ROCK FRACTURE PROCESS ZONE AND ITS INFLUENCE ON MODE Ⅱ FRACTURE BEHAVIOR①

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ABSTRACT On the basis of that rock material usually has a larger fracture process zone, a new fracture criterion which is different from that of linear elastic fracture theory was presented. On this basis, the fracture behavior and influencing factors under mode Ⅱ or compressive shear loading were investigated.

Key words fracture process zone shear fracture initiation angle

1 INTRODUCTION

It's well known that rock fracture experimental results existed very apparent non-linear problem. The reason can be attributed to the influence of rock FPZ (fracture process zone) which is analogous to the plastic zone in metals. Up to now a large number of researchers have investigated the FPZ both in theory and experiment. Labuz[1] presented a three processes zone model on the basis of experimental observation and ascertained that the length of FPZ is $r = 0.88(K_{Ic}/\sigma_t)^2$. On the basis of maximum normal stress criterion and non-part strength fracture criterion, Schmidt[2] and Liu[3] concluded that the size of FPZ is $r = (2\pi)^{-1} \cdot (K_{Ic}/\sigma_t)^2$. No matter which criterion, the calculated size of FPZ was about several milimeters to several tens milimeters. With experimental observation and analysis, it is found that the size of FPZ in short rod gabbro specimens was about 10 mm[4]. With the help of laser speckle interferometry[5], it is found that the length of FPZ was 4～6 mm in marble and granite specimens (the width is 40 mm and notch length is 20 mm). Thus the theory and experimental facts have all been proved that the size of FPZ has the same orders of magnitude as specimen size. This is very different from metal fracture test that the ratio of plastic zone size to crack length must be less than 0.02 to enable the specimen to meet the linear elastic conditions[6]. In fact, more and more experiments have showed that rock fracture experimental results sometimes are very different from that determined by LEFM. The major behavior in mode I fracture about this difference is size effect of fracture toughness; and the fracture behavior in mode Ⅱ may be appeared in initial fracture anlge or other factors. In order to connect the experiment result with engineering rockmass closely, the influence of FPZ to crack propagation behavior in mode Ⅱ or compressive shear loading will be studied.

2 THEORETICAL ANALYSIS OF FPZ INFLUENCE ON FRACTURE BEHAVIOR

In LEFM, the value r/a is generaly assumed to near zero, so r/a and its higher rank terms can all be ignored. But in rock materials, as r/a is a large value usually, it must be considered. With Westgard function method, the stress field at crack tips can be written as following equations and shown in Fig. 1.

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where
$$\sigma_{xx} = [f_1(\theta, r) - f_2(\theta, r)]K_I/\sqrt{2\pi r} + [f_3(\theta, r) + f_4(\theta, r)]K_{II}/\sqrt{2\pi r} + \sigma_x - \sigma_y$$
$$\sigma_{yy} = [f_1(\theta, r) + f_2(\theta, r)]K_I/\sqrt{2\pi r} - f_4(\theta, r)K_{II}/\sqrt{2\pi r}$$
$$\tau_{xy} = -f_4(\theta, r)K_I/\sqrt{2\pi r} + [f_1(\theta, r) - f_2(\theta, r)]K_{II}/\sqrt{2\pi r} \quad (1)$$
$$f_1(\theta, r) = \cos\frac{\theta}{2} + \frac{3r}{4a}\cos\frac{\theta}{2} - \frac{1}{4}(\frac{r}{a})^2\cos\frac{\theta}{2} + \frac{3}{32}(\frac{r}{a})^2\cos\frac{3\theta}{2} + \cdots\cdots$$
$$f_2(\theta, r) = \frac{1}{2}\sin\theta\sin\frac{3\theta}{2} - \frac{3r}{8a}\sin\theta\sin\frac{\theta}{2} - \frac{15}{16}(\frac{r}{2a})^2\sin\theta\sin\frac{\theta}{2} + \cdots\cdots$$
$$f_3(\theta, r) = -2\sin\frac{\theta}{2} + \frac{2r}{a}\sin\frac{\theta}{2} - \frac{1}{2}(\frac{r}{a})^2\sin\frac{3\theta}{2} + \cdots\cdots$$
$$f_4(\theta, r) = -\frac{1}{2}\sin\theta\cos\frac{3\theta}{2} + \frac{3r}{8a}\sin\theta\cos\frac{\theta}{2} - \frac{15r^2}{32a^2}\cdot\sin\theta\cos\frac{\theta}{2} + \cdots\cdots$$
$$K_I = \sigma_y\sqrt{\pi a},\ K_{II} = \tau\sqrt{\pi a}$$

When FPZ size r is small, such as $r/a \leqslant 0.1$, then the two rank small terms in the stress field equations can be ignored, and the tangential or shear stress at crack tips can be

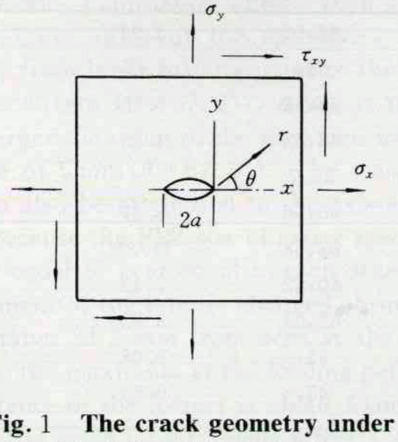

Fig. 1 The crack geometry under biaxial and shear load

writen as follows:
$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}[\frac{1}{2}(1+\cos\theta) + \frac{3}{4}(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2})\frac{r}{a}] - \frac{K_{II}}{\sqrt{2\pi r}}\cdot \sin\theta\{\frac{3}{2}\cos\frac{\theta}{2} + [2\cos\frac{3\theta}{2} + \frac{3}{8}(2\sin\frac{\theta}{2}\sin\theta\cos 2\theta + \cos\frac{\theta}{2})]\frac{r}{a}\} + (\sigma_x - \sigma_y)\sin^2\theta \quad (2)$$
$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\sin\theta(\frac{1}{2}\cos\frac{\theta}{2} - \frac{3r}{8a}\cos\frac{3\theta}{2}) + \frac{K_{II}}{\sqrt{2\pi r}}[\frac{1}{2}(3\cos\theta - 1) + \frac{r}{a}(\frac{3}{4}\cos\frac{5\theta}{2} - \frac{3}{8}\sin\theta\sin\frac{3\theta}{2})] + (\sigma_x - \sigma_y)\sin\theta\cos\theta \quad (3)$$

According to the maximum tangential stress criterion, a crack will propagate along the direction in which $\sigma_{\theta\theta}$ is maximum at a critical value σ_t. Based on this criterion, the fracture initiation angle can be determined by following equations.
$$\partial\sigma_{\theta\theta}/\partial\theta = 0,\ \partial^2\sigma_{\theta\theta}/\partial\theta^2 < 0 \quad (4)$$

In the process of rock fracture toughness measurement, the fracture toughness which determined by LEFM usually only include the singular stress ($r/a \to 0$) at crack tips. So the measured fracture toughness is less than the real value, and the measured results of rock fracture toughness usually need non-linear correction.

In pure mode Ⅱ loading conditions, with the formula (2) and (4), it can be evaluated that when $r/a = 0.01$, initial angle $\theta = 69.5°$; when $r/a = 0.1$, $\theta = 59.8°$ and when $r/a = 0.2$, $\theta = 53.6°$. So we can conclude that the existence of FPZ will cause the decrease of mode Ⅱ crack initiation angle.

Considering another condition that $\sigma_{\theta\theta} < \sigma_t$ but the maximun shear stress $\tau_{r\theta}$ has reached its critical value, then the crack will propagate under mode Ⅱ and the initiation direction will be determined by following equations
$$\partial\tau_{r\theta}/\partial\theta = 0,\ \partial^2\tau_{r\theta}/\partial\theta^2 < 0 \quad (5)$$

3 INFLUENCE OF FPZ ON MODE Ⅱ CRCACK PROPAGATION BEHAVIOR

3.1 *Experimental Method and Results*

Four points bend shear specimen is most widely used in mode Ⅱ or mixed mode fracture test. In this paper, the marble specimen size is 20 cm×6 cm×1.5 cm, 16 cm×6 cm×1.5 cm and 22 cm×10 cm×1.5 cm. In the upper and lower part, double notch or single notch is cut. The width of the notch is 0.6 mm. The loading apparatus see Fig. 2. The loading rate was 1.25 μm/s and some mode Ⅱ fracture test results are listed on Table 1 and Table 2.

3.2 *Experimental Results Analysis*

As listed in Table 1, it can be concluded that large specimen (width equal to 10 cm) has a larger initiation angle than that of small specimens (width equal to 6.2 cm). Therefore, mode Ⅱ crack initiation angle has apparent size effect. The reason for this can be attributed to the existence of FPZ. Acording to maximum normal stress criterion, the size of FPZ at mode Ⅱ crack tips was proportational to the value of $(K_{\mathrm{Ic}}/\sigma_t)^2$, so the FPZ size of

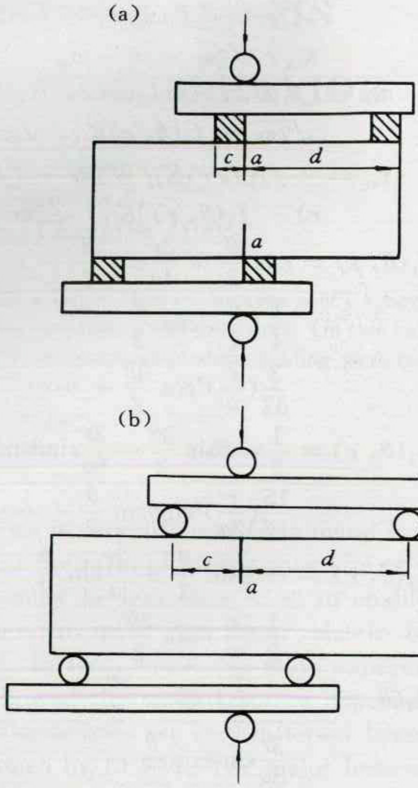

Fig. 2 Loading configuration for shear test
(a)—double notch specimen;
(b)—single notch specimen

Table 1 Mode Ⅱ fracture test results for double notch marble specimen

Specimen No.	Double notch depth a /mm	Specimen width W /mm	$\alpha = a/W$	initiation angpe θ /(°)	failure shear load Q /kN
C_1	19.3×2	62.4	0.31×2	64,53	3.94
C_2	19.0×2	62.0	0.31×2	64,	4.05
C_3	17.0×2	60.0	0.28×2	33,52	5.53
C_4	18.5×2	61.7	0.30×2	42,58	4.23
mean			0.30×2	52.7	
E_1	32.0×2	100.0	0.32×2	60,56	5.07
E_2	36.5×2	101.3	0.26×2	58,68	11.25
E_3	32.6×2	101.0	0.32×2	60,62	7.43
E_4	34.0×2	100.0	0.34×2	62,57	3.83
E_5	33.2×2	100.0	0.33×2	65,	6.08
E_6	33.5×2	102.8	0.33×2	43,	6.53
mean			0.32×2	59.1	

Table 2 Mode Ⅱ fracture test results for single notch marble specimen

Specimen No.	Single notch depth a /mm	Specimen width W /mm	$\alpha = a/W$	initation angle θ /(°)	distance from inner load point to notch c /mm	distance from outer load point to notch d /mm	initiation fracture shear laod Q /kN	failure shear laod Q_m /kN
B_1	29.1	61.3	0.48	64.0	30	93	1.42	1.64
B_2	30.8	63.0	0.49	72.0	30	93	1.40	1.69
B_3	30.5	63.0	0.49	66.0	30	93	1.54	1.57
B_4	30.4	62.4	0.49	65.0	30	93	1.46	1.73
B_5	30.4	62.4	0.49	76.0	30	93	1.56	1.61
mean			0.49	67.8			1/48	1.65
H_1	11.2	62.0	0.18	71.0	5.5	75	2.63	5.65
H_2	11.5	62.1	0.19	66.0	5.5	75	2.59	5.47
H_3	12.0	62.8	0.19	68.0	5.5	75	2.80	5.61
H_4	12.0	52.6	0.19	70.0	5.5	75	2.37	5.61
H_5	12.5	61.7	0.20	65.0	5.5	75	2.72	6.17
mean			0.19	68.0			2.62	5.70
K_1	29.5	62.0	0.48	65.0	5.5	75	2.59	3.78
K_2	28.4	61.3	0.46	62.0	5.5	75	2.59	3.02
K_3	29.3	61.3	0.46	62.0	5.5	75	2.59	3.02
K_4	31.3	62.0	0.50	57.5	5.5	75	1.73	2.59
K_5	30.5	61.8	0.49	54.5	5.5	75	2.37	2.97
mean			0.48	57.9			2.44	3.23

the two type specimens is the same, and the values of r/a and $r/(W-a)$ in large specimen are larger than those in small specimen. As analyzed at above section, it can explain reasonably why larger apecimen has a larger initiation angle than small specimen.

As listed in Talbe 2, it can be concluded that different loading locations may cause different mode Ⅱ initiation angle, such as that the initiation anlge of the specimen, whose distance from inner loading point to the notch plane is 30 mm ($\theta = 67.8°$), is about ten degrees larger than that of the specimen with the distance of 5 mm ($\theta = 57.9°$). The reason for this can also be attributed to the existence of FPZ. Because the FPZ size of above specimens may be equal or near equal to each other, but the moment of the later is changed abruptly in the distance of 5 mm from zero at the notch plane to the maximum at the loading point and the distance of the former is about 30 mm. So the later is acted larger tensile stress (perpendicular to notch plane) in the FPZ than the former, and resulted the initiation angle of the later less than the former.

In the Table 2, it can be also found that the mode Ⅱ initiation angle is also different when loading location is the same, but the length of notch is different, such as the initiation angle whose $\alpha = 0.19 (\theta = 68°)$ is much larger than that of $\alpha = 0.5 (\theta = 57.9°)$. It can be concluded that mode Ⅱ initiation angle is decreased with the increase of notch length. To analyze this behavior, complicated numerical analysis must be made for this finit length specimen.

4 FPZ AND COMPRESSIVE SHEAR FRACTURE

Generaly, the rockmass which contains cracks is acted by triaxial stress. A large number of engineering practices have showed that the compressive stress which paralell to the crack has a large controlling function on the crack propagation behavior, but how much the influence caused is still unsolved. The reason for this is that the strength criteria or stress-strain analysis method all can't be used, because the existence of crack causes high stress concentration (acording to LEFM, stress singularity is existed). With LEFM to analyze the problem also can't solve the problem, because the fracture criterion according to LEFM can only reflect the action of the stress which has contribution to fracture toughness, but the influence to the fracture behavior which caused by compressive stress paralell to crack (has no contribution to fracutre toughness) can't be reflected. Moreover, according to LEFM, all mode crack will propagate under pure mode Ⅰ. This is certainly not accord. with the shear fracture behavior in rockmass engineering.

4.1 *Theoretical Analysis*

As rockmass is generaly under triaxial compression, the cracks are generaly closed. According to photoelasticity experiment[7], the stress intensity factor $K_{\mathrm{I}} = 0$ for completly closed crack, and the tangential stress or shear stress around fracture process zone has the following value.

$$\sigma_{\theta\theta} = \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\sin\theta \cdot \{\frac{3}{2}\cos\frac{\theta}{2} + [2\cos\frac{3\theta}{2} + \frac{3}{8}(2\sin\frac{\theta}{2}\sin\theta\cos 2\theta + \cos\frac{\theta}{2})]\frac{r}{a}\} + (\sigma_x - \sigma_y)\sin^2\theta$$

$$\tau_{r\theta} = \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}[\frac{1}{2}(3\cos\theta - 1) + (\frac{3}{4}\cos\frac{5\theta}{2} - \frac{3}{8}\sin\theta\sin\frac{3\theta}{2})\frac{r}{a}] + (\sigma_y - \sigma_x)\sin\theta\cos\theta$$

where $K_{\mathrm{II}} = (\tau - \mu\sigma_y)\sqrt{\pi a}$, μ is the frictional factor of crack surface

In order to analyze clearly, the maximum stress value of $\tau_{r\theta}$ and $\sigma_{\theta\theta}$ with different r/a values are listed in Table 3 (to make things simple, the θ which determined by pure mode Ⅱ conditions is used).

Table 3 Stress value for different FPZ size

r/a	τ_θ	$\sigma_{\theta\theta}$
0.01	$10(\tau - \mu\sigma_y)$	$11.3(\tau - \mu\sigma_y) + 0.76(\sigma_y - \sigma_x)$
0.1	$2.4(\tau - \mu\sigma_y)$	$2.5(\tau - \mu\sigma_y) + 0.62(\sigma_y - \sigma_x)$
0.2	$1.82(\tau - \mu\sigma_y)$	$1.8(\tau - \mu\sigma_y) + 0.52(\sigma_y - \sigma_x)$

From Table 3, we can find that the compressive stress parallel to crack has no influence $\tau_{r\theta}$, but can inhibit tensile propagation of the crack. This inhibitory action is strengthened with the increase of r/a. But when $\sigma_y \leqslant \sigma_x$, this action will not cause a great difference between $\tau_{r\theta}$ and $\sigma_{\theta\theta}$, so no matter how long the FPZ is, only if the shear strength is much larger than the tensile strength of rocks, the crack propagtion behavior will always be mode Ⅰ. But when σ_x is much larger than σ_y and r/a reached a certain values, then the maximum tangential stress $\sigma_{\theta\theta}$ will be much smaller than the maximum shear stress $\tau_{r\theta}$. At this conditions, shear fracture will happen if the stress $\tau_{r\theta}$ reached its critical value firstly. Now we can conclude that tensile crack propagation will not occur and shear fracture will take place under the action of both large FPZ and large compressive stress prallel to cracks.

4.2 *Main Experimental Evidnece*

A large number of mode Ⅱ or compressive shear fracture tests showed that mode Ⅱ or mixed mode crack will usually propagate in pure mode Ⅰ in the direction of a large angle to the crack plane. This is accorded with LEFM. So some researchers declared that true mode Ⅱ fracture will not happen. The marble compressive shear fracture test with the compressive model plate shear method shows that true mode Ⅱ fracture is existed under the compressive shear loading conditions[8] (the model plate angle is larger than 55°) and the tensile crack has not appeared. But when the planar angle is less than 55°, the

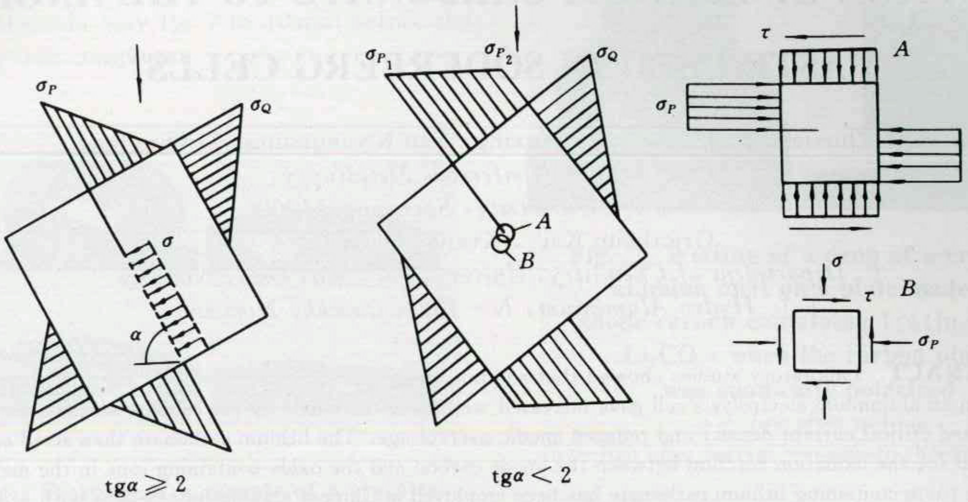

Fig. 3 Stress distribution for compressive model planar shear specimen

shear fracture will not happen. The reason is that, when the planar angle is larger than 55°, the strss σ_p which acted on the specimen at a little distance from crack surface is much larger than the stress σ which perpendicular to crack surface (it's derived that the stress ratio $\sigma_p/\sigma = 4tg\alpha + \sqrt{4tg^2\alpha - 2tg\alpha + 4}$).

Because the large stress σ_p will inhibit the mode I crack propagation greatly, the mode I crack propagation is avoided and mode Ⅱ fracture is promoted.

5 CONCLUSIONS

(1) The FPZ size of rock materials has the same orders of magnitude as specimen size. Most of rock specimens can't be analyzed the fracture behavior by LEFM theory directory.

(2) The crack will propagate in the direction of maximum tangential stress or maximum shear stress around its fracture process zone. This is determined by loading conditions and rock characters.

(3) The existence of FPZ will cause the size effect of mode Ⅱ crack initial fracture direction, and this direction is also influenced greatly by loading position and crack length.

(4) The action of high compressive stress parallel to crack surface and the existence of FPZ will cause rock fracture from mode I to true mode Ⅱ.

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