CORRECTION OF DISCONTINUITY SPACING BIAS CAUSED BY FINITE LENGTH SCANLINE SURVEY①

Chen Feng
Testing Center, Central South University of Technology, Changsha 410083

ABSTRACT For discontinuity spacing based on negative exponential probability density, a weight function method used for correction of sampling bias caused by finite scanline was proposed. The result showed that relative error improves from 0.5% to 40%. This makes it possible to estimate discontinuity spacing of rock-mass using short scanline.

Key words discontinuity spacing negative exponential probability density scanline survey

1 INTRODUCTION

The discontinuity parameters, such as discontinuity spacing, frequency and intensity, play an important role in analyzing excavation dimention[1], rock quality designation(RQD)[2]. Because of its importance, the discontinuity spacing was widely investigated by many authors, a variety of models and procedures were adopted in analyzing these discontinuity parameters. Priest and Hudson[3] pointed out through their field surveys that any combination of evenly spaced, clustered and randomly positioned discontinuities leads to a negative exponential form of frequency *vs* joint spacing value curve. Their argument was based on infinite length scanline. In engineering practice, however, one is always confronted with finite length of discontinuity at outcropes, excavations or in short boreholes, therefore the scanline readings reflect only the finite length properties, which are biased and can not be representative for large rock-mass. The biases caused by finite length of scanline must be corrected before determining any engineering design variable. For this purpose, Sen Z and Kazi A[4] modeled discontinuity spacing with short scanline measurements, which can be used to infer rock-mass characteristics from finite scanline measurements.

The precision of their procedure, however, is still a problem, especially with short scanline. The purpose of this paper is to develop an analytical procedure to cope with discontinuity spacings of rock-mass by using finite scanline length. For discontinuity spacing following the negative exponential distribution, a weight function is suggested to correct the sampling bias caused by finite scanline length. This method permits of evaluating the discontinuity spacing of rock-mass by finite scanline without losing much precision. The comparison between suggested method and that given by Sen and Kazi is also presented.

2 THEORETICAL BASIS OF WEIGHT FUNCTION CORRECTION FOR FINITE SCANLINE BIAS

Scanline survey is one of the common methods used for measuring the discontinuity spacings. Intersections of discontinuities with scanline in a given direction form a set of randomly located points along this line. The distance between an adjacent pair of points corresponds to discontinuity spacing. Apparently, discontinuity spacing is greatly controlled by the scanline direction, the length of scanline adopted, size of individual blocks of intact rock, etc. Priest and Hudson[3] pointed out through their study that unless there is a large predominance of evenly

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spaced, clustered and randomly positioned discontinuities, it will lead to a negative exponential form of frequency *vs* spacing value curve. For engineering design purpose, the knowledge of rock-mass discontinuity spacing of this kind of distribution is most important, however, one always be confronted by finite length of discontinuity on outcrops and in bore hole etc, it means that one can not use infinite scanline length to investigate discontinuity spacing, sampling bias will be introduced then. This bias should be corrected before inferring discontinuity spacing of rock-mass. For this purpose, the well known negative exponential distribution of discontinuity spacing will be considered:

$$f(x) = \lambda e^{-\lambda x} \quad (0 \leqslant x \leqslant \infty) \tag{1}$$

where $f(x)$ is the frequency of a given discontinuity spacing value x, and λ is the average number of discontinuities per meter, i. e. the frequency of discontinuities in rock-mass. For this probability density distribution function expressed in equation (1), the probability of discontinuity spacing x of rock-mass can then be expressed as:

$$\int_0^{\infty} f(x)\mathrm{d}x = 1 \quad (0 \leqslant x \leqslant \infty) \tag{2}$$

If a finite scanline length, L, is adopted in the scanline survey, the discontinuity spacings longer than L can not be observed, this will result in sampling bias. Sen and Kazi[4] suggested the following censored distribution function $f'(x)$, which can be expressed in terms of parent distribution:

$$f'(x) = \frac{f(x)}{\int_0^{\infty} f(x)\mathrm{d}x} \quad (0 \leqslant x \leqslant L) \tag{3}$$

Theoretically, eqns. (2) and (3) can be applied to any given density distribution function of discontinuity spacing x. For negative exponentially distributed intact length x, the censored discontinuity spacing frequency distribution can be obtained by substituting eqn. (1) into (3):

$$f'(x) = \frac{\lambda}{1 - e^{-\lambda L}} e^{-\lambda x} \quad (0 \leqslant x \leqslant L) \tag{4}$$

The mean discontinuity spacing $E(x)$ is derived from expected value of censored frequency distribution function $f'(x)$:

$$E(x) = \frac{1}{\lambda(1 - e^{-\lambda x})}[1 - (1 + \lambda L)e^{-\lambda L}] \tag{5}$$

The graphical expression of eqn. (5) is given in Fig. 1, which shows the effect of scanline length on the mean discontinuity spacing. It is clear from Fig. 1 that the mean discontinuity spacing estimated from finite scanline length is less than rock-mass values, i. e. finite length scanline measurements yield under-estimated mean discontinuity spacings. Generally, the shorter the scanline length we adopted, the greater the sampling bias will be for a given mean discontinuity spacing of rock-mass. Theoretically, the mean discontinuity spacing of sample would be equal to the counterpart of rock-mass if unlimited scanline length could be adopted. In fact, Sen and Kazi[4] pointed out that for scanlines longer than approximately $20/\lambda$, the rock-mass value and the scanline estimated value become almost equal . For clearness, assuming that the mean discontinuity spacing of rock-mass is 2 m, the influence of finite scanline length L on mean discontinuity spacing is given in Table 1.

It can be seen from Table 1 that discontinuity spacing is greatly affected by the adopted scanline length. Short scanline survey may cause a significant sampling bias. A close-formed weight function is therefore suggested, which

Fig. 1 Mean discontinuity spacing (E) of rock-mass

Table 1 The influence of scanline length on the mean discontinuity spacing

Adopted scanline length/ m	Rock-mass mean discontinuity spacing/ m	Sample discontinuity spacing obtained with different scanline/ m	Relative error percentage / %
1	2	0.46	77.0
2	2	0.83	59.0
3	2	1.12	44.0
5	2	1.56	22.0
10	2	1.96	2.0

can be used to correct sampling bias due to finite scanline length. The weight function with two parameters should satisfy the following two conditions: (1) It should be a decreasing function with increasing scanline length L at a given mean discontinuity spacing $1/\lambda$; (2) For a given scanline length L, the value of the weight function increases with increasing of mean discontinuity spacing $1/\lambda$.

The proposed weight function is as follows:

$$W(\lambda, L) = (1 - e^{-\lambda L})^{-1} \tag{6}$$

where L is scanline length, and λ is discontinuity frequency. The graphical expression of eqn. (6) is given in Fig. 2. It is clear from Fig. 2 that weight function (6) is satisfied by above mentioned two conditions. The mean discontinuity spacing of sample estimated using finite length scanline can then be corrected by this function. From eqn. (5), we obtain:

$$\begin{aligned} E'(x) &= E(x) \cdot W(\lambda, L) \\ &= [\lambda(1 - e^{-\lambda L})^2]^{-1} \cdot \\ &\quad [1 - (1 + \lambda L)e^{-\lambda L}] \end{aligned} \tag{7}$$

The graphical expression of eqn. (7) is given in Fig. (3), which shows the relation between mean discontinuity spacings of sample corrected by weight function and that of the rock-mass. It can be seen from Fig. (3) that the mean discontinuity spacing of sample corrected by weight function (6) is greatly increased, therefore, the relative error percentage is significantly decreased, especially for shorter scanlines. Taking the same rock-mass mean discontinuity spacing value (2 m) as an example, the corrected and uncorrected values of sample mean discontinuity spacing are tabulated in Table 2.

In addition, the unbiased estimation region for each scanline is also improved. The upper limit of unbiased mean discontinuity spacings is higher than that obtained by Sen and Kazi. For example, for 1 m length of scanline, the upper limit of unbiased mean discontinuity spacing is 0.2 m following Sen and Kazi, however, it reaches 0.3 m for the same length of scanline using present method. The relative error percentage in the mean discontinuity spacing estimation is given in the following equation[4].

$$\alpha = 100\left[\frac{1/\lambda - E(x)}{1/\lambda}\right] \tag{8}$$

Substituting eqns. (5) and (7) into eqn.

Fig. 2 Relationship between weight function(W) and mean discontinuity spacing (λ^{-1})

Table 2 Comparison of sample mean discontinuity spacings between corrected and uncorrected values

Adopted scanline length / m	Rock-mass mean discontinuity spacing / m	Uncorrected values / m	Corrected values / m	Decreased relative error percentage / %
1	2	0.46	1.26	40.0
2	2	0.83	1.43	30.0
3	2	1.12	1.67	27.0
5	2	1.56	1.88	16.0
10	2	1.96	1.97	0.50

(8) yields

$$\alpha = 100\left\{1 - \frac{1}{1 - e^{-\lambda L}} \cdot \left[1 - (1 + \lambda L)e^{-\lambda L}\right]\right\} \quad (9)$$

and

$$\alpha' = 100\left\{1 - \frac{1}{(1 - e^{-\lambda L})^2} \cdot \left[1 - (1 + \lambda L)e^{-\lambda L}\right]\right\} \quad (10)$$

The graphical representation of eqn. (10) is given in Fig. 4. It is clear from Fig. 4 that the relative error percentage is greatly decreased by using weight function correction, especially for short scanlines.

Fig. 4 Relative error percentage of λ^{-1} corrected by weight function (α') *vs* scanline length(L)

For the decision of scanline length at a specified error percentage level , say, 5%, a shorter scanline can be selected for the same mean discontinuity spacings in Fig. 4 than that used by Sen and Kazi. Since the engineering situations are usually complicated and the actual sites are not always long enough to permit using longer scanline survey, besides, the cost is another reason that should be considered, therefore, these characteristics of weight function is of practical importance. We can even use short scanline to evaluate rock-mass values of mean discontinuity spacing without losing much accuracy.

3 SUMMARY AND CONCLUSION

Based on negative exponential distribution of mean discontinuity spacing, a close-formed weight function method for correction of sampling bias caused by finite scanline length is proposed. The mean discontinuity spacing of sample will increase through this method, especially for shorter scanline length. The relative error percentage decreases, thus at a given relative error percentage one can use shorter scanline for the same mean discontinuity spacing. Another advantage of this method is that the upper limit of unbiased mean discontinuity increaes too.

Theoretically, this analytical procedure can also be applied in other discontinuity spacing survey provided that proper weight function can be constructed.

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