A NEW METHOD FOR SOLUTING TRANSIENT RESPONSE OF GENERALLY VISCOUS DAMPING MULTI-DEGREE SYSTEM①

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ABSTRACT By means of the theory of composite-modality, the superposition principle of the vibration mode of the linear system, and the analytical method of the original coordinate, a mathematical model of transient response to any stimulus for generally viscous damping multi-degree system was established. This method not only solves the problem of the transient response of displacement, but also calculates the transient response of the elastic force or the elastic couple of the system.

Key words damping multi-degree transient response

1 INTRODUCTION

So far, in the study of a damped vibration system, all kinds of damps are simplified as a viscous damp, i. e. damped force increases with increasing velocity. When using analytical method of the modality to solve the problem of the system response, all kinds of damps are further simplified as a ratio viscous damp in order to uncouple the coordinates of the differential movement equations, thereby obtain the parameters of the real-modality. But strictly speaking, the ratio damp could not appear really. In fact, the generally viscous damp is more popular, thus the composite-modality parameters of the system are caused. Of the methods of using the theory of composite-modality to solve the problem of the transient response of generally viscous damping multi-degree system, so far, the method of state-space is most successful. The method of the state-space refers to the method that people change differential movement equations into the form of the state equation, using the method of the calculation of the characteristic value of really general matrix to get the parameters of composite-modality, similar to the method of solving the problem of ratio damped system to uncouple state-equation, finally solve for movement coordinate of the system by conversion.

This paper applies the theory of composite-modality and establishes a mathematical model for solving the transient response of generally viscous damping multi-degree system based on the original coordinate method for solving the transient response of unviscous damped multi-degree system.

The basic theory of the method this paper provided is that by means of the mothod for solving the composite characteristic value and composite characteristic vector of the really general matrix, we can calculate the composite-modality parameters of the vibration system, and using the superposition principle of the vibration mode of the linear system, we can obtain the natural decay vibration of the system, i. e. the common solution of the homogeneously differential equation, and further get the special solution of the nonhomoge-

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neously differential equation, then obtain the common solution of the nonhomogeneous differentiation equation, i. e. the transient response of the system. The character of the method is that we needn't do coordinate-uncoupling and coordinate conversion of the differential equations, needn't ask the factor matrix of the differential equations to have the nature of symmetry. We can solve the problem of differential movement equations and the one of the elastic force or elastic couple as well. The process of solving is simple, and this method can be used widely. The established mathematical model can be a good way for solving transient response of generally linear discretization system.

2 BASIC MATHEMATICS MODEL

Because there is a linear conversion between displacement and elastic force or between angular displacement and elastic couple. So differential movement equation and differential elasticity force equations with generally viscous damping multi-degree system can be expressed as:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{\boldsymbol{x}\}=\{\boldsymbol{Q}(t)\} \quad (1)$$

where as to differential movement equations, $\{\boldsymbol{x}\}$ stands for vector of displacement, $[M]$ for mass matrix, $[C]$ for damp matrix, $[K]$ for matrix of digity, $\{\boldsymbol{Q}(t)\}$ for matrix of interruptions outside. Among these matrixes, $[M]$, $[C]$, and $[K]$ are symmetrical. As to differential elasticity force equations, $\{\boldsymbol{x}\}$ stands for vector of elastic force, $[M]$ for Wide-Sense mass matrix, $[C]$ for Wide-Sense damp matrix, $[K]$ for Wide-Sense matrix of digity, $\{\boldsymbol{Q}(t)\}$ for Wide-Sense vector of interruptions outside. $[C]$ and $[K]$ are unsymmetrical matrices.

Homogeneous differentiation equations that matches eqn. (1) is

$$[M]\{\ddot{\boldsymbol{x}}\}+[C]\{\dot{\boldsymbol{x}}\}+[K]\{\boldsymbol{x}\}=\{0\} \quad (2)$$

Suppose that the form of the solution of natural vibration is

$$\{\boldsymbol{x}\}=\{z\}e^{ut} \quad (3)$$

Substituting eqn. (3) into eqn. (2), we can get

$$(u^2[M]\{z\}+u[C]\{z\}+[K]\{z\})e^{ut}=0 \quad (4)$$

Generally speaking: $e^{ut}\neq 0$, so the expression of the characteristic value problem is

$$(u^2[M]+u[C]+[K])\{\boldsymbol{z}\}=\{0\} \quad (5)$$

as to the $\{\boldsymbol{z}\}$ that does not equal zero, the characteristic equation is

$$|u^2[M]+u[C]+[K]|=0 \quad (6)$$

In eqns. (5) and (6), u stands for characteristic value, $\{z\}$ for characteristic vector. Because $[C]$ and $[K]$ may be unsymmetric matrix, so eqn. (5) belongs to the problem of characteristic value of really general matrix.

As to really general metrix, u and $\{z\}$ are complex numbers and they stand for composite characteristic value and composite characteristic vector, respectively. By means of the program that solves the problem of the characteristic value of really general matrix, we can obtain characteristic value and characteristic vector as follows:

$u=\alpha+\mathrm{i}\beta$ (α and β are real numbers, i is unit of imaginary number)

$\{\boldsymbol{z}\}=\{\boldsymbol{x}\}+\mathrm{i}\{y\}$

$\{\boldsymbol{x}\}$ and $\{\boldsymbol{y}\}$ are real number vector

Suppose that $\bar{u}$ and u are coupled, complex numbers, $\{\bar{\boldsymbol{z}}\}$ and $\{\boldsymbol{z}\}$ are coupled vectors, it can be proved that $\bar{u}$ stands for characteristic value too, and $\{\bar{\boldsymbol{z}}\}$ stands for characteristic vector that matches $\bar{u}$.

3 NUMBERICAL SOLUTION OF EQUATION (1)

3.1 *Solution of Natural Vibration System*

Suppose that the form of the solution of the natural vibration system of homogeneously differential equations, i. e. eqn. (2) is

$$\{\boldsymbol{x}\}=\mathrm{e}^{ut}\{z\}$$

Because u and $\bar{u}$ are both characteristic value, $\{\boldsymbol{z}\}$ and $\{\bar{\boldsymbol{z}}\}$ are characteristic vector, $e^{-ut}\{\bar{\boldsymbol{z}}\}$ is the solution of the equations, i. e. eqn. (2), so the solution of the homogeneously differential equations is

$$\{\boldsymbol{x}\}=C_1e^{ut}\{\boldsymbol{z}\}+C_2e^{-ut}\{\bar{\boldsymbol{z}}\} \quad (7)$$

where C_1 and C_2 stand for complex numbers respectively, and C_1 and C_2 are coupled

According to the superposition principle of the vibration mode[1], we may know that the response of the natural vibration of the system is

$$\{x'\}=\sum_{k=1}^{n}(C_k e^{u_k t}\{z_k\}+\bar{C}_k e^{\bar{u}_k t}\{\bar{z}_k\}) \quad (8)$$

where α_k stands for the rate of decay of the natural vibration; ω_k stands for the fixed frequence with damp. And it is supposed that

$u_k=-\alpha_k+i\omega_k$

$C_k=a_k+ib_k$ (a_k and b_k are real numbers)

$\{z_k\}=\{x_k\}+i\{y_k\}$ ($\{x_k\}$ and $\{y_k\}$ are real vectors)

So eqn. (8) can be changed into

$$\{x'\}=2[X][E][U]\{A\}-2[X][E][S]\{B\}-2[Y][E][S]\{A\}-2[Y][E][U]\{B\} \quad (9)$$

where $[X]=[\{x_1\},\{x_2\},\cdots,\{x_n\}]$;

$[Y]=[\{y_1\},\{y_2\},\cdots,\{y_n\}]$;

$\{A\}=(a_1,a_2,\cdots,a_n)^T$;

$\{B\}=(b_1,b_2,\cdots,b_n)^T$

$$[E]=\begin{bmatrix} e^{-\alpha_1 t} & & & 0 \\ & e^{-\alpha_2 t} & & \\ & & \ddots & \\ 0 & & & e^{-\alpha_n t}\end{bmatrix}$$

$$[S]=\begin{bmatrix} \sin\omega_1 t & & & 0 \\ & \sin\omega_2 t & & \\ & & \ddots & \\ 0 & & & \sin\omega_n t\end{bmatrix}$$

$$[U]=\begin{bmatrix} \cos\omega_1 t & & & 0 \\ & \cos\omega_2 t & & \\ & & \ddots & \\ 0 & & & \cos\omega_n t\end{bmatrix}$$

3.2 *Linearization of Any Stimulus by Cutting*

Any stimulus may be treated as a linear force function according to the period of time. When the period of time is cut into small enough, enough accuracy of the calculation can be reached. In order to simplify calculation, the period of time is cut into equal smaller ones, suppose that the equal smaller period of time equals T_1, during T_1, stimulus is regarded as constant.

According to eqn. (1), suppose that any stimulus matching Wide-Sense coordinate x_i is $Q_i(t)$, as shown in Fig. 1.

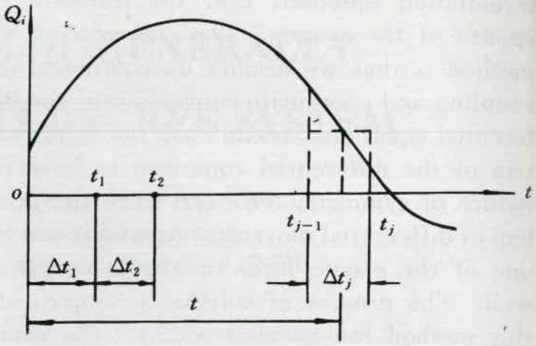

Fig. 1 Any stimulus matching

Now we take one of the equal smaller periods of time as Δt_j, suppose that stimulus force equals $Q_i^{(\Delta t_j)}$, and $Q_i^{\Delta t_j}=Q_i(\bar{t}_j)=V_i=$ Constant. where $\bar{t}_j=(t_j+t_{j-1})/2$, $t_1=t_j-t_{j-1}$

So during Δt_j, the vector of stimulus force is

$$\{Q\}_{\Delta_j}=\begin{Bmatrix} Q_1(\bar{t}_j) \\ Q_2(\bar{t}_j) \\ \cdots \\ Q_n(\bar{t}_j)\end{Bmatrix}_{(\Delta t_j)}=\begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n\end{Bmatrix}_{(\Delta t_j)}=\{V\} \quad (10)$$

3.3 *In Any Period Δt_j, the Special Solution of Nonhomogeneously Differential Equations*

In the period Δt_j, because the special solution vector to which the stimulus is correspondent is $\{x^*\}$, replacing $\{Q(t)\}$ in eqn. (1) with $\{V\}$, we can get

$$\{x^*\}=[K]^{-1}\{V\} \quad (11)$$

3.4 *In Any Period Δt_j, the Common Solution of The Nonhomogencously Differential Equations*

In any period Δt_j, suppose that the transient response vector of the system is

$$\{x\}=(x_1, x_2, x_3, \cdots, x_n)^T$$

From the character of the linear differentiating equation with constant coefficient, we can get $\{x\}=\{x'\}+\{x^*\}$ (12)

Replacing $\{x'\}$ and $\{x^*\}$ in eqn. (12) with eqns. (9) and (11), we can get

$$\{x\}=2[X][E][U]\{A\}-2[X][E][S]\{B\}-2[Y][E][S]\{A\}-2[Y][E][U]\{B\}+[K]^{-1}\{V\} \quad (13)$$

In eqn. (13), $\{A\}$ and $\{B\}$ are decided by initial conditions and special solution $\{x^*\}$ of this period Δt_j. $\{x\}$ and $\{\dot{x}\}$ at the moment t_{j-1} are initial conditions in the period Δt_j.

3.5 *In the Period Δt_j, How to Get the Expressions of the Integral Constant Vector $\{A\}$ and $\{B\}$*

In any period Δt_j, we place original point of time at the moment t_{j-1}, that is to say, when t equlas zero, original point of the time t is moment t_{j-1}.

Suppose that the initial conditions of the period Δt_j are

$$\{x_0\}=(x_{10}, x_{20}, \cdots, x_{n0})^{\mathrm{T}}$$
$$\{\dot{x}_0\}=\{\dot{x}_{10}, \dot{x}_{20}, \cdots, \dot{x}_{n0}\}^{\mathrm{T}}$$

the first derivative of eqn. (13) is

$$\{\dot{x}\}=-2[X]([\alpha][E][U]+[E][L][S])\{A\}+2[X]([\alpha][E][S]-[E][L][U])\{B\}+2[Y]([\alpha][E][S]-[E][L][U])\{A\}+2[Y]([\alpha][E][U]+[E][L][S])\{B\} \quad (14)$$

In eqn. (14)

$$[\alpha]=\begin{bmatrix}\alpha_1 & & & 0\\ & \alpha_2 & & \\ & & \ddots & \\ 0 & & & \alpha_n\end{bmatrix}$$

$$[L]=\begin{bmatrix}\omega_1 & & & 0\\ & \omega_2 & & \\ & & \ddots & \\ 0 & & & \omega_n\end{bmatrix}$$

During this period, when $t=0$, we have $[E]=[I]$; $[U]=[I]$; $[S]=[0]$.

From eqns. (13) and (14), we can get

$$\{x_0\}=2[X]\{A\}-2[Y]\{B\}+[K]^{-1}\{V\} \quad (15)$$

and $\{\dot{x}_0\}=-2[X][\alpha]\{A\}-2[X][L]\{B\}-2[Y][L]\{A\}+2[Y][\alpha]\{B\}$ (16)

From eqns. (15) and (16), we can get the vectors of the integral constant

$$\{B\}=[Q]^{-1}([F]\{G\}-\frac{1}{2}\{\dot{x}_0\}) \quad (17)$$

$$\{A\}=[X]^{-1}([Y]\{B\}+\{G\}) \quad (18)$$

where $\{G\}=\frac{1}{2}(\{x_0\}-[K]^{-1}\{V\})$;

$$[Q]=[X][\alpha][X]^{-1}[Y]+[Y][L][X]^{-1}[Y]+[X][L]-[Y][\alpha];$$
$$[F]=-([X][\alpha]+[Y][L])[X]^{-1}$$

3.6 *Numerical Solution of the Transient Response*

In any period Δt_j, replacing the time variable in eqn. (13) with a step length of time T_1, we can get the response of the moment t_j,

$$\{x\}_{t_j}=2[X][E]_T[U]_T\{A\}-2[X][E]_T[S]_T\{B\}-2[Y][E]_T[S]_T\{A\}-2[Y][E]_T[U]_T\{B\}+[K]^{-1}\{V\} \quad (19)$$

$$\{\dot{x}\}_{t_j}=-2[X]([\alpha][E]_T[U]_T+[E]_T[L][S]_T)\{A\}+2[X]([\alpha][E]_T[S]_T-[E]_T[L][U]_T)\{B\}+2[Y]([\alpha][E]_T[S]_T-[E]_T[L][U]_T)\{A\}+2[Y]([\alpha][E]_T[U]_T+[E]_T[L][S]_T)\{B\} \quad (20)$$

where

$$[E]_T=\begin{bmatrix}e^{-\alpha_1 T_1} & & & 0\\ & e^{-\alpha_2 T_1} & & \\ & & \ddots & \\ 0 & & & e^{-\alpha_n T_1}\end{bmatrix},$$

$$[S]_T=\begin{bmatrix}\sin\omega_1 T_1 & & & 0\\ & \sin\omega_2 T_1 & & \\ & & \ddots & \\ 0 & & & \sin\omega_n T_1\end{bmatrix},$$

$$[U]_T=\begin{bmatrix}\cos\omega_1 T & & & 0\\ & \cos\omega_2 T_1 & & \\ & & \ddots & \\ 0 & & & \cos\omega_n T_1\end{bmatrix}.$$

Obviously, $\{x\}_{t_j}$ and $\{\dot{x}\}_{t_j}$ are the initial conditions of the period Δt_{j+1}. In order of gaps of time, calculating gradually in the way mentioned above, we can obtain the response in the whole time.

4 THEORETICAL TEST OF THE METHOD

Example 1: for the following equation

$$\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\end{Bmatrix}+\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{Bmatrix}$$

$$+\begin{bmatrix}2 & 1 & 0\\ 1 & 3 & 0\\ 0 & 0 & 4\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\\ x_3\end{Bmatrix}=\begin{Bmatrix}2\\ 4\\ 6\end{Bmatrix}\sin t \tag{21}$$

its initial condition is

$\{x_0\}=(-1, -3, -2)^T$;

$\{\dot{x}_0\}=(1, 2, 3)^T$.

The accurately theoretical solution of eqn. (21) is

$$\begin{Bmatrix}x_1\\ x_2\\ x_3\end{Bmatrix}=\begin{Bmatrix}-\cos t+\sin t\\ -3\cos t+2\sin t\\ -2\cos t+3\sin t\end{Bmatrix} \tag{22}$$

Suppose that the stride of time T_1 is $\pi/80$, using the program and calculating $\{x_1, x_2, x_3\}$ at the 120 different points of time, we can get the result of the calculation. Comparing it with the accurately theoretical solution, we can find that the derivation between them ranges from 0 to 0.043%. This prove that the theory is correct.

Example 2: How to get the dynamic response of a tilting machinery of the 20 oxygen turning stove in the zero position and in the starting process with gap.

The mechanical model is shown in Fig. 2. The results of calculation and test are shown in Fig. 3.

The relative deviation of the biggest torsional couple is

$$\left|\frac{M_{2\max}-M_{1\max}}{M_{2\max}}\right|\times 100\% = 2.69\%$$

which is caused mainly by the simplification of the mechanical model and measurement deviation. The calculation basically agrees with the experiment.

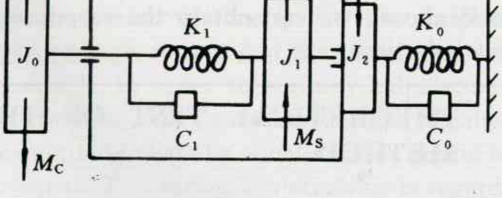

Fig. 2 Mechanical model

J_0 —the whole turning inertia that the hung gear box, motor and main reduction device make while turning around the axis of the ear-axis;

J_1 —all equivalent turning inertia that the rotor of the motor, coupling, and rotative parts of the main reduction device produce around the axis of the ear-axis;

J_2 —all the turning inertia of furnace body, the liquid in the furnace, and the large gear;

K_0 —the equivalent module of rigidity of torsion of the twistproof equipment of the hung gear box;

K_1 —the equivalent module of rigidity of torsion that transmission system of the tilting device and ear-axis produce in the axis of the ear-axis;

C_0 —the equivalent damping coefficient of the twist-proof equipment;

C_1 —the damped coefficient at the ear-axis.

φ_0, φ_1, φ_2 —the absolute displacement of J_0, J_1, J_2;

M_s —the driving couple of the motor;

M_c —the rolling couple that resist the turning of the furnace

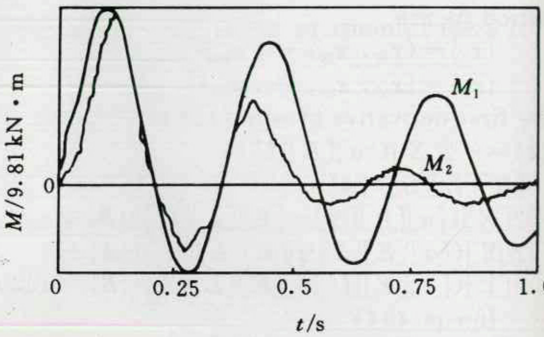

Fig. 3 The results of calculation and test

M_1 —the calculation value of the twist-vibration couple of the ear-axis;

M_2 —the test value of the twist-vibration couple of the ear-axis

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