SURFACE INTEGRAL OF THREE-DIMENSIONAL VELOCITY FIELD FOR SQUARE BAR DRAWING THROUGH CONICAL DIE①

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ABSTRACT The kinematically admissible three-dimensional continuous velocity field of square bar drawing through a conical die was established. Its divergence has been proved to be zero. Then, the integral as a function of the upper limit and surface integral were used respectively in deformation and friction powers, and an upper bound analytical solution of the square bar drawing stress was obtained by the upper-bound theorem.

Key words square bar drawing three dimensional velocity field surface integral

1 INTRODUCTION

The square bar drawing is different from round and flat bar drawing in that the drawing is not axisymmetrical and plane deformation problem but a three dimensional deformation. So, how to establish three dimensional velocity field of square bar and make it be kinematically admissible, as the method for round and strip drawing in Ref. [1- 2], is first priority of this paper. What kind of mathematical integration will be used to make above velocity field get analytical solution is put on emphasis of the paper.

2 VELOCITY FIELD

The deforming zone of drawing square bar through a conical die is shown in Fig. 1(a), (b) and the die in (c). If the die is dissected in longitudinal direction by plane XOZ, let point O be origin of the coordinates at the exit, then on the cross section at the distance x from the exit, the horizontal velocity, thickness and width are v_x, $h(x)$ and $b(x)$. The equations of the die profile are

$$h(x) = 2x\,\mathrm{tg}\,\alpha \qquad (1)$$

$$b(x) = h(x) = 2x\,\mathrm{tg}\,\alpha \qquad (2)$$

where α is the half die angle, as shown in Fig. 1(b).

From Von Karman's assumption, let x, y, z be the principal directions of stress and σ_x, v_x be uniformly distributed on the cross section, thus

$$h_0 b_0 v_0 = h_1 b_1 v_1 = h(x)\,b(x)\,v_x = C \qquad (3)$$

$$v_x = \frac{C}{b(x)\,h(x)} \qquad (a)$$

From Cauchy Equations and the direction of v_x is opposite from x, we can get

$$\dot{\varepsilon}_x = -\frac{\partial v_x}{\partial x} = \frac{C[b'(x)h(x) + b(x)h'(x)]}{b^2(x)h^2(x)}$$

$$\dot{\varepsilon}_x - \frac{Cb'(x)}{b^2(x)h(x)} - \frac{Ch'(x)}{b(x)h^2(x)} = 0$$

Thus, the strain rate and velocity fields are

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Fig. 1 The square bar drawing

$$\left.\begin{aligned} \dot{\varepsilon}_x &= \frac{C[b'(x)h(x) + b(x)h'(x)]}{b^2(x)h^2(x)}; \\ \dot{\varepsilon}_y &= \frac{-Cb'(x)}{b^2(x)h(x)}; \\ \dot{\varepsilon}_z &= \frac{-Ch'(x)}{b(x)h^2(x)} \end{aligned}\right\} \quad (4)$$

$$\left.\begin{aligned} v_y &= \int \dot{\varepsilon}_y \partial y = \frac{-Cb'(x)y}{b^2(x)h(x)}; \\ v_z &= \int \dot{\varepsilon}_z \partial z = \frac{-Ch'(x)2}{b(x)h^2(x)} \end{aligned}\right\} \quad (b)$$

From Eqn. (2) and substituting $b'(x) = h'(x) = 2\mathrm{tg}\,\alpha$, $h(x) = b(x)$ into Eqns. (a), (b) and (4), we get

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$$\left.\begin{aligned} v_x &= \frac{C}{4x^2\mathrm{tg}^2\alpha}; \\ v_y &= \frac{-Cy}{4x^3\mathrm{tg}^2\alpha}; \\ v_z &= \frac{-Cz}{4x^3\mathrm{tg}^2\alpha} \end{aligned}\right\} \quad (5)$$

$$\left.\begin{aligned} \dot{\varepsilon}_x &= \frac{C}{2x^3\mathrm{tg}^2\alpha}; \\ \dot{\varepsilon}_y &= \frac{-C}{4x^3\mathrm{tg}^2\alpha}; \\ \dot{\varepsilon}_z &= \frac{-C}{4x^3\mathrm{tg}^2\alpha} \end{aligned}\right\} \quad (6)$$

It can be seen from Fig. 1(b) that substituting $x = x_0$, $x = x_1$ and $y = 0$, $z = 0$ into Eqn. (5) yields $v_x = v_0$, $v_x = v_1$, $v_y = 0$, $v_z = 0$. Since the divergence of three dimensional velocity vector is

$$\vec{v} = v_x i + y_y j + v_z k$$

and

$$\mathrm{div}\,\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z = 0$$

we know Eqns. (5) and (6) are kinematically admissible.

3 UPPER BOUND POWER

3.1 Integration of plastic deformation power

Substituting Eqn. (6) into the equation

$$\dot{W}_i = \int_V \sigma_s \sqrt{\frac{2}{3}} \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}\, \mathrm{d}v = \sigma_s \sqrt{\frac{2}{3}} \int_V \sqrt{\dot{\varepsilon}_x^2 + \dot{\varepsilon}_y^2 + \dot{\varepsilon}_z^2}\, \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,$$

and noting $h(x)/2$, $b(x)/2$ are still the functions of x, integrating x, y, z in turn yields

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$$\dot{W}_i = \sigma_s \sqrt{\frac{2}{3}} \int_{x_1}^{x_0} \mathrm{d}x \int_{-\frac{h(x)}{2}}^{\frac{h(x)}{2}} \mathrm{d}y \cdot \int_{-\frac{h(x)}{2}}^{\frac{h(x)}{2}} \frac{\sqrt{6}C}{4x^3\mathrm{tg}^2\alpha} \mathrm{d}z$$

$$= 2\sigma_s C \ln x \mid_{x_1}^{x_0} = \sigma_s C \ln\left(\frac{x_0}{x_1}\right)^2 = \sigma_s C \ln\left(\frac{h_0}{h_1}\right)^2 = \sigma_s C \ln \lambda \quad (7)$$

where $x_0 = \frac{h_0}{2\mathrm{tg}\,\alpha}$, $x_1 = \frac{h_1}{2\mathrm{tg}\,\alpha}$; thus $\frac{x_0}{x_1} = \frac{h_0}{h_1}$.

$\lambda = \left(\frac{h_0}{h_1}\right)^2$ is called as the elongation coefficient of square bar drawing.

3.2 Shear power

Substituting $x = x_0$ into Eqn. (5), the velocity field at entry becomes

$$\left.\begin{aligned} v_x &= v_0 \\ v_y &= \frac{-Cy}{4x_0^3 \mathrm{tg}^2\alpha} \\ v_z &= \frac{-Cz}{4x_0^3 \mathrm{tg}^2\alpha} \end{aligned}\right\} \tag{c}$$

Since outside of entry is rigid region, the tangential velocity discontinuity along the section is

$$\begin{aligned} |\Delta v_t| &= \sqrt{\Delta v_y^2 + \Delta v_z^2} = \sqrt{v_y^2 + v_z^2} \\ &= \frac{C}{4x_0^3 \mathrm{tg}^2\alpha}\sqrt{y^2+z^2} \end{aligned} \tag{d}$$

The domain of integration is shown in Fig. 2, where the equation of straight line OB is $y = z$. As long as the shear power in triangle area is found out, the total power in whole square is obtained from following equation

$$\begin{aligned} \dot{W}_{s_0} &= 8\int_F k\,|\Delta v_t|\,\mathrm{d}F \\ &= 8k\int_F \frac{C}{4x_0^3\mathrm{tg}^2\alpha}\sqrt{y^2+z^2}\,\mathrm{d}y\,\mathrm{d}z \\ &= \frac{2kC}{x_0^3\mathrm{tg}^2\alpha}\int_0^{\frac{b_0}{2}}\mathrm{d}y\int_0^{z=y}\sqrt{y^2+z^2}\,\mathrm{d}z \end{aligned}$$

Considering Ref. [3] and noting $b_0 = h_0$, $h_0 = 2x_0\mathrm{tg}\alpha$, above integral becomes

ig. 2 Integral domain at entry

$$\begin{aligned} \dot{W}_{s_0} &= \frac{2kC}{x_0^3\mathrm{tg}^2\alpha}\int_0^{\frac{b_0}{2}}\left[\frac{z}{2}\sqrt{z^2+y^2} + \frac{y^2}{2}\ln\left(z+\sqrt{z^2+y^2}\right)\right]_0^{z=y}\mathrm{d}y \\ &= \frac{2kC}{x_0^3\mathrm{tg}^2\alpha}\int_0^{\frac{b_0}{2}}\left[\frac{\sqrt{2}}{2}y^2 + \frac{y^2}{2}\ln\left(y+\sqrt{2}y\right) - \frac{y^2}{2}\ln y\right]\mathrm{d}y \\ &= \frac{\sqrt{2}}{3}Ck\mathrm{tg}\alpha + \frac{2kC}{x_0^3\mathrm{tg}^2\alpha}\left\{\int_0^{\frac{b_0}{2}}\ln\left(1+\sqrt{2}\right)\frac{y^2}{2}\mathrm{d}y\right\} \\ &= \frac{Ck\mathrm{tg}\alpha}{3}\left[\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right] \end{aligned} \tag{e}$$

With the same procedure, substituting $x = x_1$ into Eqn. (5), the velocity field at exit is

$$v_x = v_1;\quad v_y = \frac{-Cy}{4x_1^3\mathrm{tg}^2\alpha};\quad v_z = \frac{-Cz}{4x_1^3\mathrm{tg}^2\alpha}$$

and the tangential velocity discontinuity is

$$\begin{aligned} |\Delta v_t| &= \sqrt{\Delta v_y^2 + \Delta v_z^2} = \sqrt{v_y^2 + v_z^2} \\ &= \frac{C}{4x_1^3\mathrm{tg}^2\alpha}\sqrt{y^2+z^2} \end{aligned} \tag{f}$$

The integral domain is just the exit section, as shown in Fig. 3. The equation of OA is $y = z$ and $b_1 = h_1 = 2x_1\mathrm{tg}\alpha$, thus the sum of the shear powers in whole domain is

$$\begin{aligned} \dot{W}_{s_1} &= 8k\int_F |\Delta v_t|\,\mathrm{d}F \\ &= \frac{8kC}{4x_1^3\mathrm{tg}^2\alpha}\int_F\sqrt{y^2+z^2}\,\mathrm{d}F \\ &= \frac{2kC}{x_1^3\mathrm{tg}^2\alpha}\int_0^{\frac{b_1}{2}}\mathrm{d}y\int_0^{z=y}\sqrt{y^2+z^2}\,\mathrm{d}z \\ &= \frac{Ck\mathrm{tg}\alpha}{3}\left[\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right] \end{aligned} \tag{g}$$

The results of Eqns. (g) and (e) show that the shear power consumed at exit is the same as that at entry. The sum of the shear powers is

$$\begin{aligned} \dot{W}_s &= \dot{W}_{s_0} + \dot{W}_{s_1} \\ &= \frac{2Ck\mathrm{tg}\alpha}{3}\left[\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right] \end{aligned} \tag{8}$$

3.3 Surface integral of friction power

Suppose the friction stress at the contact

Fig. 3 Integral domain at exit

surface is $\tau_f = mk$ and the tangential velocity vector of deformed metal along it is, $\boldsymbol{v}_t = v_x i + v_y j + v_z k$. Noting the die does not move, we multiply the friction power in $AABB$ by 4 and find the sum of them

$$\begin{aligned}\dot{W}_f &= 4\int_F \tau_f \mid \Delta \boldsymbol{v}_t \mid \mathrm{d}F \\ &= 4\int_F mk \mid 0 - \boldsymbol{v}_t \mid \mathrm{d}F \\ &= 4mk\int_F \mid \boldsymbol{v}_t \mid \mathrm{d}F \qquad \text{(h)}\end{aligned}$$

Substituting $Z = \frac{h(x)}{2}$ into Eqn. (5), and $v_z = \frac{-C}{4x^2 \mathrm{tg}\,\alpha}$ into modulus of the tangent velocity vector and arranging, we get

$$\begin{aligned}\mid \boldsymbol{v}_t \mid &= \sqrt{v_x^2 + v_y^2 + v_z^2} \\ &= \frac{C}{4x^2 \mathrm{tg}^2\alpha}\sqrt{1 + \mathrm{tg}^2\alpha + \left(\frac{y}{x}\right)^2}\end{aligned}$$

$$\mathrm{d}F = \sqrt{1 + \left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2}\,\mathrm{d}x\,\mathrm{d}y$$

Substituting above Equations into Eqn. (h) yields the double integral as follows

$$\dot{W}_f = 8mk\int_{x_1}^{x_0}\int_0^{\frac{h(x)}{2}} \frac{C}{4x^2\mathrm{tg}^2\alpha} \cdot \sqrt{1 + \mathrm{tg}^2\alpha + \left(\frac{y}{x}\right)^2} \cdot \sqrt{1 + \left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2}\,\mathrm{d}x\,\mathrm{d}y$$

Because $y = \frac{b(x)}{2} = x\,\mathrm{tg}\,\alpha$, $z = \frac{h(x)}{2} = x\,\mathrm{tg}\,\alpha$, and $\frac{\mathrm{d}z}{\mathrm{d}x} = \mathrm{tg}\,\alpha$, thus

$$\begin{aligned}\dot{W}_f &= 8mk\int_{x_1}^{x_0}\left[\int_0^{x\cdot\mathrm{tg}\,\alpha} \frac{C\sqrt{1+\mathrm{tg}^2\alpha}}{4x^2\mathrm{tg}^2\alpha} \cdot \sqrt{1 + \mathrm{tg}^2\alpha + \left(\frac{y}{x}\right)^2}\,\mathrm{d}y\right]\mathrm{d}x \\ &= 8mkC\int_{x_1}^{x_0} \frac{\sqrt{1+\mathrm{tg}^2\alpha}}{4x^2\mathrm{tg}^2\alpha} \cdot \left\{\frac{y}{2}\sqrt{1+\mathrm{tg}^2\alpha + \left(\frac{y}{x}\right)^2} + \frac{x(1+\mathrm{tg}^2\alpha)}{2} \cdot \ln\left[\frac{y}{x} + \sqrt{1+\mathrm{tg}^2\alpha+\left(\frac{y}{x}\right)^2}\right]\right\}_0^{x\cdot\mathrm{tg}\,\alpha}\mathrm{d}x \\ &= 8mkC\int_{x_1}^{x_0} \frac{\sqrt{1+\mathrm{tg}^2\alpha}}{4x^2\mathrm{tg}^2\alpha} \cdot \left\{\frac{x\,\mathrm{tg}\,\alpha}{2}\sqrt{1+2\mathrm{tg}^2\alpha} + \frac{x(1+\mathrm{tg}^2\alpha)}{2} \cdot \ln\frac{\mathrm{tg}\,\alpha + \sqrt{1+2\mathrm{tg}^2\alpha}}{\sqrt{1+\mathrm{tg}^2\alpha}}\right\}\mathrm{d}x \\ &= \left[\frac{\sqrt{1+\mathrm{tg}^2\alpha}\cdot\sqrt{1+2\mathrm{tg}^2\alpha}}{\mathrm{tg}\,\alpha} + \frac{(1+\mathrm{tg}^2\alpha)^{\frac{3}{2}}}{\mathrm{tg}^2\alpha}\ln\frac{\mathrm{tg}\,\alpha + \sqrt{1+2\mathrm{tg}^2\alpha}}{\sqrt{1+\mathrm{tg}^2\alpha}}\right] \cdot mkC\int_{x_1}^{x_0}\frac{\mathrm{d}x}{x}\end{aligned}$$

Owing to $\frac{x_0}{x_1} = \frac{h_0}{h_1}$ and from Ref. [5], the result is arranged into the following

$$\dot{W}_f = mkC\ln\frac{h_0}{h_1}\left\{\sqrt{\csc^2\alpha + 2\sec^2\alpha} + \frac{\ln(\sin\alpha + \sqrt{1+\sin^2\alpha})}{\cos\alpha\sin^2\alpha}\right\} \qquad (9)$$

Let the drawing power in exit be $\sigma_f h_1^2 v_1 = \sigma_f C = \dot{W}_i + \dot{W}_s + \dot{W}_f$, substitute Eqns. (7), (8), (9) into it, note $k = \frac{\sigma_s}{\sqrt{3}}$ and rearrange, we obtain

$$\frac{\sigma_f}{\sigma_s} = \ln\lambda + \frac{2\mathrm{tg}\,\alpha}{3\sqrt{3}}\left[\sqrt{2} + \ln(1+\sqrt{2})\right] + \frac{m}{\sqrt{3}}\ln\frac{h_0}{h_1}\left[\sqrt{\csc^2\alpha + 2\sec^2\alpha} + \frac{\ln(\sin\alpha + \sqrt{1+\sin^2\alpha})}{\cos\alpha\cdot\sin^2\alpha}\right] \qquad (10)$$

Eqn. (10) is the upper bound analytical solution of drawing stress for square bar, where λ is elogation coefficient, α is half-die angle, $h_0 = b_0$ and $h_1 = b_1$ are the sides of the square at entry and exit respectively.

The limit reduction for square bar drawing is

$$\ln\lambda + 0.88\mathrm{tg}\,\alpha + \frac{m}{\sqrt{3}}\ln\frac{h_0}{h_1}\cdot\left[\sqrt{\csc^2\alpha + 2\sec^2\alpha} + \frac{\ln(\sin\alpha + \sqrt{1+\sin^2\alpha})}{\cos\alpha\cdot\sin^2\alpha}\right] \leqslant 1 \quad (11)$$

Here, neglecting the effect of the die bearing and back tension, the equation of the drawing stress deduced by Avitzur is

$$\frac{\sigma_f}{\sigma_s} = 2f(\alpha)\ln\left(\frac{R_0}{R_1}\right) + \frac{2}{\sqrt{3}}\left[\frac{\alpha}{\sin^2\alpha} - \cot\alpha + m(\cot\alpha)\ln\left(\frac{R_0}{R_1}\right)\right] \quad (12)$$

where the functional expression $f(\alpha)$ is given in the Ref. [6].

4 CALCULATING RESULTS

An annealed aluminium square bar with section of 15 mm × 15 mm is drawn into the section of 13 mm × 13 mm through a conical square die. If $\alpha = 12°$ and $f \approx m = 0.1$, the relative drawing stress can be calculated.

Substituting $\lambda = 1.33$, $m = 0.1$, $h_0 = 15$, $h_1 = 13$, $\alpha = 12°$ into Eqn. (10), we obtain

$$\frac{\sigma_f}{\sigma_s} = 0.285 + 0.188 + 0.00826[5.022 + 4.88] = 0.55$$

Calculating approximately by Eqn. (12), changing the entry and exit sections into rounds with the same areas, we obtain $h_0^2 = 15^2 = \pi R_0^2$. Substituting $R_0 = 8.463$, $h_1^2 = 13^2 = \pi R_1^2$, $R_1 = 7.334$, $f(\alpha) = 1.00093$[6], $\alpha = 12° = 0.209$ rad, $m = 0.1$ into Eqn. (12), then

$$\frac{\sigma_f}{\sigma_s} = 0.53$$

The relative error of calculated results between the two formulas is

$$\Delta = \frac{0.55 - 0.53}{0.55} = 3.6\%$$

Above comparison shows that for the same friction, elongation and deforming conditions, the drawing stress of square bar is higher than that of a round bar. To the example in this paper the error between the both is only about 3.6%.

5 CONCLUSIONS

(1) The three dimensional kinematically admissible continuous velocity field of drawing square bar through conical die satisfies Eqn. (5). The divergence of the field is zero and the strain rate field satisfies Eqn. (6).

(2) Using upper-bound theorem, an analytical solution of drawing stress Eqn. (10) and corresponding maximum possible reduction Eqn. (11) are obtained by the curvilinear integral and the integral as a function of the upper limit. It shows that the drawing stress is the function of λ, m, α and σ_s.

(3) The same shear powers consumed cross entry and exit sections satisfy the Eqns. (e) and (g).

(4) To square and round bars, the drawing stress of square is higher than that of round. The error of example in this paper is 3.6%.

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