Ti(C, N) BASED CERAMETS AND ITS FRACTAL ANALYSIS OF FRACTURE SURFACES①

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ABSTRACT The fractal Brownian model of the secondary electron image of fracture surfaces has been founded and the quantitative analysis method on the three dimensional morphology of surfaces by using digital image processing (DIP) system as a means of information processing proposed. It has been proven theoretically and experimentally that the new quantitative treatment can measure the three dimensional morphology of fracture surfaces exactly and is independent of the linear transformation of the intensity of secondary electron image. The relationships among the fractal dimension of fracture surfaces, transverse rupture strength and microstructure of Ti(C, N) based ceramets have been studied. The microfracture fractal model of tested materials has been founded and the estimated fractal dimension formula of fracture surfaces proposed. The experimental results have shown that the common logarithm values of transverse rupture strength, $\lg \sigma_{bb}$, increase linearly with the increase of fractal dimension, D_F. The measured fractal dimension depends on the choice of measuring unit. There is an upper critical measuring unit and its value is 0.07 μm.

Key words Ti(C, N) based ceramets fracture surface fractal analysis

1 THE FRACTAL MODEL OF SECONDARY ELECTRON IMAGES OF FRACTURE SURFACES

Based on the typical Brownian motion, Mandelbrot[1] defined the fractal Brownian motion ——FBM. To use it as a model in researching many natural scenes, a good effect is obtained. The definition of FBM is as follows: Let H be in the range of $0 < H < 1$, and let b_0 be an arbitrary real number. $B_H(t, w)$ is defined by

$$B_H(0, w) = b_0,$$

$$B_H(t, w) - B_H(0, w) = \frac{1}{\Gamma(H + 1/2)} \times \{\int_{-\infty}^{0} [(t - s)^{H-\frac{1}{2}} - (-S)^{H-\frac{1}{2}}] dB(s, w) + \int_{0}^{t} (t - s)^{H-\frac{1}{2}} dB(s, w)\} \quad (1)$$

We call the function $B_H(t, w)$ fractal Brownian motion. Pentland[2] provided extension to two or more topological dimensions and got fractal Brownian random field ——FBR field.

Let a random function $I(x)$ be a fractal Brownian function for all x and Δx in R^n,

$$Pr\left\{\frac{I(x + \Delta x) - I(x)}{\| \Delta x \|^H} < y\right\} = F(y) \quad (2)$$

We define function $I(x)$ as fractal Brownian random function. Where $F(y)$ is a cumulative distribution function for which the average vector is zero. H is estimated fractal dimension. To the discrete function $I(x)$, we call it as discrete fractal Brownian random field ——DFBR field. DFBR field has an important characteristic, that is, its $E[| I(x + \Delta x) - I(x) |^2]$ and $| \Delta x |$ satisfy Eq. (3), i.e.

$$E[| I(x + \Delta x) - I(x) |^2] = C | \Delta x |^{2H} \quad (3)$$

where C is a constant. Using Eq. (3), We obtain

$$\lg E[| I(x + \Delta x) - I(x) |^2] = 2H \lg | \Delta x | + \lg C \quad (4)$$

Through the calculation of $E[| I(x + \Delta x)$

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$-I(x)|^2]$ under different increments of $|\Delta x_i|$, using Eq. (4) we can obtain the estimated values of H based on the least squares technique.

2 THE MICROFRACTURED FRACTAL MODEL OF Ti(C, N) BASED CERAMETS

2.1 Ti(C, N)/Ti(C, N) *Boundaries Intergranular Brittle Fracture*

Lung[3] proposed two kinds of fractal models of intergranular fracture as shown in Fig. 1.

Fig. 1 Fractal models of intergranular brittle fracture

(a) $-N = 2$, $\gamma = \frac{1}{1.732}$, $d_f = \frac{\lg 2}{\lg 1.732} = 1.262$

(b) $-N = 4$, $\gamma = \frac{1}{3}$, $d_f = \frac{\lg 4}{\lg 3} = 1.262$

we can consider the Ti(C, N) particles as the grains, hence, the fractal dimension of Ti(C, N)/Ti(C, N) boundaries intergranular fracture d_f equals 1.262.

2.2 *Transgranular Cleavage Fracture in* Ti(C, N) *Particles*

Xie[4] proposed the estimated fractal model of transgranular fracture as shown in Fig. 2. So we obtain $N = 3$, $\gamma = 1/\sqrt{5}$, $d_f = \lg 3/\lg\sqrt{5} = 1.365$.

2.3 Ti(C, N) *Particles Diverged From Ni Boundaries*

When Ti(C, N) particles diverge from Ni boundaries, the binder phase Ni will be tensiled in local area and get some plastic deformation, in this case, the fracture path is not along the 120°, it may undulate around 120° in the range of 100° to 140°. Based on Su *et al*'s work[5], the random fractal dimension of intergranular fracture d_f equals 1.325.

Fig. 2 Fractal model of transgranular cleavage fracture in Ti(C, N) particles

2.4 *Transgranular Ductile Fracture in Binder Phase* Ni

The experimental results show that when the crack propagates in the binder phase the fracture path is zigzag, so we can use the fractal model as shown in Fig. 3 to simulate the transgranular ductile fracture. In this case $N = 2$, $\gamma = \frac{1}{\sqrt{2}}$, $d_f = \frac{\lg 2}{\lg\sqrt{2}} = 2$.

Fig. 3 Fractal model of transgranular ductile fracture in binder phase

2.5 *Synthetical Fractal Model in* Ti(C, N) *Based Ceramets*

In any fracture surface of Ti(C, N) based ceramets, we find the four kinds of morphologies mentioned above more or less. The fractal dimension of fracture surfaces should be the general contribution of all fracture morphologies. The

surface roughness R_S is equal to the ratio of the fracture surface to its projection in the normal direction of the mean plane of fracture[6,7], i. e.

$$R_S = S/A \quad (5)$$

So we have

$$R_S = S/A = (S_{Ti(C,N)/Ti(C,N)} + S_{Ti(C,N)} + S_{Ti(C,N)/Ni} + S_{Ni})/A \quad (6)$$

From Eq. (6), we obtain

$$R_S A/S = f_{Ti(C,N)/Ti(C,N)} + f_{Ti(C,N)} + f_{Ti(C,N)/Ni} + f_{Ni} = 1 \quad (7)$$

where f is the area fraction in fracture surfaces. A and S are constants. Since the fractal dimension, D_F, is based on the general contribution of area fraction of fracture morphology, we may have the following Eq. (8):

$$D_F = K + d_{Ti(C,N)/Ti(C,N)} f_{Ti(C,N)/Ti(C,N)} + d_{Ti(C,N)} f_{Ti(C,N)} + d_{Ti(C,N)/Ni} f_{Ti(C,N)/Ni} + d_{Ni} f_{Ni} \quad (8)$$

where d is the fractal dimension; f is its area fraction in fracture surfaces; K is a constant to be decided. Considering the fractal dimension of three dimension surface, we have $2 \leqslant D \leqslant 3$[8], so the fractal dimension mentioned in section 2. 1 to 2. 4 should be added with one, in this case, we obtain

$$D_F = K + 2.262 f_{Ti(C,N)/Ti(C,N)} + 2.365 f_{Ti(C,N)} + 2.325 f_{Ti(C,N)/Ni} + 3 f_{Ni} \quad (9)$$

3 EXPERIMENTAL METHOD

The tested materials are of composition (%) as follows: 30~ 40 Ni—13 Mo—10 TiN — TiC(rest). The specimens with the size of 5 mm × 5 mm × 30 mm are manufactured using a conventional powder metallurgy technique. The transverse rupture strength tests were performed using three point bend method with a 10 t universal testing machine. The step is 25 mm and the tested data are the average of twelve test pieces. The fracture surfaces were examined by SEM and the photograph was transformed into a discrete digitizing grey image by means of S600 DIP system. The grey level for each pixel is 256. In the calculation of H, we divide the image into many small windows, the number of which is $m \times n$. With the calculated values of H, we can calculate the fractal dimension D_F using Eq. (10):

$$D_F = 2 + \frac{1}{m \times n}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij}\right]^r \quad (10)$$

where H_{ij} is the calculated value of small window number ij.

4 RESULTS AND DISCUSSIONS

The relationship between the logarithm of transverse rupture strength and the fractal dimension of surfaces is shown in Fig. 4.

Fig. 4 $\lg \sigma_{bb}$ *vs* D_F **for different** ε **values**
(a) — ε = 8.47 μm; (b) — ε = 0.34 μm; (c) — ε = 0.07 μm

Fig. 4 shows that the measured results are different under different measuring unit conditions. Since the fracture surface has only a certain value of D_F, so the rest are only the results of calculation by DFBR method and not the real fractal dimension of surfaces. The correlation between the D_F and yardstick is shown in Fig. 5. Parts of the SEM micrographs of fracture surfaces are shown in Fig. 6. It has been denoted that D_F increases with the decrease of yardstick, but becomes stable when the yardstick is less than 0. 07 μm. Only when $\varepsilon \leqslant 0.07$ μm, the measured D_F is the real fractal dimension of fracture surfaces.

Yang[9] pointed out that it remained an upper critical point ε_k when anyone calculated the fractal dimension of an actual body. The

critical measuring yardstick in Fig. 5 is 0.07μm. Pentland[2] pointed out that when the projected size of a pixel is large with respect to the largest components, we observe the familiar Nyquist sampling behavior, i. e. the surface appears to become smoother as the pixel size is increased, and it will decrease the D_F. This well explains the results of Fig. 5.

Fig. 5 Measuring yardstick ε *vs* D_F of surfaces

By means of linear regression analysis in Fig. 4, we can obtain Eq. (11):

$$\lg \sigma_{bb} = -14.5214 + 5.8958 D_F \quad (11)$$

In Eq. (9) with the D_F values and area fraction of fracture morphologies measured, we can calculate the value of K, so we have Eq. (12):

$$D_F = 0.25 + 2.262 f_{Ti(C,N)/Ti(C,N)} + 2.365 f_{Ti(C,N)} + 2.325 f_{Ti(C,N)/Ni} + 3 f_{Ni} \quad (12)$$

From Eqs. (11) and (12), we have founded the relationship among σ_{bb}, D_F and the fracture morphology. For the study of fracture characteristics and the accident analysis of materials, our result would be very useful.

Fig. 6 SEM micrographs showing the morphology of fracture surfaces
(a) — D_F = 2.824; (b) — D_F = 2.768

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