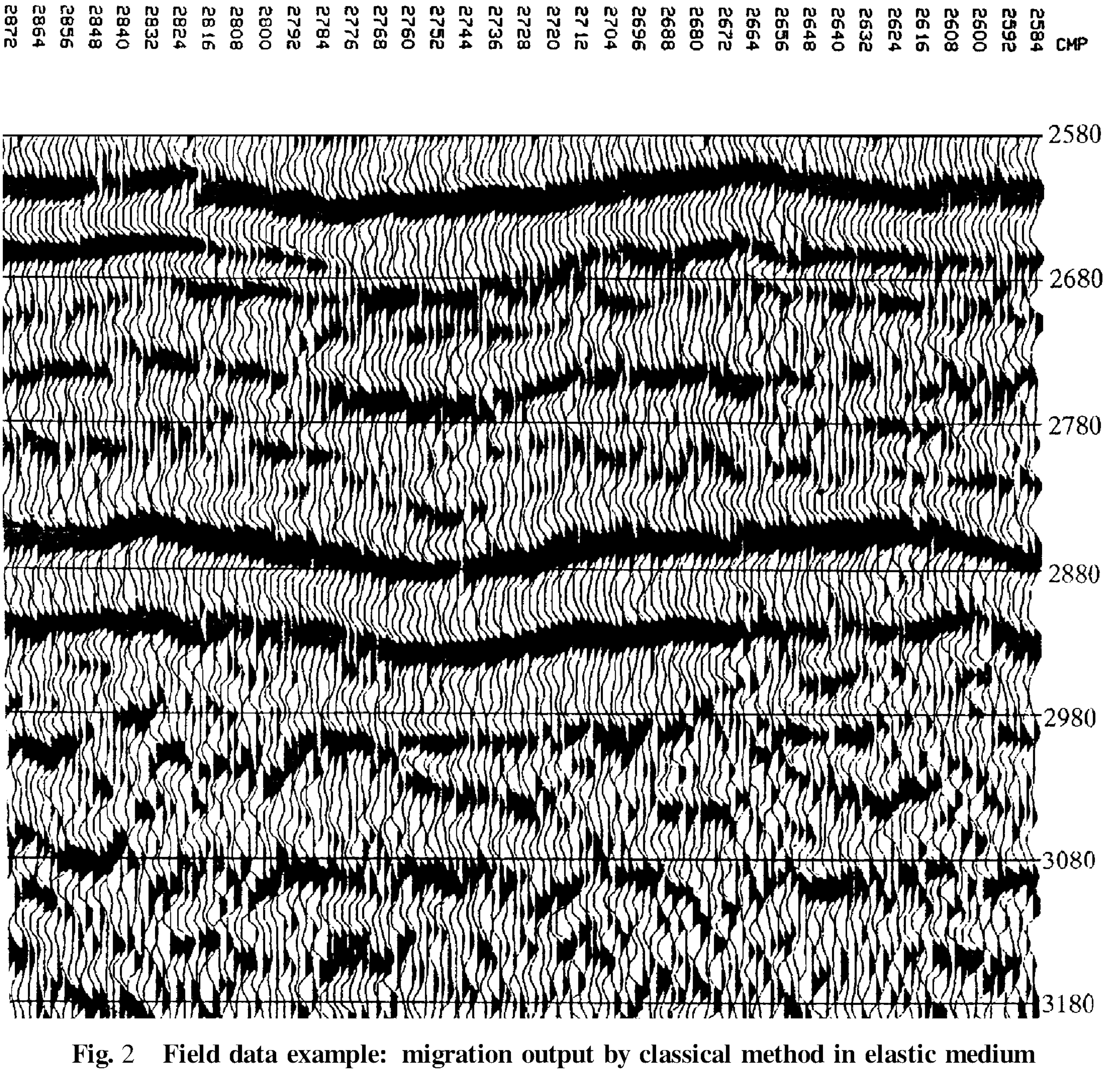

Fig. 2 Field data example: migration output by classical method in elastic medium

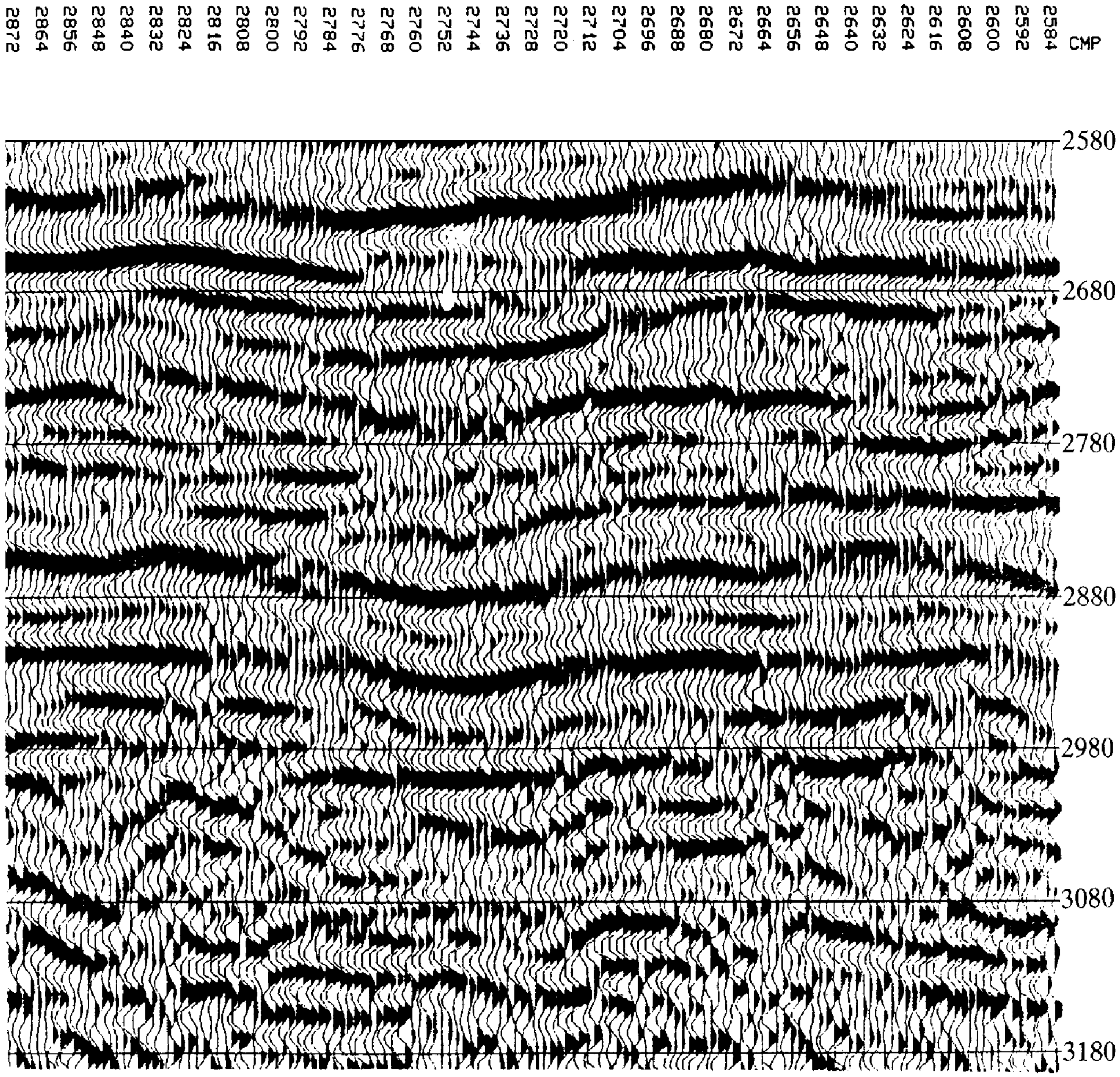

Fig. 3 Field data example of Fig. 2 after resolution enhancement using focus method

lecture (1992, in China), but is different with his linearization method. In this paper, the Born's approximation is not needed. Next step is naturely to combine the method in this paper with the velocity inversion, which is very convenient. This problem will be discussed in another paper.

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