NATURAL FREQUENCY OF COMPONENT UNDER INFLUENCE OF WELDING RESIDUAL STRESS①

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ABSTRACT According to the distribution feature of welding residual stress on plate with welding seam along axis, a mathematical model of the welding residual stress is established. A formula of natural frequency of four boundary simply-supported quadrate thin plate with welding residual stress is developed. Some conclusions have been obtained. They are: (1) if there is welding residual stress, all natural frequencies of component are in increase, (2) change of high rank natural frequency under the influence of residual stress is larger than that of low rank natural frequency, and the higher the rank number is, the larger the absolute amount of change of natural frequency is.

Key words mathematical model residual stress natural frequency welding quadrate thin plate

1 INTRODUTION

Problems of residual stress are now studied earnestly both at home and abroad. There has been some literatures on production mechanism, survey, readjustment of residual stress. But the conditions of production have great influence on residual stress and the cause of production is very complex. Moreover, amount of residual stress changes with time. Hence, distribution and amount of residual stress varies randomly. Therefore, there is little literature of theory research on natural frequency of component under the influence of residual stress. In this paper, an object of study is the welding residual stress. According to the distribution feature of welding residual stress on plate with welding seam along axis, a mathematical model of welding residual stress is established and a formula of natural frequency of component under the influence of residual stress is developed. We will discuss that residual stress has influence on natural frequency of component from the formula and acquire some useful conclusions.

2 A MATHEMATICAL MODEL OF WELDING RESIDUAL STRESS

We discuss free vibration of quadrate same thickness thin plate.

Suppose that there is a welding seam along axis. Hence, there is distribution of residual stress in the plate as shown in Fig. 1[1]. Fig. 1(a) shows residual stress on parallel join direction. The residual stress is large. Large drawing residual stress is produced in the middle of the join. Fig. 1(b) shows the distribution of stress on vertical join direction.

Because of thin plate, we suppose that residual stress σ_x, σ_y are only a function of x, y. Residual stress σ_x is parallel to join direction. Residual stress σ_y is vertical to join direction.

According to Fig. 1 which shows distribution of welding residual stress, we suppose that σ_x is only a function of y and σ_y is only a functon of x for simple computation. Welding residual stress

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does not change with small deformation.

According to the suppositions above and the distribution as shown in Fig. 1, we can establish a mathematical model of welding residual stress, that is,

$$\left.\begin{aligned}\sigma_x &= F\left[\cos\left(\frac{2\pi}{b}\left(y-\frac{b}{2}\right)\right)+\cos\left(\frac{4\pi}{b}\left(y-\frac{b}{2}\right)\right)\right]\\ \sigma_y &= G\cos\left(\frac{2\pi}{b}\left(x-\frac{a}{2}\right)\right)\end{aligned}\right\} \tag{1}$$

Figure of the formula (1) is shown by Fig. 2.

Because residual stress exists in object with balance condition when there is not outer force, it equals to zero on any section that the join force and the join moment of force are produced by residual stress, that is,

$$\int \sigma \, \mathrm{d}A = 0 \tag{2}$$

$$\int \mathrm{d}M = 0 \tag{3}$$

We prove that the formula (1) is satisfied with formulas (2) and (3). Thus, we have

$$\int_{-z}^{z}\int_{0}^{b} F\left[\cos\left(\frac{2\pi}{b}\left(y-\frac{b}{2}\right)\right)+\cos\left(\frac{4\pi}{b}\left(y-\frac{b}{2}\right)\right)\right]\mathrm{d}y\mathrm{d}z = 0$$

$$\int_{-z}^{z}\int_{0}^{a} G\cos\left(\frac{2\pi}{a}\left(x-\frac{a}{2}\right)\right)\mathrm{d}x\mathrm{d}z = 2zG\frac{a}{2\pi}\sin\left(\frac{2\pi}{a}\left(x-\frac{a}{2}\right)\right)\Big|_0^a = 0$$

Thus, formula (1) has been satisfied with formula (2). Similarly, we have

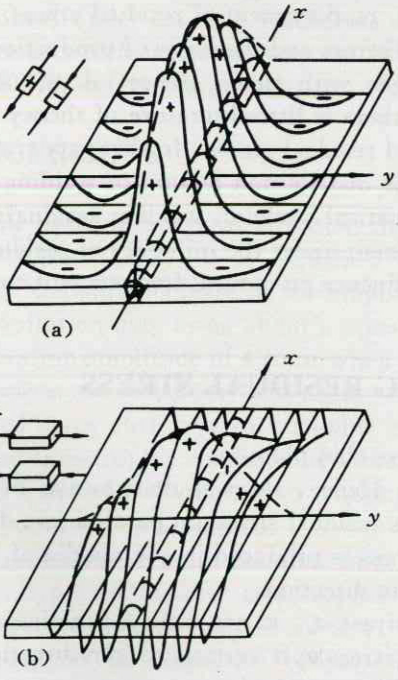

Fig. 1 Distribution of residud stress

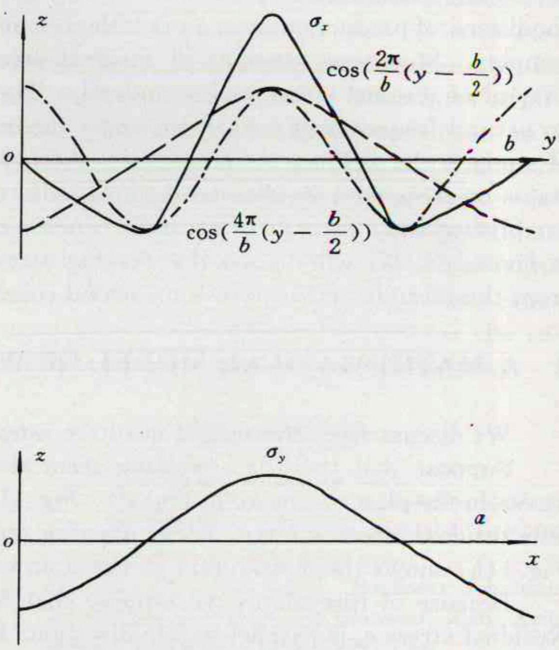

Fig. 2 Line of mathematical model

$$M_y = \int_{-z}^{z}\int_0^b zF[\cos(\frac{2\pi}{b}(y - \frac{b}{2})) + \cos(\frac{4\pi}{b}(y - \frac{b}{2}))]\mathrm{d}y\mathrm{d}z = 0$$

$$M_z = \int_{-z}^{z}\int_0^b yF[\cos(\frac{2\pi}{b}(y - \frac{b}{2})) + \cos(\frac{4\pi}{b}(y - \frac{b}{2}))]\mathrm{d}y\mathrm{d}z = 0$$

Because of the similarity, we have,

$$M_x = \iint zG\cos(\frac{2\pi}{a}(x - \frac{b}{2}))\mathrm{d}x\mathrm{d}z = 0,$$

$$M_z = \iint xG\cos(\frac{2\pi}{a}(x - \frac{b}{2}))\mathrm{d}x\mathrm{d}z = 0$$

Similarly, formula (1) has been satisfied with formula (3). Hence, we may think that the formula (1) can be a mathematical model of the welding residual stress.

3 A FORMULA OF NATUREL FREQUENCY OF COMPONENT UNDER THE INFLUENCE OF RESIDUAL STRESS

Suppose quadrate thin plate in to be simply-supported on four boundaries. Its condition of boundary is

$$\left.\begin{aligned}
&(W)_{x=0} = 0,\ (\frac{\partial^2 W}{\partial x^2})_{x=0} = 0 \\
&(W)_{x=a} = 0,\ (\frac{\partial^2 W}{\partial x^2})_{x=a} = 0 \\
&(W)_{y=0} = 0,\ (\frac{\partial^2 W}{\partial y^2})_{y=0} = 0 \\
&(W)_{y=b} = 0,\ (\frac{\partial^2 W}{\partial y^2})_{y=b} = 0
\end{aligned}\right\} \tag{4}$$

According to Ref. [2], differential equation of vibration of thin plate is

$$D(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}) + \rho h \ddot{W} = q \tag{5}$$

where $D = \dfrac{Eh^3}{12(1-\mu^2)}$ is bend rigid of the plate, h is thickness of the plate (h is smaller than other measurement), ρ is mass of unit volume of the palte, q is crosswise load of unit face of the plate, W is deflection and μ is a constant called Poisson's ratio.

Partial forces on z direction are produced by residual stress on parallel and vertical x axis in the plate when the plate is bend, as shown in Fig. 3. We have

$$\sigma_x \sin\theta = \sigma_x \theta,\ \theta = \frac{\partial W}{\partial x};$$

$$\sigma_y \sin\alpha = \sigma_x \alpha,\ \alpha = \frac{\partial W}{\partial y}$$

Thus, total partical force f_z which acts on $\mathrm{d}x\mathrm{d}y$ face and in unit thickness on z direction is

$$f_z = f_{xz} + f_{yz}$$

where f_{xz} and f_{yz} are given by

$$f_{xz} = -\sigma_x \mathrm{d}y(\theta + \frac{\partial\theta}{\partial x}\mathrm{d}x) + \sigma_x \mathrm{d}y\theta$$

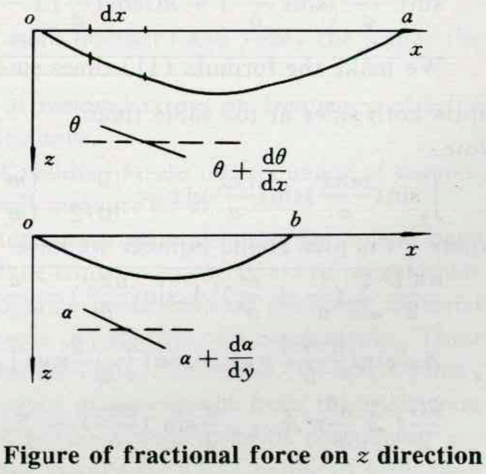

Fig. 3 Figure of fractional force on z direction

$$= -\sigma_x \frac{\partial \theta}{\partial x} dx dy = -\sigma_x \frac{\partial^2 W}{\partial x^2} dx dy$$

$$f_{yz} = -\sigma_y \frac{\partial^2 W}{\partial y^2} dx dy$$

therefore, we obtain

$$f_z = -\sigma_x \frac{\partial^2 W}{\partial x^2} dx dy - \sigma_y \frac{\partial^2 W}{\partial y^2} dx dy$$

and

$$q = \int_{-h/2}^{h/2} (-\sigma_x \frac{\partial^2 W}{\partial x^2} - \sigma_y \frac{\partial^2 W}{\partial y^2}) dz = -h(\sigma_x \frac{\partial^2 W}{\partial x^2} + \sigma_y \frac{\partial^2 W}{\partial y^2}) = -h\{[\cos(\frac{2\pi}{b}(y - \frac{b}{2})) + \cos(\frac{4\pi}{b}(y - \frac{b}{2}))]F \frac{\partial^2 W}{\partial x^2} + G\cos(\frac{2\pi}{a}(x - \frac{a}{2})) \frac{\partial^2 W}{\partial y^2}\} \tag{6}$$

using the formulas (5) and (6), main vibrational equation is given by

$$D(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}) + \rho h \ddot{W} = -h\{[\cos(\frac{2\pi}{b}(y - \frac{b}{2})) + \cos(\frac{4\pi}{b}(y - \frac{b}{2}))] \times F \frac{\partial^2 W}{\partial x^2} + G\cos(\frac{2\pi}{a}(x - \frac{a}{2})) \frac{\partial^2 W}{\partial y^2}\} \tag{7}$$

W is defined by

$$W = \overline{W}(x, y)\cos(\omega_{mn} t + \beta) \tag{8}$$

When we apply the formula (8) to formula (7), main equation of vibrational form is obtained

$$D\nabla^4 \overline{W}(x, y) - \rho h \omega_{mn}^2 \overline{W}(x, y) = -h\{[\cos(\frac{2\pi}{b}(y - \frac{b}{2})) + \cos(\frac{4\pi}{b}(y - \frac{b}{2}))F \frac{\partial^2}{\partial x^2} + G\cos(\frac{2\pi}{a}(x - \frac{a}{2})) \frac{\partial^2}{\partial y^2}\}\overline{W}(x, y) \tag{9}$$

$\overline{W}(x, y)$ is defined by

$$\overline{W}(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \tag{10}$$

obviously, formula (10) has been satisfied with condition of boundary as shown by formula (4). When we apply the formula (10) to the equation (9), we have

$$\pi^4 D \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (\frac{m^2}{a^2} + \frac{n^2}{b^2})^2 A_{mn} \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) - \rho h \omega_{mn}^2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \times \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) = h[\cos(\frac{2\pi}{b}(y - \frac{b}{2})) + \cos(\frac{4\pi}{b}(y - \frac{b}{2}))]F \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2}{a^2} \pi^2 A_{mn} \times \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) + hG\cos(\frac{2\pi}{a}(x - \frac{a}{2})) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{n^2}{b^2} A_{mn} \pi^2 \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \tag{11}$$

We make the formula (11) times $\sin(\frac{i\pi x}{a})$, then we integrate formula (11) over x from 0 to a on its both sides at the same time.

Note:

$$\int_0^a \sin(\frac{m\pi x}{a}) \sin(\frac{i\pi x}{a}) dx = \begin{cases} 0 & (m \neq i) \\ a/2 & (m = i) \end{cases} \tag{12}$$

where i is plus round number we have

$$\frac{a\pi^4 D}{2} \sum_{m=1}^{\infty} (\frac{i^2}{a^2} + \frac{n^2}{b^2})^2 A_{in} \sin(\frac{n\pi y}{b}) - \frac{a}{2} \rho h \omega_{in}^2 \sum_{n=1}^{\infty} A_{in} \sin(\frac{n\pi y}{b}) = \frac{-hF}{2} \pi^2 \sum_{n=1}^{\infty} \frac{i^2}{a^2} \frac{a}{2} \times A_{in}[\sin(\frac{n+2}{b}\pi y) + \sin(\frac{n-2}{b}\pi y)] + \frac{hF}{2} \pi^2 \sum_{n=1}^{\infty} \frac{i^2}{a^2} \frac{a}{2} A_{in}[\sin(\frac{n+4}{b}\pi y) + \sin(\frac{n-4}{b}\pi y)] - \frac{hG}{2}(\sum_{n=1}^{\infty} \frac{n^2}{b^2} \pi^2 A_{i-2,n} \frac{a}{2} \sin(\frac{n\pi y}{b}) + \sum_{n=1}^{\infty} \frac{an^2}{2b^2} \pi^2 A_{i+2,n} \sin(\frac{n\pi y}{b}) \tag{13}$$

We make the formula (13) times $\sin(\frac{j\pi y}{b})$, then we integrate formula (13) over y from 0 to b

on its both sides at the same time.

Note:

$$\int_0^b \sin(\frac{n\pi y}{b})\sin(\frac{j\pi y}{b})\mathrm{d}y = \begin{cases}0 & (n \neq j) \\ b/2 & (n = j)\end{cases} \tag{14}$$

where j is plus round number, we have

$$\frac{ab}{4}\pi^4 D(\frac{i^2}{a^2} + \frac{j^2}{b^2})^2 A_{ij} - \frac{ab}{4}\rho h\omega_{ij}^2 A_{ij} = -\frac{Fh}{2}\pi^2 \frac{i^2}{a^2}\frac{ab}{4}(A_{i,j-2} + A_{i,j+2}) +$$

$$\frac{Fh}{2}\pi^2 \frac{i^2}{a^2}\frac{ab}{4}(A_{i,j-4} + A_{i,j+4}) - \frac{hG}{2}\pi^2 \frac{j^2}{b^2}\frac{ab}{4}(A_{i-2,j} + A_{i+2,j})$$

and hence, we have

$$\omega_{ij}^2 = \frac{\pi^4 D}{\rho h}(\frac{i^2}{a^2} + \frac{i^2}{b^2})^2 + \frac{G\pi^2 j^2}{2\rho b^2}(\frac{A_{i-2,j} + A_{1+2,j}}{A_{ij}}) + \frac{F\pi^2 i^2}{2\rho a^2} \times$$

$$(\frac{A_{i,j-2} + A_{i,j+2} - A_{i,j-4} - A_{i,j+4}}{A_{ij}}) \tag{15}$$

The formula (15) is applied to natural frequency of component under the influence of residual stress computation.

According to the process of development of the formula (15), we put negative sign in front of A_{mn} when angular sign of A_{mn} appear negative round number in formula (15) and angular sign of A_{mn} is plus round number. For exmple, $A_{1,-2}$ can be written as $-A_{i,2}$. Amount of A_{mn} is zero when an angular sign of A_{mn} is zero.

We have obtianed the function of vibrational form on quadrate thin plate when four boundary of the thin plats is simply-supported in elasticity

$$\overline{W}_{mn} = \sin(\frac{m\pi x}{a})\sin(\frac{n\pi x}{b}) \tag{16}$$

and a formula of natural frequency

$$\omega_{mn}^2 = \pi^4(\frac{m^2}{a^2} + \frac{n^2}{b^2})^2 \frac{D}{\rho h} \tag{17}$$

Through contrasting formula (15) with formula (17), we know that second item and third item of formula (15) show influence of residual stress on natural frequency of component.

4 CONCLUSIONS OF THEORY

(1) If other factors are not changed, the larger the residual stress F and G is, the larger the influence of residual stress is.

(2) If other factors are not changed, the higher the rank number i and j are, the larger the influence of residual stress on frequency is.

(3) If other factors are not changed, the influence of residual stress on frequency of large density component is smaller than that of the small density ones.

(4) If other factors are not changed, the influence of residual stress on frequency of component with large measure a, b is smaller than that with small measure a, b.

These conclusions of theory are of great value of reference to general component, and have been proved by experiments of modal analysis. We have done contrast experiments of modal analysis on contrast plate with the welding residual stress to same measures and the same material plate with no welding residual stress. There are twenty pieces of experimental components. Their measures are 1 000 mm×100 mm×4 mm, 600 mm×100 mm×4 mm, 550 mm×100 mm×5 mm, 600 mm×100 mm×6 mm. We have obtained some conclusions of experiment from these contrast experiments: (1) If there is welding residual stress, all natural frequencies of component are changed and the larger the residual stress is, the larger the change is. (2) Change of high rank natural frquency under the influence of residual stress is larger than that of low rank natural frequency and the higher the rank number is, the larger the absolute change of natural frequency is. (3) If other factors are not changed, the influence of the residual stress on frequency of component with large measure a, b is smaller than on those with small measure a, b. The conclusions of theory are proved by conclusions of experiments.

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(1) High control accuracy. Through muliple parameter's comprehensive control, system's control mode achieve the zero-drop control from the original drop control, and the control accuracy increases and attains system's check accuracy from original dead zone control value.

(2) Fast response. System can begin its compensation as MCB's helix angle far below the original dead zone control value by checking the inclination angular velocity. Since the inertia of MCB is rather high, the increasing of the response speed is advantageous to decrease the maximum helix angle of MCB.

(3) High reliability. Because adotping the pressure signal protection, even MCB deflects reversely under the inertia effect, it will not cause vibration.

(4) The system can be operated more smoothly and steadily.

(5) High compensation efficiency. The maximum helix angle of MCB decreases greatly and the framework's force condition is improved enomously.

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