AN APPROACH FOR CONTROLLING OSCILLATION IN DYNAMIC STRESS-STRAIN MEASUREMENT①

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ABSTRACT The reasons why oscillation exists in the measurement of dynamic stress-strain curves of the SHPB subjected to a rectangular loading wave have been analyzed and a conclusion of that the oscillation can be controlled by the loading stress waveforms was made. Then, the Impact Discrete Inversion Technique proposed by the authors was introduced to design the rams based on expected stress waveforms. The confirmative experimentation results obtained using the designed ram which can produce an approaching sine wave showed that the oscillations were effectively controlled.

Key words dynamic experiment technique impact stress waveform

1 INTRODUCTION

The Split Hopkinson Pressure Bar (SHPB) has been widely used to measure the dynamic properties of solid materials, such as metals and rocks. The loading stress wave in the SHPB is initiated by impact between a bar and a cylindrical ram of the same diameter as the bar, which produces a rectangular loading stress wave. Numerous investigators have found that oscillations exist in the incident waves, reflection waves, strain rate curves and stress-strain curves obtained by the SHPB subjected to a rectangular loading wave. Owing to the existence of oscillations it becomes very difficult to obtain the exact dynamic parameters of the materials. As Pochhammer and Chree first analyzed the above phenomenon, this oscillation has been named as Pochhammer-Chree oscillation (P-C oscillation). The key reasons of such phenomenon may be the specimen is in a state of one dimensional stress and couldn't be mantained well. Kolsky, Davies and Hunters studied this phenomenon and gained the approximate corrections for radial inertia and the criteria for the SHPB specimen design[1][2]. Bertholf and Karnes examined the previous study using a comprehensive two-dimensional numerical study of the SHPB including the effects of the inertia and friction[3]. Li and Gu discussed it based on experimental results[4]. As the moduli of the elasticity of rock are smaller than that of the metals, more serious oscillations exist in the measurement of the dynamic parameters of rocks by the SHPB subjected to a rectangular loading stress wave, see Fig. 1. Thus, it is more important to control the P-C oscillations in the measur-

Fig. 1 The dynamic stress-strain curves of granite specimens subjected to rectangular loading wave
RG1, RG2, RG3 — Number of specimens

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rement of the dynamic properties of rocks. It is the objective of this paper to present a reasonable loading waveform and its corresponding loading system that can restraint the P-C oscillations.

2 THEORETICAL ANALYSIS

In order to present the method of controlling P-C oscillations, it is necessary to make a brief explanation on the effects of inertia in the SHPB based on energy principles.

Denoting the specimen kinetic energy by E_k, the deformation energy by E_d, we have

$$\left.\begin{aligned} \sigma\frac{d\varepsilon}{dt} &= \frac{\partial E_d}{\partial t} + \frac{\partial E_k}{\partial t} \\ E_d &= \frac{1}{2}E\varepsilon^2 \\ E_k &= \frac{1}{8}\rho\nu^2 d^2\left(\frac{\partial\varepsilon}{\partial t}\right)^2 \end{aligned}\right\} \tag{1}$$

where σ and ε are the stress and strain in the specimen; E, ρ, ν and d are Young modulus, density, Poissons ratio and diameter of the specimen, respectively.

Therefore

$$\sigma = E\varepsilon + \frac{1}{4}\rho\nu^2 d^2\frac{\partial\varepsilon^2}{\partial t^2} \tag{2}$$

Substituting eqn(2) into motion equation

$$\rho\frac{\partial v}{\partial t} = \frac{\partial\sigma}{\partial t}$$

yields

$$\frac{\partial^2 u}{\partial t^2} - \frac{1}{4}\nu^2 d^2\frac{\partial^4 u}{\partial t^2\partial x^2} = C_0^2\frac{\partial^2 u}{\partial x^2} \tag{3}$$

where $C_0 = \sqrt{E/\rho}$

The above equation is the wave equation including the effects of radial inertia. Assuming a simple loading stress of sine wave with circular frequence ω, its phase velocity c can be expressed as

$$C = C_0 - \frac{1}{4}\nu^2 d^2\omega^2 \tag{4}$$

The above equation shows that the stress waves with different frequences transmit with different phase velocities. Because every loading stress wave consists of some componental sine waves with various frequences, the dispersion phenomenon occurs, which results in P-C oscillations. It is apparent that the wider the frequence range of principal-componental sine waves, the more serious the dispersion and oscillations. Therefore, the control of the oscillations could be realized by the control of the loading stress waveforms. More specifically, (half) a single sine wave does not result in the dispersion and oscillations, i. e., the reasonable loading waveform should be a single sine wave.

3 IMPACT DISCRETE INVERSION TECHNIQUE

The loading stress wave is generated by impact with the ram, Datta studied the determination of stress waveforms produced by the ram of various geometrical designs[5]; Gupta studied the impact between a finite conical ram and a long cylindrical bar, using three-dimensional finite element model and experiments[6]. Their studies showed that the variation of the ram impact velocity produces a direct variation in amplitude of stress wave, while the ram geometry determined the profile of the stress wave. Numerous investigators have attempted to analyze the relationships between the stress waveform and the geometry of the ram. To the writers' knowledge, all published analyses have been only solved the problem that the stress waveform is determined based on the given geometry of the ram; while its inverse problem, in which the geometry of the ram is designed based on the expected stress wavefom, have been left to be solved.

In order to design a ram that can produce a loading stress of single sine wave. The Impact Discrete Inversion Technique proposed by the authors is introduced here (for detailed discussions of the technique see Ref. [7]).

Assuming that the profile function of the ram is $f(x)$; the waveform function is $\varphi(t)$, i. e.

$$R = f(x) \qquad 0 \leqslant x \leqslant L \tag{5}$$

$$P = \varphi(t) \qquad 0 \leqslant t \leqslant 2L/a \tag{6}$$

Throughout the discretization the ram profile and waveform are divided into n segments.

$$R_i = f(i\Delta x), \quad i = 1, \cdots n;$$

$$P_j = \varphi(j\Delta t), \quad j = 1, \cdots 2n-1$$

where $\Delta x = L/n;$

$\Delta t = \Delta x/a$

Based on the wave equation and characteristics method, the following equations are obtained, which can be used to calculate the radius of the ram based on the stress waveform.

$$m_i = F_{i\,n+1-i}/(v_{i\,n+1-i} - v_0) \tag{7}$$

$$R_i = \sqrt{\frac{m_i}{\pi\rho a}} \tag{8}$$

$$F_{i-1\,2k-1} = [F_{i\,2k-2} + F_{i\,2k} + m_i(v_{i\,2k-2} - v_{i\,2k})]/2 \tag{9}$$

$$v_{i-1\,2k-1} = [v_{i\,2k-2} + v_{i\,2k} + (F_{i\,2k-2} - F_{i\,2k})/m_i]/2 \tag{10}$$

$$(i = n, \cdots 1;\ k = 1, \cdots n)$$

where the initial conditions and the boundary conditions are now changed as follows.

$F_{i0} = 0 \quad (i = 0 \cdots n+1)$

$v_{i0} = v_0 \quad (i = 0 \cdots n)$,

$v_{n+i0} = 0$

$F_{nj} = P_j$

$v_{nj} = P_j/m_0 (j = 1 \cdots 2n-1)$

Here P is the force applied on elastic bar (N), R the radius of the ram (m), m_0 the characteristic impedance of the bar (M·s/m), v_0 the impact velocity (m/s), $L \cdot \rho$ and a are the length, ρ and a the density and longitudinal wave velocity of the ram, F_{ij} and v_{ij} the force and velocity of the ith segment of the ram during the jth period, m_i the characteristic impedance of the ith segment of the ram, n is the number of the segments.

Assuming that the force waveform function $\varphi(t)$ is given, the profile function of the ram $f(x)$ can be calculated by the above equations.

Using a computer program, the profile function of the ram that can produce a single semi-sine wave is calculated. It must be mentioned that only half of the wave function ($0 \leqslant t \leqslant 2L/a$) is the exact sine function and the other of waveform is approching sine function.

4 EXPERIMENTAL RESULTS

Using the Impact Discrete Inversion Technique, a new ram, which produces an approching semi-sine stress wave, is designed and made. The shape of the new ram and the corresponding stress waveform are shown in Figs. 2~3. The practical wave is in good agreement with the desired wave.

A series of confirmative experiments were performed to measure the dynamic stress-strain curves of many kinds of rocks. Several experiment results for granite specimens are illustrated in Fig. 4. Comparing with the experimental results obtained from the experiments using rectangular loading wave shown in Figure 1, it is

Fig. 2 The outline drawing of the designed ram

Fig. 3 The stress waveform generated by the designed ram

Fig. 4 The dynamic stress-strain curves of granite specimens subjected to approching sine loading wave
SG1, SG2, SG3 —number of specimens

seen that the P-C oscillations have been effectively controlled.

5 CONCLUSIONS

From the above analyses and experimental results, it can be concluded:

(1) In the measurement of rock behaviours by the SHPB subjected to rectangular loading wave, serious P-C oscillations exist in the dynamic stress strain curves.

(2) A reasonable loading stress wave should be a single sine wave which can eliminate P-C oscillation.

(3) The designed ram based on the Impact Discrete Inversion Technique can produce an aproching semi-sine loading wave and can control the P-C oscillations effectively.

It is suggested that the loading system of the SHPB should be improved by using the new ram and the Impact Discrete Inversion Technique should also be used to examine the behaviors of matrials subjected to different kinds of stress waveforms by the SHPB.

REFERENCES

1 Kolsky H. Stress Waves in Solids, Oxford: Clarendon Press, 1953.

2 Davies C D, Hunter S C. J Mech Phys Solids, 1963, 11: 155- 179.

3 Bertholf L D, Karnes C H. J Mech Phys Solids, 1975, 23: 1- 19.

4 Li Xibing, Gu Desheng. Rock Tmpact Dynamics, (in Chinese). Changsha: Central South University of Technology press, 1993.

5 Dutta P K. J Rock Mech Min Sci, 1968, 5: 501- 518.

6 Gupta R B, Nilsson L. J of Sound and Vibration, 1978, 60(4): 553- 563.

7 Liu Deshun, Yang Xiangbi. The Chinese Journal of Nonferrous Metals, (in Chinese). 1995, 5(1): 14- 17.

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