DETERMINING THICKENER UNDERFLOW CONCENTRATION AND UNIT AREA①

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ABSTRACT The relationship between the characteristics and the sediment curve in a batch sedimentation has been discussed, and the origination of characteristics has been analysed by two phases flow wave theory. According to the sedimentation theory and experimental data, two mathematical equations for calculating underflow concentrations and unit area of a continuous thickener were derived. And a new procedure to design a continuous thickener was developed. The procedure is reasonable and valid to calculate underflow concentration and unit area of a continuous thickener.

Key words batch sedimentation curve thickener unit area underflow concentration

1 INTRODUCTION

Thickeners are used industrially to reduce the amount of liquid in slurries containing such materials as concentrates, tailings and sewage. Traditional design procedures for sizing gravity thickeners have been based on works by Coe and Clevenger[1], Kynch[2], Talmage and Fitch[3]. The shortcomings of those methods are the particle settling is only involved and effects of the rising compression zone upon action of suspension settling are not considered. Recently many scholars have revised Kynch theory. Tiller[6] has developed an equation correlating the solid cocentration of the layers that reach the supernatant-suspension interface as a function of the variations of its height and the compression discontinuity height vs time. Font[7] developed and discussed a mathematical model that may be used to work out the relation between the settling rate and the solids concentration, but this procedure is complicated to calculate the underflow concentration and the unit area of a continuous thickener.

The aims of this work are as follows:

(1) To analyze origination of characteristics theoretically.

(2) To deduce mathematical models that may be used to work out underflow concentration and unit area of a continuous thickeners.

(3) To develop a new procedure to design continuous thickeners.

(4) To calculate underflow concentration and unit area of a continuous thickener by various methods and compare theirs results.

2 ORIGINATION OF CHARACTERISTICS

Characteristics are lines, whose points correspond to layers with the same solids concentration and the same settling rate. Kynch[2] assumed that all characteristics originate from origin of the coordinates(height vs time). Tiller[5] recognized that characteristics rise from a sediment surface curve in height vs time plot. Fitch[6] and Font[7] deduced that every characteristic rises from the sediment tangentially. Fitch does not explain the reasonable result. And Font only interprets origination of characteristics geometrically, not theoretically.

In uncompressible two phases flow, there are continuous waves and stimulated waves. The velocity of continuous waves and stimulated waves is respectively:

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$$V_w = G + \frac{\partial G}{\partial C} \quad (1)$$

$$V_s = G + \frac{G_2 - G_1}{C_2 - C_1} \quad (2)$$

and the total flux of a batch sedimentation is null, i. e.

$$G = 0 \quad (3)$$

therefore,

$$V_w = \frac{\partial G}{\partial C} \quad (4)$$

$$V_s = \frac{G_2 - G_1}{C_2 - C_1} \quad (5)$$

In Fig. 1, H (height of supernatant- suspension interface) and L (sediment height) are plotted against time. The coordinates($t + dt$, $H + dH$) and (t, H) of two adjacent points in the straight line portion of the curve of H *vs* t are related by the following equation:

$$C_1 = C_2 \quad (6)$$

i. e.

$$\frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial H} \mathrm{d}H = 0 \quad (7)$$

Consider two layers at $H + \mathrm{d}H$ and H . In time $\mathrm{d}t$, the accumulation of particles between the two layers is the difference between the flux of particles G_1 in through the upper layer and the flux G_2 out through the lower layer, per unit area.

$$\frac{\partial (C \mathrm{d}H)}{\partial t} \mathrm{d}t = (G_2 - G_1) \mathrm{d}t \quad (8)$$

divided by $\mathrm{d}H\,\mathrm{d}t$, the equation becomes,

Fig. 1 Concentration discontinuity height vs time plot

$$\frac{\partial C}{\partial t} = \frac{\partial G}{\partial H} \quad (9)$$

or,

$$\frac{\partial C}{\partial t} - \frac{\mathrm{d}G}{\mathrm{d}C} \frac{\partial C}{\partial H} = 0 \quad (10)$$

Combining eqns. (4) and (10), we derive the following equation:

$$\frac{\partial C}{\partial t} - V_w \frac{\partial C}{\partial H} = 0 \quad (11)$$

i. e.

$$\frac{\partial C}{\partial t} \mathrm{d}t - V_w \frac{\partial C}{\partial H} \mathrm{d}t = 0 \quad (12)$$

Comparing eqns. (7) and (12), we find,

$$V_w = - \frac{\mathrm{d}H}{\mathrm{d}t} \quad (13)$$

The above equation indicates that the slope of H vs t curve equals the velocity of continuous waves that are provided by solid particles passing down through liquid phase in batch settling experiments. That is to say, the curve of H vs t is the locus of the continuous waves. When a fast continuous wave catches up with a slow one, the two waves overlap and cause a stimulated wave. At the begining of particles settling, $t = 0$, suspension is uniformly successive(the gradient of the concentration is zero.). When $t > 0$, the solid particles, which exists near the bottom of the cylinder affected by gravity force, move down to the bottom and then stop there. Supposing the velocity of the first continuous wave near the cylinder bottom is V_{w1} , then

$$\lim_{t \to 0} V_{w1} = V_w \quad (14)$$

when $t > 0$, $V_{w1} = 0$.

In the front of the stimulated wave, the solid concentration is discontinuous, as shown in Fig. 2. When the velocity of the first stimulated wave reduces to zero, the following continuous waves overlap each other and become stimulated waves again and again, then move down to the bottom of the cylinder(actually the stimulated waves are caused successively). The concentration discontinuity of the front of stimulated waves propagates upwards in opposite direction of the velocity of the stimulated waves. Since the solid particles carried by the stimulated waves accumulate at the bottom, the liquid medium, which is compressed by the solid particles, moves upwards, and so the intensity of the stimulated

Fig. 2 **Sediment developing process**

waves at the upper interface of the concentration discontinuity, is weakened, and a series of continuous stimulated waves were formed. The position of the continuous stimulated waves corresponded in zone C , as shown in Figure 2. The upper interface of zone C is adjacent to the front of the continuous waves, and the lower interface is next to stimulated waves. When zone B disappears, the last continuous wave overlap with the weak stimulated wave in zone C and moves down until the sedimentation process is over.

According to the above analysis, the stimulated waves in a batch sedimentation cause the concentration discontinuity. The propagation of the concentration discontinuity results from accumulation of the solid particles carried by the stimulated waves. Eqn. (5) expresses the velocity of the stimulated waves moving down. Since the solid particles at the bottom of the cylinder is much dense and uncompressible, the velocity direction of the concentration discontinuity is opposite to that of the stimulated waves. Supposing u stands for the velocity of the concentration discontinuity, then

$$u = - V_s = - \frac{G_2 - G_1}{C_2 - C_1} \tag{15}$$

which is identical to that in Kynch theory,

$$u = \frac{G_2 - G_1}{C_1 - C_2} \tag{16}$$

Assuming the locus function of the stimulated waves is

$$L = f(t) \tag{17}$$

then

$$V_s = \frac{\mathrm{d}L}{\mathrm{d}t} = \frac{\mathrm{d}f(t)}{\mathrm{d}t} \tag{18}$$

Combining eqns. (15) and (18), we derive

$$u = - \frac{\mathrm{d}L}{\mathrm{d}t} = - \frac{\mathrm{d}f(t)}{\mathrm{d}t} \tag{19}$$

The general form of the characteristic equation[5] is

$$L = ut + L_0 \tag{20}$$

so

$$L = - \frac{\mathrm{d}f(t)}{\mathrm{d}t} t + L_0 \tag{21}$$

specially when $L_0 = 0$,

$$L = - \frac{\mathrm{d}f(t)}{\mathrm{d}t} \tag{22}$$

Above all, the characteristics rise from the sediment tangentially. Font's[7] view is correct.

3 DERIVATION OF EQUATIONS

Fig. 3 shows the suspension sedimentation curve of a batch experiment. We may choose such a point at the first falling rate period[5] as $B(t_2, H_2)$ and draw a line from this point tangential to the sediment curve. The tangetial point is $A(t_1, L_1)$, and the intercept on the axis of ordinate is H_j , then the line H_jB is a characteristic one. Supposing this characteristic

Fig. 3 **Interface and characteristics in batch sedimentation**

concentration is C_2, Font[7] derives

$$C_2 = \frac{C_0H_0}{H_{12} - L_1}\exp\left[-\int_{t_1}^{0}\frac{dt}{t_2 - t_1}\right] \quad (23)$$

In Fig. 3, the curve is drawn according to the experimental data, therefore, if points A and B have been chosen, values of t_1 and t_2 are identified, and Eqn. (23) may becomes

$$C_2 = \frac{C_0H_0}{H_{12} - L_1}\exp\left[-\frac{t_1}{t_2 - t_1}\right] \quad (24)$$

Fig. 3 shows

$$\triangle H_iBH_j \backsim \triangle CBA \quad (25)$$

then

$$\frac{1}{H_{12} - L_1} = \frac{t_2}{t_2 - t_1}\frac{1}{H_i - H_j} \quad (26)$$

Substituting Eqn. (26) into Eqn. (24), we obtain

$$C_2 = \frac{C_0H_0}{H_i - H_j}\frac{t_2}{t_2 - t_1}\exp\left[-\frac{t_2}{t_2 - t_1}\right] \quad (27)$$

According to Yoshioka construction[4], the relation of G_m, C_u and V_u is

$$G_m = V_uC_u \quad (28)$$

Fitch[6] found that when the initial conditions are identical, the downward velocity V_u imparted to the system by underflow withdrawal of a continuous thickener in a steady state must be negatively equal to the upward propagation velocity u of the compression discontinuity with respect to suspension, and consequently

$$V_u = -u \quad (29)$$

According to Kynch theory, we obtain

$$u = \frac{\triangle G}{\triangle C} \quad (30)$$

therefore

$$V_u = -u = -\frac{V_2C_2 - V_1C_1}{C_2 - C_1} \quad (31)$$

When $C_1 = C_u$, the sediment has been dense consequently, $V_1 = 0$, and Eqn. (35) becomes

$$V_u = -u = -\frac{V_2C_2}{C_2 - C_u} \quad (32)$$

i. e.

$$C_u = C_2\left(1 + \frac{V_2}{V_u}\right) = C_2\left(1 - \frac{V_2}{u}\right) \quad (33)$$

In Fig. 3,

$$V_2 = \frac{H_i - H_2}{t_2} \quad (34)$$

$$u = \frac{H_2 - H_j}{t_2} \quad (35)$$

Substituting Eqns. (34) and (35) into Eqn. (33), we derive

$$C_u = C_2\frac{H_i - H_j}{H_2 - H_j} \quad (36)$$

Combining Eqns. (28) and (36), we obtain

$$C_u = \frac{C_0H_0}{H_2 - H_j}\frac{t_2}{t_2 - t_1}\exp\left[-\frac{t_1}{t_2 - t_1}\right] \quad (37)$$

Combining Eqns. (28), (29), (35) and (37), we deduce

$$G_m = \frac{C_0H_0}{t_2 - t_1}\exp\left[\frac{t_1}{t_2 - t_1}\right] \quad (38)$$

and the unit area of a continuous thickener can be expressed as

$$q = \frac{1}{G_m} \quad (39)$$

Substituting Eqn. (38) into Eqn. (39), we acquire

$$q = \frac{t_2 - t_1}{C_0H_0}\exp\left[\frac{t_2 - t_1}{t_1}\right] \quad (40)$$

From the above derivation, we obtain two equations to calculate the underflow concentration and the unit area of a continuous thickener according to a batch sedimentation curve.

4 CALCULATION PROCEDURE

The new procedure is as follows:

(1) Suspension should be mixed, whose initial properties are identical with those fed to a continuos thickener being designed. After a batch sedimentation experiment is over, the sedimentation curve can be drawn out(H and L vs t), shown in Fig. 4.

(2) Any point such as $B(t_2, H_2)$ at the first falling rate period of the settling curve is chosen. A line tangential to the sediment curve is drawn from point B, the tangential point is $A(t_1, L_1)$, and the intercept on the axis of ordinates is H_j.

(3) Substituting values of t_1, t_2, L_1, H_2, C_0 and H_0 into Eqns. (37) and (40), values of C_u and q can be worked out.

(4) Repeating processes (2) and (3), choosing several other points and calculating val-

Fig. 4 Discontinuity height vs time

ues of C_u and q at every point.

(5) A plot of C_u vs q can be drawn out according to values of (C_u, q), and the reasonable values of q is obtained from relative values of C_u designed in a new continuous thickener.

(6) The total areas can be worked out when the thickener's capability is identified.

5 BATCH EXPERIMENTAL RESULTS

In order to verify the new model , magnetite tailings suspension was chosen. Experiments were separated into two series: flocculated and unflocculated suspensions. Every series included six various initial volume concentrations: 1.95% 、4.08% 、6.29% 、8.70% 、11.2% and 14.1%.

The − 200 mesh weight percent of the material is 80.0%. The solid density was 2610 kg/m^3, measured with distilled water using a pycnometer.

The tests were carried out at 25 ℃ or so in a graduated plexiglass cylinder, whose valid height is 600mm and the diameter of cross section is 55mm.

The curve of discontinuity heights vs time at different initial concentrations are plotted in Fig. 5.

6 COMPARISON OF PROCEDURES TO CALCULATE C_u AND q

In order to verify the new procedure validity, values of C_u and q for a continuous thickener calculated by this method were compared with those by Talmage-Fitch[3] and Oltmann method which are usually used in designing a continuous thickener. However, much practical experience

Fig. 5 Discontinuity heights vs time

1− C_0= 1.95% ; 2− C_0= 4.08%; 3− C_0= 6.29%;

4− C_0= 8.70%; 5− C_0= 11.2%; 6− C_0= 14.1%

(a) − unflocculated suspension; (b) − flocculated suspension shows that values of q worked out by Talmage-Fitch method are greater than the real results, and smaller by Oltmann method.

The procedures of Talmage- Fitch and Oltmann methods are shown in Fig. 6. The unit area of a continuous thickener is worked out by each of the two methods based on Eqns. (41) and (42):

$$C_0 H_0 = C_u H_u \tag{41}$$

$$q = \frac{t_u}{C_0 H_0} \tag{42}$$

Fig. 6 Construction of Talmage- Fitch and Oltmann methods

The underflow concentration of a continuous thickener was worked out by the new procedure based on Eqn. (37) according to experimental data of the various initial concentrations: 1.95%, 6.29% and 14.1%, respectively. Unit area of a continuous thickener is calculated as follows: (1) by the new procedure based on Eqn. (40), (2) by Talmage- Fitch method and Oltmann method, respectively, based on Eqns. (41) and (42). Comparison of the results by three methods is shown in Fig. 7.

In view of Fig. 7, we may observe that values of unit area of a continuous thickener obtained by the new method lie between those worked out by Talmage- Fitch method and Oltmann method. These phenomena prove that the new procedure is more valid in designing a continuous thickener than Talmage- Fitch and Oltmann methods. We find the higher underflow concentration is, the smaller unit areas is. When the initial concentration is identified, there is a maximum value for the underflow

Fig. 7 Unit areas vs underflow concentration for a continuous thickener (C_0 = 6.29%)

1 —Talmage- Fitch method; 2 —the new method; 3 —Oltmann method.
(a) - unflocculated suspension;
(b) - flocculated suspension

concentration of a thickener. When initial and underflow concentrations are identified, the unit area of a continuous thickener processing the flocculated suspension is much smaller than that doing unflocculated suspension. For example, when C_0 = 6.29%, C_u = 53.6%, the unit area of the former is $2.40 \times 10^4 m^2/(m^3 \cdot s)$ and the latter is $2.24 \times 10^5 m^2/(m^3 \cdot s)$. In other words, the ability of a continuous thickener for handling flocculated suspension is 9.35 times greater than that for doing the unflocculated suspension.

By this new procedure, we can work out the underflow concentration of a continuous thickener, and identify the optimun values of C_u and q for a continuous thickener being designed with computer.

7 NOTATION

V_w —continuous wave velocity, m/s;

V_s —stimulated wave velocity, m/s;

G —volumetric settling flux of solids, $m^3/(m^2 \cdot s)$;

G_1 —lower interface flux of the concentration discontinuity, $m^3/(m^2 \cdot s)$;

G_2 —upper interface flux of the concentration discontinuity, $m^3/(m^2 \cdot s)$;

H —height of a descending interface, mm;

C_1, C_2 —volume percent of solids for sedimentation curve at points 1 and 2, %;

u —upward propagation rate of the compression discontinuity, m/s;

H_{12} —the ordinate of point C in Fig. 3, mm;

L_1 —sediment height at t_1, m;

H_2 —value of H at t_2, m;

t_1, t_2 —time, s;

H_i —intercept height of a tangent to the curve $H_2 = g(t_2)$ on H axis, mm;

H_j —intercept height of a tangent to the curve $L_1 = f(t_1)$ on L axis, mm;

H_0 —initial value of H at $t = 0$, mm;

G_m —the maximum flux of solids, $m^3/(m^2 \cdot s)$;

V_u —downward velocity of pulp in a continuous thickener resulting from underflow withdrawal, m/s;

C_0 —initial feed concentration, %;

C_u —underflow volume fraction of solids, %;

q —unit area of a continuous thickener, $m^2/(m^3 \cdot s)$;

t_u —value of t by Talmage-Fitch method and Oltmann method.

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