THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS AND ALLOWABLE DEFECT ASSESSMENT OF THE CENTRIFUGE BOWL BODY①

Xu Zhenxing, Chen Fanjun, Wang Zhaoqi*, Xu Yan*, Liu Yaodong**, Chen Yijun**

Mechanical Engineering Department, Xiangtan University, Xiangtan 411105

ABSTRACT For the first time, an allowable defect analysis of the bowl body of XZZ 1200-G Centrifuge was made by using fatigue fracture mechanics method; and, again for the first time, the design fatigue curve of the dangerous scction of the bowl body was given out, which may form the basis for the design and use of the centrifuge.

Key words bowl body of centrifuge fatigue fracture mechanics method crack

1 INTRODUCTION

Cracks, containing sludge and gas holes inevitably occur in the bowl body of the centrifuge in the course of its casting and heat treatment. So far there have been little deep-going and systematic researches on how these defects affect the safety of the cenfrifuge. The technical standards for the cenfrifuge set in 1950's continue to be in use up to now, and such regulations are expressly provided as "in the surfaces of the casting elements of the centrifuge there shall be no cracks and other defects likely to reduce strength and affect their surface quality". Therefore, in centrifuge manufactories quite a number of bowl body products have to be discarded as useless. (see Fig. 1)

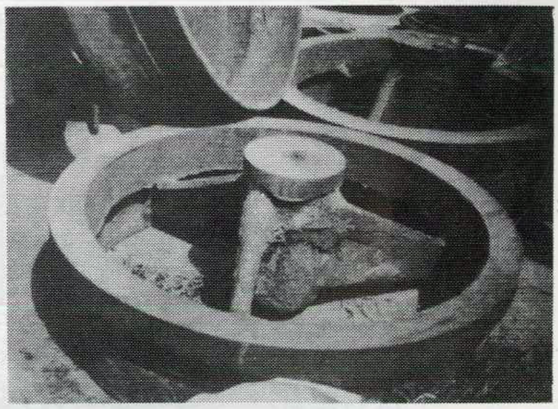

Fig. 1 Bowl body products discarded as useless

However, investigations indicated that it is rare that bowl bodies of centrifuges are destroyed by defects. Will the defects of the bowl bodies affect their safety, then? The authors of this article, for the above purpose, have made a three-dimensional finite-element calculation about the bowl body by using the analysing method of fatigue fracture mechanics, conducted a series of experimental determinations about the crack growth rate of the bowl body material. And by applying the stress intensity factor equation of surface cracks put forward by Newman and Raju[1], which is internationally acknowledged to be most exact, the authors worked out "Surface

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* Shanghai Branch of 701 Research Institute of General Co. of Shipping Industry of China

** Xiangtan Centrifuge Plant

Crack Analysis System Programme (SCAS)", thus gave the design fatigue curve of the centrifuge bowl body, which may form the basis for the design and use of the centrifuge.

2 THREE-DIMENSIONAL FINITE-ELEMENT ANALYSIS

Because of the comparative complication of the shape of the XZZ 1200-G centrifuge bowl body (see Fig. 1), the stress analysis can only be carricd out with the aid of the general three-dimensional finite-element. The rotational speed of the centrifuge bowl $n = 1\,000\,r/min$, power $N = 45\,kW$, the maximum feed weight is 4.9 kN. The load that the centrifuge bowl body bears is mainly the centrifugal force, and the edge moment and edge shear force of feed, support screen, spacer screen and so on, are transferred through the bowl. This edge force can be calculated with the aid of the design data concerned.

$M = 4.266\,kN \cdot m/m$

$P = 118.66\,kN/m$

The division of the three-demensional finite-element mesh (see Fig. 2) is achieved by using 8-node isoparametric element, the elements being 612, and the nodes being 1 494. When the grand programme ADINA is employed and the 386 Micro Computer (Coprocessor) is in motion, the time for motion is 30 min.

The stress analysis shows that the stress distribution of the bowl body is extremely uneven: the tensile stress that the edgy of ring body bears is comparatively great; there is a relatively strong stress conecntration (the relevant data of stress analysis will be given in another article) on the section where the ring body and rib joint, of which the drawing stress (σ_t) and bending stress (σ_w) are respectively as follows:

$\sigma_t = 3.412\,MPa$; $\sigma_w = 59\,MPa$

It is thus evident that the bending stress

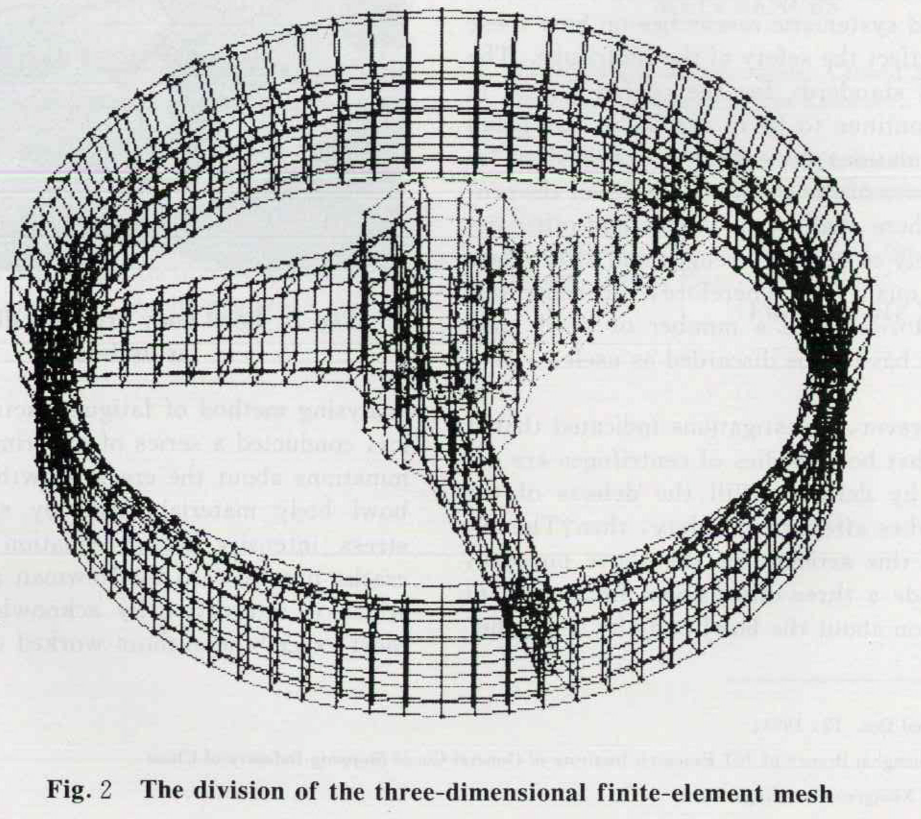

Fig. 2 The division of the three-dimensional finite-element mesh

is far bigger, and therefore the component form at this place is quite complicated and the cracks often occur. So it can be regarded as a dangerous area.

3 THE TEST OF GROWTH RATE OF SURFACE CRACKS

While the centrifuge is working, frequent stops and starts are unavoidable, so it is necessary to test the growth rate of surface cracks of bowl body materials under alternating load. Suppose the surface crack is semielliptical (see Fig. 3), the depth of the crack is a, the length is $2c$. And further, suppose that the crack growth rate at points A and B at the crack front and the stress intensity factor ranges at these points (ΔK_A and ΔK_B) independently obey Paris relationship, then we have:

$$\frac{da}{dN} = C_A(\Delta K_A)^{n_A} \quad (1)$$

$$\frac{dc}{dN} = C_B(\Delta K_B)^{n_B} \quad (2)$$

Applying the surface crack stress intensity factor equation put forward by Newman and Raju[1], with the joint function of the drawing stress and bending stress which are vertical to the crack place, we have:

$$K_I = M_I(\sigma_t + H\sigma_w)\sqrt{\frac{\pi a}{Q}} \quad (3)$$

$$(\frac{a}{c} \leqslant 1,\ \frac{a}{B} < 1,\ \frac{2c}{w} < 0.5,\ 0 \leqslant \theta \leqslant \pi)$$

where B is the plate thickness,

$$M_I = [M_1 + M_2(\frac{a}{B})^2 + M_3(\frac{a}{B})^4] f_\theta \cdot g \cdot f_w, \text{ or}$$

$$M_1 = 1.13 - 0.09(\frac{a}{c})$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + (a/c)}$$

$$M_3 = 0.5 - \frac{1.0}{0.65 + (a/c)} + 14(1.0 - \frac{a}{c})^{24}$$

$$g = 1 + [0.1 + 0.35(\frac{a}{B})^2](1 - \sin\theta)^2$$

$$f_\theta = [(\frac{a}{c})^2 \cos^2\theta + \sin^2\theta]^{1/4}$$

$$f_w = [\sec(\frac{\pi c}{w}\sqrt{\frac{a}{B}})]^{1/2}$$

$$H = H_1 + (H_2 - H_1)\sin^p\theta$$

$$p = 0.2 + \frac{a}{c} + 0.6\frac{a}{B}$$

$$H_1 = 1 - 0.34\frac{a}{B} - 0.11\frac{a}{c}(\frac{a}{B})$$

$$H_2 = 1 + G_1(\frac{a}{B}) + G_2(\frac{a}{B})^2$$

$$G_1 = -1.22 - 0.12\frac{a}{c}$$

$$G_2 = 0.55 - 1.05(\frac{a}{c})^{3/4} + 0.47(\frac{a}{c})^{3/2}$$

$$Q = \{E(k)\}^2$$

$$E(k) = \int_0^{\frac{\pi}{2}} [1 - k^2\sin^2\theta]^{1/2} d\theta$$

$$k = (1 - a^2/c^2)^{1/2}$$

Being the second kind of complete elliptical integral, there is the empirical formula:

$$Q = 1 + 1.464(\frac{a}{c})^{1.65} \quad (a/c \leqslant 1)$$

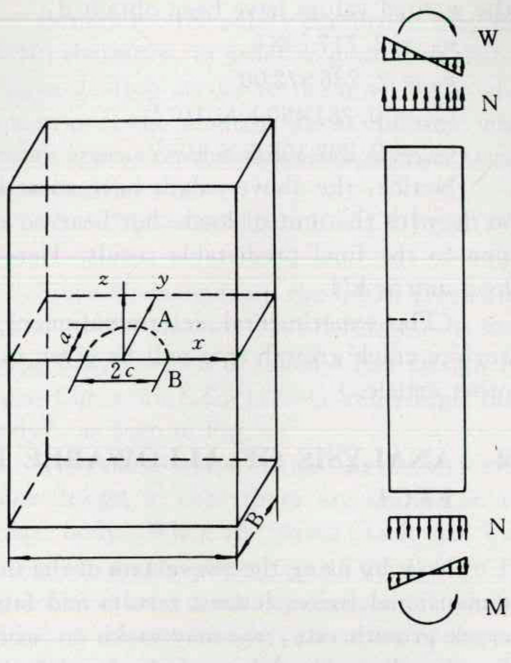

Fig. 3 Surface crack in a plate

The specimen material is cast steel, identical to that of the bowl body. The yield stress of the material has been determined to be $\sigma_s = 359.2$ MPa, and its ultimate stress to be $\sigma_b = 631.18$ MPa. The initial crack size is set by consulting the state standard; an electricdischarge machined notch is used as a crack starter. The fatigue test is carried out in order to determine the exact value of the corresponding crack length ($2c$) and depth (a) with cyclic loading for varying numbers N of cycles. Since it is rather easy to have an exact determination of the crack length, the key problem here is the determination of the depth. After several repeated explorations, with the aid of a special load marker, the authors eventually got a good determination. The whole experiment was made under the American Material Test System (MTS), whose maximum load is 200 kN. Through repeated loading and unloading and marking, the performance has been proved to be steady and the work went on smoothly. By using SCAS programme, the test data was analysed and sorted out by a computer, and finally the bending fatigue cracks were expanded, and the wanted values have been obtained:

$n_A = 3.7177080$

$n_B = 2.23697200$

$C_A = 0.2814501 \times 10^{-3}$

$C_B = 0.2023086 \times 10^{-3}$

Notice: the above values have something to do with the unit of load, but bear no relation to the final predictable result. Here the load unit is kN.

(The experimental determination of the surface crack growth rate will be given in another article.)

4 ANALYSIS OF ALLOWABLE DEFECT

Now by using the above data of the three-dimensional finite-element results and fatigue crack growth rate, we may make an analysis for the allowable defect of the bowl body of XZZ 1200-G Centrifuge.

4.1 *The Estimation of Remnant Life*

We can suppose that there is an initial surface crack on the dangerous section of the bowl body, its length being $2c$ and its depth being a. Let the crack surface be subjected to a drawing stress (σ_t) and a bending stress (σ_w) from the vertical direction. Applying the equation (1) to this, we'll have the remnant life (N_c) of the bowl body:

$$N_c = \int_{a_0}^{a_{max}} \frac{da}{C_A(\Delta K_A)^{n_A}} \tag{4}$$

The upper limit of the integral (a_{max}) should be the minimal value between the critical value of the crack depth (a_c) at the time when the crack is in instable growth and the plate thickness (B), namely, $a_{max} = \min(a_c, B)$ and a_c is determined according to the fracture toughness of the surface crack. However, because of the complication of the K_I in equation (3), the evaluated value (a_c) for the given K_{Ie} is not sole, which brings about some difficulty to life analysis. After carrying out the fatigue crack growth test, the authors discovered that, when the thickness $B \geqslant 16$ mm, instable growth invariably occurred after some cyclic loading. In the course of instable growth the crack depth (a_c) changes as show in Table 1.

The average value of a_c/B is 0.739; to assure safety, we choose $a_{max} = 0.7B$.

Table 1 The data of the crack depth(a_c)

Serial number of specimens	1	2	3	4	5	6
Thickness B /mm	19.8	20	17.8	17.2	18	20
a_c /mm	14.5	14	12.3	14.4	13	15
a_c/B	0.732	0.7	0.69	0.84	0.72	0.75

Since ΔK_A in the equation (4) has a great deal to do not only with the crack depth (a) but also with the instantaneous value of the crack semilength(c), numerical is called for:

$$N_c = \sum_{i=1}^{n} \frac{\Delta a_i}{C_A[\Delta K_A(a_{i-1}, c_{i-1})]^{n_A}}, \quad i = 1, 2, \cdots, n \tag{5}$$

where $\Delta a_i = (a_{max} - a_0)/n$.

Notice: in the above equation (5), each a_i, but c_i is unknown. Therefore we again apply the equations (1) and (2) which are obtained through experiments, and have:

$$\frac{da}{dC_i} = \frac{C_A \Delta K_A^{n_A}}{C_B \Delta K_B^{n_B}} \quad (6)$$

When Δa_i is minor enough, then

$$\frac{\Delta a_i}{\Delta c_i} = \frac{C_A[\Delta K_A(a_{i-1}, c_{i-1})]^{n_A}}{C_B[\Delta K_B(a_{i-1}, c_{i-1})^{n_B}} \quad (7)$$

So c_i corresponding to a_i can be evalunted. For example:

when $i = 1$, $\Delta a_1 = (a_{max} - a_0)/n$, $a_{i-1} = a_0$, $c_{i-1} = c_0$, applying equations (3) and (7), Δc_1 can be evaluated, then

$$a_1 = a_0 + \Delta a_1 \quad c_1 = c_0 + \Delta c_1$$

when $i = 2$, similarily we can evaluate Δc_2; then

$$a_2 = a_0 + \Delta a_1 + \Delta a_2$$

$$c_2 = c_0 + \Delta c_1 + \Delta c_2$$

when i is arbitrary, we have:

$$a_i = a_0 + \sum_{j=1}^{i} \Delta a_j,$$

$$c_i = c_0 + \sum_{j=1}^{i} \Delta c_j \quad (8)$$

Substitute equation (8) for (5) item by item, and we can evaluate the remnant life (N_c) of the centrifuge bowl body on the condition that the initial crack size with the function of a given outside load is a_0 or c_0. Calculations show that, in equation (5) if n is too small in value, there will be a great difference in the value of N_c for various values of n; this holds true especially of the initial crack when $a_0/B = 0.6$, $a_0/C_0 = 1$. But when $n \geqslant 500$, the value difference of N_c is less than 2%, which holds true of any size of initial crack and of various n value; and this shows all the engineering requirement has been met.

4.2 *The Service Life of the Centrifuge*

The life needed during which the centrifuge can be put to work is referred to as a service life, which depends upon the working condition and using requirement of the centrifuge. The work time of XZZ 1200-G centrifuge is from 8 to 10 years: each year, it is in motion for 3 to 4 months; when being very busy, each day consists of 3 shifts, each hour the machine starts and stops for as many as 6 times, and every start is divided into 4 levels:

0~110 r/min; 110~200 r/min; 220~500 r/min; 500~1 000 r/min.

For the sake of safety, each start may be counted as two cyclic loadings. Therefore, the service life of the whole time will be:

$$N_s = 2 \times 6 \times 24 \times 31 \times 4 \times 10 = 357\,120 (r)$$

4.3 *The Design Fatigue Curve*

Now comes the problem of how to design fatigue curve if cracks occur on the dangerous section of the bowl body. As the drawing stress on the section is so small, even smaller than 6% of the bending stress, that it can be neglected. With the aid of SCAS programme, we can evaluate respectively the remnant life N_c of different initial cracks a_0 and c_0 under the function of the equation $\sigma_w = 59$ MPa, and give a chart of remnant life. According to the service life of the centrifuge, considering a certain safety coefficient k, we think the fatigue strength is

$$N_s k \leqslant N_c \quad (9)$$

Generally, the safety coefficient may be $k = 10$; therefore, in order to guarantee that no fatigue destroy occurs to the bowl body while it is in use, the remnant life at the time when cracks appear on the dangerous section should be no smaller than

$$N_c = N_s k = 357\,120 \times 10 = 3\,571\,200 (r) \quad (10)$$

Since N_c depends on the initial crack sizes a_0 and c_0, the equation (10) determines a curve (of a_0 & c_0) which is called "The Design Fatigue Curve of XZZ 1200-G centrifuge Bowl Body", as seen in Fig. 4.

The above curve forms the basis for fatigue design in case there are cracks on the bowel body. When the initial crack size (a_0, c_0) is set on the lower left of the Design Fatigue Curve, the crack is permissable; when a_0 and c_0 on the upper right, the crack is not permissble.

5 CONCLUSION

With the three-dimensional finite-element analysis of the bowl body of XZZ 1200-G centrifuge and the surface crack growth rate determination of the bowl body material, the authors have, by applying the surface crack stress intensity factor equation put forward by Newman and Raju[1], worked out surface crack analysis system programme, and made an allowable defect analysis of the bowl body.

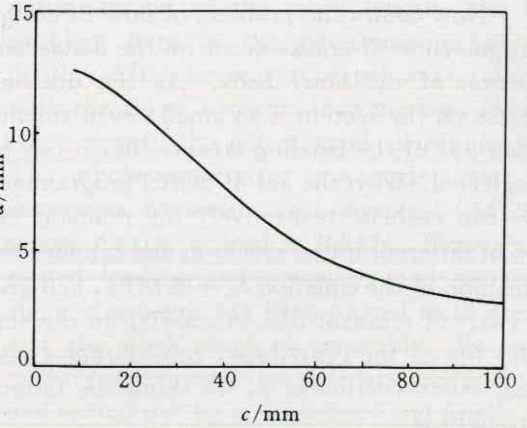

Fig. 4 Design fatigue curve

The following conclusions have been drawn:

(1) The bending fatigue test with specimens made of bowl body material obviously shows that, when the specimen thickness $B \geqslant 16\,mm$, after a certain amount of fatigue loading, there will generally be instable growth, and that, when instable growth occurs, the average value of the radio of the crack depth and specimen thickness is 0.739. Therefore, it is possible and not negligible, that the crack instable growth occurs on the bowl body with cracks because of long alternation loading function.

(2) The design fatigue curve is given about the high stress area of XZZ 1200-G centrigue bowl body with surface cracks. When the sizes of initial cracks are on the lower left of the curve, these cracks are permissable. Because in this case the safety of the bowl body can be guaranteed during the work period. When the sizes on the upper right, the cracks are not permissable. Table 1 shows that such cracks are all big in size. So most cracks on real bowl bodies will not threaten the safety of centrifuges.

(3) It is neither scientific nor practical that certain standards and technical requirements about centrifuges which are still in practice deny any surface cracks on bowl bodies of centrifugues. Therefore it should be brooked no delay to work out for instead, a new and more scientific "Standards for the Evaluation of Centrifuge Detect" should be setup, the design of centrifuge should also be laid on the basis of modern science and technology.

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