EXTRAPOLATING METHOD IN PLOT FOR FATIGUE LIFE PREDICTION OF COMPONENTS①

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ABSTRACT How to insert calculation extrapolation in plot, a new fatigue life prediction method, was studied in detail. The "step by step approach" handling method, combined with plot look-up and extrapolation, was put forward, which not only made it possible to extrapolate in plot, but also retained the advantages of simple and convenient application. Generally by repeating plot look-up for three times and extrapolating calculation, the satisfying results of predicted fatigue life could be obtained.

Key words extrapolation plot fatigue life

1 INTRODUCTION

To simplify the process of predicting the fatigue life of components and decrease the operating errors, a kind of plot method aided by computer has been established[1]. The plot forming is that the parameters such as material, load, the shape index n of the Load Accumulated Frequency Distribution (LAFD) and so on, are expressed in unified mathematic equations, then we can work out the results by computer and make up the plot. The fatigue life of component can be looked up in the plot only by the element parameters.

To make the application of plot more practicable, a method is studied, which introduces the extrapolation into plot look-up; and a procedure named "step by step approach", which combines the plot look-up with extrapolation, is put forward in this paper. This method will play an important role in bringing the effectiveness of plot into full play.

2 NECESSITY OF EXTRAPOLATION IN THE APPLICATION OF PLOT

According to the standard LAFD[2] and Miner Rule[3], the accumulated damage S to component can be expressed as

$$S = \frac{2}{N_w(\sigma_{max}/\sigma_w)^{-K}} + \sum_{i=0}^{30} \frac{H_0^{1-\left(\frac{i\Delta\sigma}{\sigma_{max}}\right)^n} - H_0^{1-\left[\frac{\Delta\sigma(i+1)}{\sigma_{max}}\right]^n}}{N_w\left[\frac{(i+1)\Delta\sigma}{\sigma_w}\right]^{-K'}} \qquad (1)$$

where H_0—maximum accumulative frequency of standard LAFD, σ_{max}—the maximum load, n = shape index of LAFD, $\Delta\sigma$—load increment, σ_w—fatigue limit, N_w—fatigue life of S-N curve at the corner, K = slope of the oblique line of S-N curve.

$$K' = \begin{cases} K & (i+1)\Delta\sigma \geqslant \sigma_w \\ 2K-1 & (i+1)\Delta\sigma < \sigma_w \end{cases}$$

Here the modified Miner Rule is adopted. Eqn. (1) was obtained by classifying equally the LAFD in 32 grades (Fig. 1), and assuming that the maximum loads will appear two times in the component's work life.

According to the theory determining H_0 on the basis of the accumulated damage $S = 1$[4], in Eqn. (1), H_0 can be substituted by the predicted fatigue life L of component, and the mathematic calculation equation for plot forming is gotten:

$$1 - \frac{2}{N_w(\sigma_{max}/\sigma_w)^{-K}} =$$

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$$\sum_{i=0}^{30} \frac{L^{1-\left(\frac{i\Delta\sigma}{\sigma_{max}}\right)^n} - L^{1-\left[\frac{\Delta\sigma(i+1)}{\sigma_{max}}\right]^n}}{N_w\left[\frac{(i+1)\Delta\sigma}{\sigma_w}\right]^{-K'}} \tag{2}$$

For example, the fatigue plot of material 9Cr2Mo can be drawn (Fig. 2) as a roll.

Fig. 2 Plot for material 9Cr2Mo

1 — n = 6 (Original Miner Rule)
1′ — n = 6 (Modified Miner Rule)
2 — n = 8 (Original Miner Rule)
2′ — n = 8 (Modified Miner Rule)

It is known from Eqn. (2) and Fig. 2 that the maximum load σ_{max} of LAFD is an important and decisive factor in fatigue life prediction by means of plot. The different predicting life values will appear when the different values of σ_{max} are used in handling. This means that it is very important to improve the predicting precision to determine the maximum load σ_{max} in LAFD. On the other hand, because the LAFD is usually obtained by measuring and counting the practical load, and the measuring time period is usually very short comparing to the component's work life, the maximum load appearing in work life has very little chance to be included in the measuring loads. In order to solve this problem in general predicting process, the maximum load in the LAFD is often determined by extrapolating method. It is known by analysing Eqns. (1) and (2) that the relationship between σ_{max} and H_0 or L is expressed, but the equations do not mention how to choose the value σ_{max} and what the value limit is. If the component's fatigue life L is looked up in the plot only with the measured maximum load σ_{max}, it shows probably the tendency that the predicted life is not safe in engineering (the fatigue life tends to longer than the practical life). Therefore it is necessary to extrapolate in plot application. Furthermore, because the plot method is simple and practicable, the corresponding extrapolating method should be simple and convenient too. Only so can the advantages of plot be given full play.

3 "STEP BY STEP APPROACH" METHOD OF PLOT LOOK-UP COMBINED WITH EXTRAPOLATION

Because common extrapolating method includes a very complicated process[5], it can not be used in plot obviously. A new calculating extrapolating method is adopted, which was put forward by the author[6]. The calculating equations are as follows:

$$\sigma_{max.2} = \sigma_{max.1}\left[\frac{\lg H_2}{\lg H_1}\right]^{\frac{1}{n}} \tag{3a}$$

or

$$\sigma_{max.2} = \sigma_{max.1}\left[1 + \frac{\lg\frac{H_2}{H_1}}{n\lg H_1}\right] \tag{3b}$$

Eqn. (3) has reversibility, that is, when $H_1 < H_2$, it can give $\sigma_{max.2} > \sigma_{max.1}$ and if $H_2 < H_1$, then $\sigma_{max.2} < \sigma_{max.1}$. This feature has provided great convenience for the application of extrapolation in plot.

Assuming that the LAFD, which is gotten through practical measurement and replaced with a most approaching curve of standard LAFD, is expressed as curve a in Fig. 3, how to get the component's fatigue life with the original LAFD a is to be studied.

(1) From σ_0 to H_1

As shown in Fig. 3, the component's fatigue life L can be looked up in plot (Fig. 2) according to the σ_0 in the original curve a under the prerequisite that σ_w, N_w, and n are known, and it is the accumulative frequency H_1 in Fig. 3. From σ_0 to H_1 a curve can be formed (curve b). This is the accumulative frequency curve whose maximum load is σ_0 and can make the accumulated damage equel to 1.

Fig. 3 Procedure of plot look-up combining with extrapolation

(2) From H_1 to σ_1

It is obvious that in above handling the H is increased from H_0 to H_1 without considering that the maximum load may increase with the increasing of accumulated loads. In most cases, this is impractical. So we must consider the effect of the rised load. In this step the extrapolation is done taking the H_0 as the original accumulated frequency and the H_1 as the goal accumulated frequency. Through extrapolotion the maximum load is increased from σ_0 to σ_1. The extrapolating formula is Eqn. (3).

(3) From σ_1 to H_2

Due to extrapolation the maximum load is increased greatly. The increased load certainly affects the predicting fatigue life. Therefore we must look up the predicting life H_2 again in plot according to the new maximum load σ_1. Obviously H_2 is less than H_1 because σ_1 is greater than σ_2.

(4) From H_2 to σ_2

Because the gotten H_2 in above step is less than H_1 and σ_1 was deduced by taking H_1 as parameter, it must be a little bigger to take σ_1 as the maximum load. For this reason, the maximum load should be decreased properly. In this case the extrapolation is also adopted, but in the extrapolating process the original accumulated frequency is H_1, and the terminal one is H_2. In fact, because $H_1 > H_2$, the extrapolation does not increase the maximum load but decrease it. According to the reversibility of Eqn. (3), this handling is possible. By this calculation σ_2 can be obtained.

(5) From σ_2 to H_3

The σ_2 obtained through reversing extrapolation must be lower than σ_1, so the new predicted fatigue life L must be looked up in the plot according to σ_2, that is to look up H_3 in Fig. 3. Keeping on the step as above, the differences of σ_{i-1}-σ_i and H_{i+1}-H_i decrease little by little. At last σ and H can tend to the fixed value σ^* and L^*. This L^* is the component's predicted fatigue life obtained on the condition that the maximum load reaches σ^* after finishing extrapolation. Through test calculation the satisfactory result can be obtained by repeating calculation for three times.

4 AN EXAMPLE OF PREDICTING CALCULATION

In a large aluminium-processing factory, the 2800 rolling line had been reformed and expanded, and the four-roll rolling mill was planed to implement strengthening rolling. The fatigue life of the back-up rolls in the dangerous roll-necks required predicting under the strengthened condition. The material of the back-up roll is 9Cr2Mo. By analysis and comparison, its σ_b was about 700 MPa; the stress of turning point in S-N curve σ_w was equal to 163.46 MPa under the condition that the destruction probability was fifty percents; the circulation life of turning point in S-N curve $N_w = 10^7$; the slope of the S-N curve on oblique part $K = 5$. The plot for material 9Cr2Mo has been shown in Fig. 2. The LAFD for the roll necks is drawn in Fig. 4 according to the materials and weight (tons) that the rolling mills will process. Its shape index $n = 6$. The procedure of plot look-up combining with extrapolation is shown in Table 1.

At last the obtained fatigue life of the dangerous point on back-up rolls was 3. 736 years and met the demands.

5 CONCLUSIONS

(1) The plot method is a simple one on the fatigue life prediction which is deduced from complicated theories. But its defect lies in the absence of extrapolation. The procedure which combines the plot look-up with extrapolation has beed put forward in this paper to make up for the defect.

(2) All equations in this paper are based on the standard LAFD, so to choose the suitable shape index n for the standard LAFD is an important means to improve predicting precision. The accuracy of shape index n appears to be more important when the extrapolating equations (Eqn. (3)) are used repeatedly. How to choose n can refer to reference[2].

(3) To some components, their maximum load can be limited in a certain extent. For example some components have overload protection device, to this components in the forming of the LAFD, with which the accumulated damage S is equal to 1, the maximum load in LAFD must be the maximum load that the component may be borne in its work life. In this case the extrapolation is done by the forming of LAFD. In the plot look-up it is not required to extrapolate again.

Table 1 Procedure of predicting example

Procedure	Results	Methods or Formulas
Original data	$H_1 = 10^5$, $n = 6$ $\sigma_{max.1} = 220.01$ $\sigma_w = 163.46$	Calculation and experience
$H_1 \rightarrow H_2$	$\sigma_w / \sigma_{max.1} = 0.743$ $H_2 = 10^8$	consult Fig. 2
Extrapolating $\sigma_{max.2}$	237.937	Eqn. (3)
$H_2 \rightarrow H_3$	$\sigma_w / \sigma_{max.2} = 0.685$ $H_3 = 3 \times 10^7$	consult Fig. 3
Extrapolating $\sigma_{max.3}$	235.27	Eqn. (3)
$H_3 \rightarrow H_4$	$\sigma_w / \sigma_{max.3} = 0.69$ $H_4 = 3.2 \times 10^7$	consult Fig. 2
Life (years)	3.736	$\frac{\text{Total of Circulation Frequency}}{\text{Year's Circulation Frequency}}$

Fig. 4 LAFD for the roll necks

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