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DESCRIPTION AND PREDICTION OF CATASTROPHE OF VIBRATION STATE FOR FAULTY ROTORS①

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ABSTRACT A sudden increase of vibration amplitude with no foreboding often results in an abrupt breakdown of a mechanical system. The catastrophe of vibration state of a faulty rotor is a typical nonlinear phenomenon, and very difficult to be described and predicted with linear vibration theory. On the basis of nonlinear vibration and catastrophe theory, the catastrophe of the vibration amplitude of the faulty rotor is described; a way to predict its emergence is developed.

Key words mechanical fault diagnosis and prediction cusp catastrophe model nonlinear vibration rotor

1 INTRODUCTION

Mechanical faults can be divided into two types, namely gradual type faults and abrupt type faults. The former means that the operating condition of a machine or its parts is becoming worse and worse till the machine can't work normally. Its feature is that the values of the characteristic parameters of the fault continuously increase or decrease till reaching and breaking through threshold values during the operation of the machine. The latter means that the fault suddenly arises with no foreboding during the operation of the machine. Its feature is that the characteristic parameters of the fault change suddenly and break through threshold values. Whether it is of gradual type faults or abrupt type faults, it is of great significance to predict accurately and effectively. Nevertheless, because of existing "grey areas" of fault mechanism, complexity of mechanical systems, randomness of material performance and uncertain environmental disturbance to machines and their monitoring systems, the study of this field is very difficult. In recent researches of fault prediction, time series models, grey forecast models, statistical models and artificial neural net techniques are used to forecast the tendencies of gradual type faults. However, so far no paper concerning the mechanisms and predicting methods of abrupt type faults has been published. There are two difficulties in this field. Firstly, because the existing methods of fault diagnosis and prediction are generally based on linear vibration theory, and linearization is only an approximation of a real system, it is difficult to reveal the mechanisms of abrupt type faults and make an effective forecast with them. Secondly, the mathematical methods applied in engineering were developed mainly in order to make quantitative analyses of various continuous processes, so we need perfect and practical theories and methods to describe the abrupt change of the system state resulting from the continuous change of some control parameters.

The abrupt change of the vibration state of a rotor in a machine during the continuous change of excitation parameters really exists in engineering practice, and is one of the principal cause to result in an abrupt breakdown of a mechanical system. As for state monitoring and fault diagnosis of mechanical systems,

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great attention should be paid to this kind of catastrophe phenomenon. Because a sudden increase of the amplitude of abnormal vibration excited by faults means that the working condition of a machine worsens suddenly, and even brings about a pernicious accident, it is very important to predict effectively the catastrophe of vibration amplitude of the rotor in key and expensive equipment. Amplitude catastrophe is a typical sort of performance of nonlinear systems, so the fault diagnosis and prediction methods based on linear vibration theory are powerless to it. Catastrophe theory founded by Thom in 1970s, which aims at noncontinuity, makes it possible to describe and predict quantitatively the amplitude catastrophe of abnormal vibration excited by faults. Based on nonlinear vibration theory and catastrophe theory, the amplitude catastrophe of forced vibration of the rotor excited by faults is described by cusp catastrophic model in this paper, its generating condition is derived, and a prediction way is developed.

2 CATASTROPHE MODELS AND CATASTROPHE FEATURES

For function group V: $X \times C \rightarrow R$ (where, X is an n-dimensional manifold, and C is an r-dimensional manifold, catastrophe maniflod M is a subset of $R^n \times R^r$, which is defined by $DV(x, c) = 0$. Catastrophe mapping χ is natural projection subject to M, π: $R^n \times R^r \rightarrow R^r$, i.e., $\pi(x, c) = c$. Singularity set S is the set of singular points of χ in M. Bifurcation set B is image $\chi(S)$ of singularity set S in C.

The number of possible, discontinuous structures depends on number r of control variables, but not on number n of state variables[1]. If $r \leqslant 4$, there are seven types of distinct catastrophes, namely fold, cusp, swallowtail, elliptically umbilic, hyperbolically umbilic, butterfly and parabolically umbilic. Each type of catastrophes has its own potential function, catastrophic manifold and bifurcation set. For example, the potential function, catastrophic manifold and bifurcation set of cusp catastrophe are respectively:

$$V(x) = x^4 + ux^2 + vx \tag{1}$$

$$4x^3 + 2ux + v = 0 \tag{2}$$

$$8u^3 + 27v^2 = 0 \tag{3}$$

Generally speaking, the catastrophe has five features. Take cusp catastrophe shown in Fig. 1 as an example to explain catastrophic features as follows: (1) Multi-modality. It means that the system has several different, possible states (see modality A and B in Fig. 1 (a)). (2) Kick. When control trajectory crosses the bifurcation set, state variables may change abruptly (① and ② in Fig. 1(b)). (3) Inaccessibility. It means that the system has unstable balanced states (unstable region shown in Fig. 1 (a)). (4) Divergency. It means unstablilty of the system to path per-

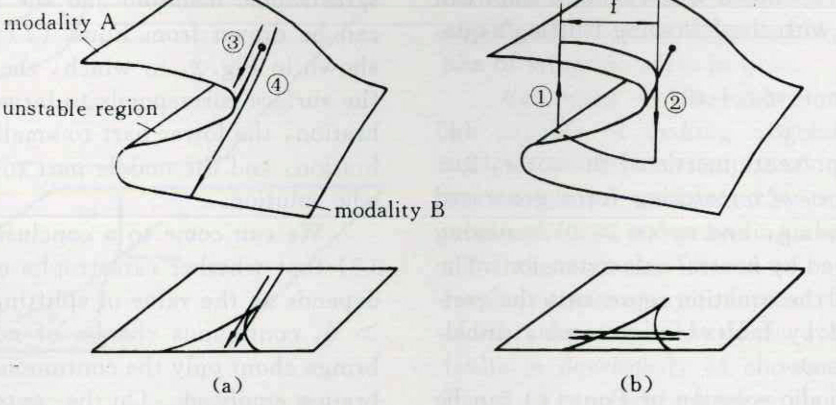

Fig. 1 Features of cusp catastrophe

turbation of control parameters (③ and ④ in Fig. 1 (a)) (5) Hysteresis. It is the phenomenon that may appear when the system is strictly irreversible. As shown in Fig. 1(b), the value of control variables corresponding to the catastrophe from a state to another and the one to its converse are unequal, and this is expressed by $f > 0$ in Fig. 1(b).

3 EXPLANATION OF AMPLITUDE CATASTROPHE OF THE FAULTY ROTOR WITH CATASTROPHE THEORY

Naturally, the features above-mentioned are associated with the abrupt type faults of machines. The paper tries to apply catastrophe theory to diagnosis and prediction of mechanical abrupt type faults. The authors's train of thought is as follows: at first, to construct a mathematical system, which can give a good description of all or requisite majority of features of the process of an abrupt type fault, then to analyse this system and derive some conclusions, which are considered fit for the real faulty process. According to analysis and observation of certain abrupt type fault, in combination with the characters of each type of catastrophes, we can find a catastrophic model capable of describing accurately or approximately this abrupt type fault, and then make a forecast on the basis of this model.

Transverse forced-vibration of a rotor can be described with the following Duffing's quation:

$$\ddot{x} + 2\mu\dot{x} + \omega_0^2 x + \varepsilon x^3 = K\cos(\omega t + \varphi) \tag{4}$$

where $\ddot{x}$ represents inertia of the rotor, $2\mu\dot{x}$ damping force, $\omega_0^2 x$ restoring force generated by shaft bending, and $\varepsilon x^3 (\varepsilon > 0)$ restoring force generated by neutral axis extension. The right term of the equation represents the excitation caused by faults (for example, unbalance and so on).

The periodic solution of Eqn. (4) can be achieved using the method of harmonic balance[2]. Suppose that zero-order approxiamte solution is:

$$x_0 = a\cos\omega t \tag{5}$$

Substituting it into Eqn. (4), and omitting higher-harmonic terms, we can get the frequency-response equation of Eqn. (4).

$$[(\omega_0^2 - \omega^2)a + \frac{3}{4}\varepsilon a^3]^2 + 4\mu^2 a^2 \omega^2 = K^2 \tag{6}$$

Obviously, Eqn. (6) is a cubic equation concerning a^2, and corresponds to cusp type catastrophic manifold. For the purpose of this paper, the frequency-response equation (6) can be regarded as a cusp type catastrophic manifold with state variable a^2 and control variables ω and K. let

$$C = \omega^2 - \omega_0^2,$$

$$y = a^2 - \frac{8}{9}\frac{C}{\varepsilon}$$

Eqn. (6) can be transformed into the normal form of cusp type catastrophic manifold:

$$4y^3 + 2uy + v = 0 \tag{7}$$

$$u = \frac{32}{9\varepsilon^2}[4\mu^2\omega^2 - \frac{1}{3}(\omega^2 - \omega_0^2)^2] \tag{8}$$

$$v = \frac{64}{9\varepsilon^2}[\frac{8}{81}\frac{(\omega^2 - \omega_0^2)^3}{\varepsilon} + \frac{32\mu^2\omega^2}{9\varepsilon} - K^2] \tag{9}$$

According to catastrophe theory, the equation of bifurcation set is:

$$8u^3 + 27v^2 = 0 \tag{10}$$

Potential amplitude catastrophe may emerge only when Eqn. (10) is satisfied.

The geometric figure of the amplitude catastrophic manifold and the bifurcation set can be drawn from Eqns. (7) and (10), as shown in Fig. 2, in which, the upper part of the surface corresponds to large amplitude vibration, the lower part to small amplitude vibration, and the middle part to unstable periodic solution.

We can come to a conclusion from Eqn. (7) that whether catastrophe emerges or not depends on the value of splitting factor u. If $u > 0$, continuous change of normal factor v brings about only the continuous change of vibration amplitude. On the contrary, if $u < 0$, continuous change of v may result in uncontinuous change, namely amplitude catastrophe. The condition for catastrophe to emerge can be derived from Eqn. (8) as

$$4\mu^2\omega^2 < \frac{1}{3}(\omega^2 - \omega_0^2)^2$$

Also $$\frac{|\omega^2 - \omega_0^2|}{\omega} > 2\sqrt{3}\,\mu \tag{11}$$

During normal operation of a machine, ε, μ, ω and ω_0 approximately keep constant; K, amplitude of excitation, increases continuously and monotonously with gradual aggrevating of faults, and is a slowly varying function of time. For certain values of ε, μ, ω and ω_0, the values K_1 and K_2 at which catastrophe of vibration amplitude may take palce can be gotten from Eqn. (10). If $K < K_1$ or $K > K_2$, only one steady periodic solution exsits, and vibration of the rotor is stable; if $K_1 < K < K_2$, two stable steady periodic solutions and one unstable steady solution exist, and the unstable one corresponds to impossible vibration in practice, so it can't be observed in real-time monitoring. During the period when the faults are gradually aggravated (such as unbalance increasing), K increases continously and slowly. When $K < K_2$, the vibration amplitude of the rotor is increasing continuously and slowly with K. When $K = K_2$, the vibration amplitude increases abruptly. At this moment, if the reached value of the amplitude catastrophe exceeds the threshold value, the mechanical system would lose working ability in a short time, even break down suddenly. When $K > K_2$, the vibration amplitude continuously increases again with K.

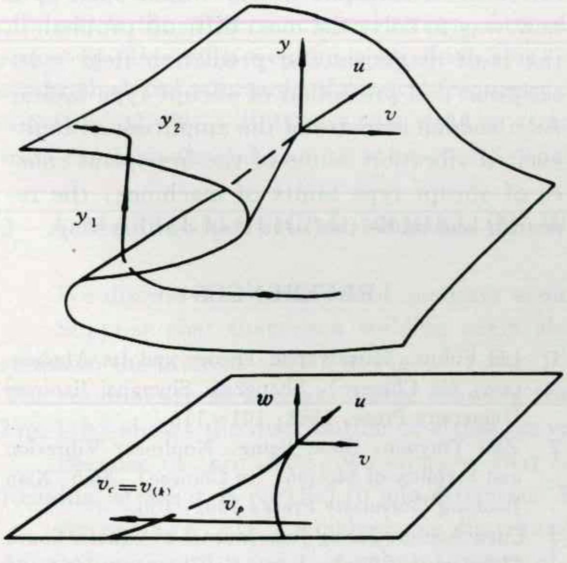

Fig. 2 Amplitude catastrophic manifold and bifurcation set

Although the analyses above aims at single fault excitation, it can be applied to other situations. In fact, for two or more fault excitations with different frequencies, in the resonance domains of the rotor system, the steady-state solution of the vibration equation consists of two parts, namely the solution for free vibration and the solution for forced-vibration, and the frequency-response equation of steady-state free vibration is also cubic equation concerning the square of amplitude. According to the same reasons as the above-mentioned, catastrophe of vibration amplitude exists and can be analysed in the same way.

4 PREDICTION OF AMPLITUDE CATASTROPHE FOR FAULT-EXCITED ROTORS

In operation process of mechanical systems, there are many faults that are gradually aggrevated (for example, unbalance, abrasion, loosening and so on). They result in continuous and monotonous increase of excitation amplitude with time. When certain conditions are satisfied, the catastrophe of abnormal vibration state will occur. It is of great significance to forecast accurately the catastrophe of vibration state in time.

According to the above-mentioned, we are capable of making a good quantitative analysis of the catastrophe of vibration state for a nonlinear mechanical system with catastrophe theory, and providing a way to forecast some abrupt faults. Based on cusp type catastrophe model, a way to forcast the catastrophe of vibration amplitude of the rotor excited by faults is developed, as shown in Fig. 3. In Fig. 3, $\hat{a}$ represents forecasting values, a_t the threshold value of vibration amplitude, v_c the

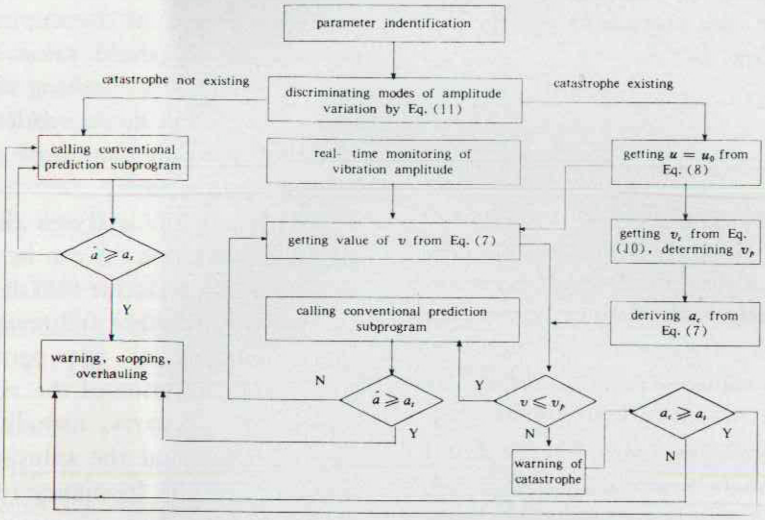

Fig. 3　Programme of predicting amplitude catastrophe

value of normal factor v when catastrophe occurs, a_c the reached value of the amplitude catastrophe, v_p the warning value of normal factor v. Obviously, $v_p > v_c$. Safety margin of forecast is $|v_p - v_c|$. Warning value v_p can be predetermined under concrete conditions such as the rate at which faults are aggrevated and the demand for safety margin etc.

5　CONCLUSIONS

On the basis of nonlinear vibration theory and catastrophe theory, it is explained that continuous and slow increase of the amplitude of fault-excitation may bring about a catastrophe of vibration amplitude of a rotor when certain conditions are satisfied. This abrupt fault is described using cusp type catastrophic model, and the catastrophic condition of vibration amplitude is derived. Futhermore, a quantitative prediction way is presented.

It is necessary to point out that there are just seven types of distinct catastrophes if r (the number of control variables) $\leqslant 4$, and they correspond to respective characteristic polynomials. Because a lot of physical phenomena can be described with polynomials, and the set of all single-variable polynomials is dense in the set of all continuous functions defined in a certain interval, i. e. , a continuous function can be approximated by an appropriate polynomial, catastrophe theory has a good prospect in the field of fault diagnosis and prediction. The authors believe that primary catastrophic models are good mathematical expressions of a good abrupt type faults of machines, even though may be they can also be expressed in other ways[3]. Therefore, application of catastrophe theory would open up a new way to solve the most difficult problem in the fault diagnosis and prediction field — description and prediction of abrupt type faults. As a sudden increase of the amplitude of fault-excited vibration is one of the important causes of abrupt type faults of machines, the research has made the first step on this way.

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