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SENSITIVITY AND SEPARABILITY OF DEFORMATION MODELS WITH REGARD TO PRIOR INFORMATION①

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ABSTRACT Based on Bayesian hypothesis test, the method of making use of the prior information to improve the sensitivity and separability of deformation models was studied. The formulae for computing the sensitivity and separability with regard to prior information have been derived, and the relationship between the sensitivity and separability was also studied. The measurement of the sensitivity and the separability, which is very useful for the design of the deformation network, has been established for the situation with regard to prior information. The results show that the sensitivity and the separability of deformation model will be largely improved if reasonable prior information is used. Finally an example is given to show the improvement.

Key words sensitivity separability prior information Bayesian hypothesis test

1 INTRODUCTION

The concept of sensitivity of monitoring networks was first introduced by Pelzer in 1972[1]. Zheng and Zhu in 1986 established the sensitivity criteria for deformation networks[2]. Lu in 1987 studied the sensitivity of different deformation models[3]. The separability of models was first discussed by Forstner[4]. Lu applied his approach in deformation surveying[5]. Chen and Chrzanowski made a further study on it[1]. All these studies are based on the classical statistics, and only the information contained in the observations can be used.

In practical situations, some information about the state of the deformation body is usually known, that is, the prior information based on the known geological and mechanical condition. In deformation analysis, we usually hope to use as possible as much the prior information. In this paper, the sensitivity and the separability of deformation models with regard to prior information will be studied. The aim is trying to make use of the prior information in the design of deformation networks. All the work is based on the work of Lu in 1987[3, 5], Chen and Chrzanowski in 1994[4] and Zhu in 1995[6].

2 SENSITIVITY WITH REGARD TO PRIOR INFORMATION

Let $\boldsymbol{d}$ be the displacement vector between the two epoches of a monitoring network with weight matrix $\boldsymbol{p}_d$, and the deformation model be $E(\boldsymbol{d}) = \boldsymbol{Bc}$, where $\boldsymbol{B}$ is a coefficient matrix, whose elements are functions of position and time, $\boldsymbol{c}$ the deformation parameters to be determined. In deformation analysis or deformation hypothesis test, the null hypothesis and the alternative hypothesis usually are

H_o: $\boldsymbol{Bc} = 0$

H_1: $\boldsymbol{Bc} \neq 0$

Respectively, the unknown parameter $\boldsymbol{c}$ can be estimated using the least squares technique from

$$\boldsymbol{d} + \boldsymbol{v} = \boldsymbol{Bc} \tag{1}$$

that is

$$\boldsymbol{c} = (\boldsymbol{B}'\boldsymbol{p}_d\boldsymbol{B})^{-1}\boldsymbol{B}'\boldsymbol{p}_d\boldsymbol{d} \tag{2}$$

and the statistic is

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$$T = \frac{c' Q_c^{-1} c}{\sigma_0^2 h} \sim F(h, f, \delta^2) \quad (3)$$

where σ_0 is the estimate of the variance factor determined by the two epochs, $h = rk(Q_c)$, f the freedom, δ^2 the noncentrering parameter. On the basis of the significance level α, the critical value T_α can be found, and we can decide which hypothesis is tenant on the following formula:

$$T \leqslant T_\alpha \quad (4)$$

let c be expressed as

$$\boldsymbol{c} = a\boldsymbol{g} \quad (5)$$

where a is a scalar and $\boldsymbol{g}$ is a unit vector. From Eqns. (3) and (4), we usually can get the sensitivity of the deformation model (see Ref. [3]):

$$a = \sigma_0 \sqrt{\frac{\delta_0}{\lambda_{\min}(\boldsymbol{M})}} \quad (6)$$

where $\boldsymbol{M} = \boldsymbol{B}' \boldsymbol{p}_d \boldsymbol{B}$, $\lambda_{\min}$ is the minimum eigenvalue of the matrix $\boldsymbol{M}$, δ_0 the boundary value of δ and is a function of significance level α, power of the test β and degree of freedom f, and can be expressed as

$$\delta_0 = \delta(\alpha, \beta, f, \infty) = \delta(\alpha_0, \beta, 1, \infty)$$

or

$$\delta_0 = \delta(T_\alpha, \beta, f, \infty) = \delta(T_{\alpha 0}, \beta, 1, \infty) \quad (7)$$

where α_0 is the significance level corresponding to degree of freedom 1 (see Ref. [1])

If the single point movement is considered, the statistic can be adopted (see Ref. [1]).

$$T = \frac{d'_i Q_{d_i}^{-1} d_i}{\sigma_0^2 h} \sim F(h, f, \delta^2) \quad (8)$$

where $\boldsymbol{d}_i$ denotes the displacement of point i. $\boldsymbol{Q}_{d_i}$ the variance matrix of $\boldsymbol{d}_i$. For this model, Zheng and Zhu in 1986[2] gave out another measure of the sensitivty

$$a_z = \sigma_0 \sqrt{\frac{\delta_0}{\lambda_{\min}(\boldsymbol{Q}_{d_i}^{-1})}} \quad (9)$$

The value of a_z in Eqn. (9) is usually different from that of a in Eqn. (6).

If prior information is considered, the test can be showed by (see Ref. [6])

$$T = \frac{c' Q_c^{-1} c}{\sigma_0^2 h} \leqslant T_k \quad (10)$$

For the case that the variance factor is known, the value of T_k is

$$T_k = (1 + 2\alpha_1)\ln(1 + \frac{1}{2\alpha_1}) + \frac{2(1 + 2\alpha_1)}{m}\ln(\frac{\pi}{1 - \pi}) \quad (11)$$

For the case that the value of the variance factor is unknown, the value of T_k is

$$T_k = \frac{kn}{(1 - k)m} \quad (12)$$

$$k = (1 + 2\alpha_1)[1 - (1 + \frac{1}{2\alpha_1})^{-\frac{m}{n+m}}] \quad (13)$$

α_1 can be found by following equations

$$(1 + 2\alpha_1)\ln(1 + \frac{1}{2\alpha_1}) = F_\alpha(h, \infty) \quad (14)$$

$$\frac{kn}{(1 + k)h} = F_\alpha(h, f) \quad (15)$$

α_1 also can be found approximately by

$$\alpha_1 = \frac{\frac{m}{36}}{1 + \frac{12}{n} + (1 - \frac{2m}{3})\exp(-\frac{m}{3}) + \frac{3m}{2n} + \frac{1}{3m}} \quad (16)$$

Similar to Eqn. (7), the boundary value δ_{B_0} of the noncentrering parameter δ in the test Eqn. (8) will be

$$\delta_{B_0} = \delta(T_k, \beta, f, \infty) \quad (17)$$

The minimum detectable deformation, that is, the sensitivity of the model should be

$$a_B = \sigma_0 \sqrt{\frac{\delta_{B_0}}{\lambda_{\min}(\boldsymbol{M})}} \quad (18)$$

If δ_0 or a is known, δ_{B_0} or a_B can be found approximately by

$$\delta_{B_0} = \delta_0 + (T_k - T_\alpha)/k_1 \quad (19)$$

where

$$k_1 = [\frac{-0.8416}{\sqrt{\delta_0 + 0.5}} + 3 - 2(\frac{T_\alpha}{1 + \delta_0})^{1/3}] \times (\frac{T_\alpha}{1 + \delta_0})^{2/3}$$

$$a_B = \sigma^2 \sqrt{\frac{\delta_{B_0}}{\lambda_{\min}(\boldsymbol{M})}} = a\sqrt{\frac{\delta_{B_0}}{\delta_0}} \quad (20)$$

According to Zhu in 1995[6], if the prior inform-ation is considered, the critical value T_k will be smaller than that of T_α. This means that the value of δ_{B_0} will be smaller than that of δ_0 and the value of a_B will be smaller than a. The sensitivity with regard of prior information will be better than that without regard of the information.

3 MODEL SEPARATION WITH REGARD PRIOR INFORMATION

Assume that there exist two possible models $\boldsymbol{B}_1\boldsymbol{c}_1$ and $\boldsymbol{B}_2\boldsymbol{c}_2$, the prior possibility of the model $\boldsymbol{B}_1\boldsymbol{c}_1$ occurring is π and that of $\boldsymbol{B}_2\boldsymbol{c}_2$ is $1-\pi$. According to Chen and Chrzawoski in 1994[1], the null hypothesis and alternative hypothesis can be expressed as

$$H_0:\ E(\boldsymbol{d}) = \boldsymbol{B}_1\boldsymbol{c}_1$$

$$H_1:\ E(\boldsymbol{d}) = \boldsymbol{B}_2\boldsymbol{c}_2$$

Where the unknown coefficient $\boldsymbol{c}_1$ can be estimated by least squares technique:

$$\boldsymbol{d} + \boldsymbol{v} = \boldsymbol{B}_1\boldsymbol{c}_1 \qquad \boldsymbol{Pd}$$

The appropriateness of the deformation model is tested by

$$T = \frac{\boldsymbol{v}'\boldsymbol{p}_d\boldsymbol{v}}{r\sigma_0^2} \tag{21}$$

where r, the degree of freedom of $\boldsymbol{v}$, is calculated by $r = rk(\boldsymbol{p}_d) - \dim(\boldsymbol{c}_1)$. The separability of $\boldsymbol{B}_1\boldsymbol{c}_1$ from $\boldsymbol{B}_2\boldsymbol{c}_2$ can be expressed as (see Ref. [1])

$$a_{21} = \sigma_0\sqrt{\frac{\delta_0}{\lambda_{\min}(\boldsymbol{M}_{21})}} \tag{22}$$

where

$$\boldsymbol{M}_{21} = \boldsymbol{B}'_2\boldsymbol{p}_d\boldsymbol{B}_2 - \boldsymbol{B}'_2\boldsymbol{p}_d\boldsymbol{B}_1(\boldsymbol{B}'_1\boldsymbol{p}_d\boldsymbol{B}_1)^{-1}\boldsymbol{B}'_1\boldsymbol{p}_d\boldsymbol{B}_2$$

$$\delta = \frac{\boldsymbol{c}'_2\boldsymbol{M}_{21}\boldsymbol{c}_2}{\sigma_0^2} \tag{23}$$

In eqn. (22), δ_0 is the boundary value of δ. Similarly, one can get

$$a_{12} = \sigma_0\sqrt{\frac{\delta_0}{\lambda_{\min}(\boldsymbol{M}_{12})}} \tag{24}$$

where

$$\boldsymbol{M}_{12} = \boldsymbol{B}'_1\boldsymbol{p}_d\boldsymbol{B}_1 - \boldsymbol{B}'_1\boldsymbol{p}_d\boldsymbol{B}_2(\boldsymbol{B}'_2\boldsymbol{p}_d\boldsymbol{B}_2)^{-1}\boldsymbol{B}'_2\boldsymbol{p}_d\boldsymbol{B}_1$$

If the prior information is considered, the test can be shown by

$$T = \frac{\boldsymbol{v}'\boldsymbol{p}_d\boldsymbol{v}}{\sigma^2 f} \leqslant T_B \tag{25}$$

The separability of $\boldsymbol{B}_2\boldsymbol{c}_2$ from $\boldsymbol{B}_1\boldsymbol{c}_1$ can be expressed as

$$a_{B_{21}} = \sigma_0\sqrt{\frac{\delta_{B_{21}}}{\lambda_{\min}(\boldsymbol{M}_{21})}} \tag{26}$$

Similarly, the separability of B_1c_1 from B_2c_2 will be

$$a_{B_{12}} = \sigma_0\sqrt{\frac{\delta_{B_{12}}}{\lambda_{\min}(\boldsymbol{M}_{12})}} \tag{27}$$

where

$$\delta_{B_{21}} = \delta(T_{k_{21}}, \beta, f, \infty) = \delta_0 + (T_{k_{21}} - T_\alpha)/k_1$$

$$\delta_{B_{12}} = \delta(T_{k_{12}}, \beta, f, \infty) = \delta_0 + (T_{k_{12}} - T_\alpha)/k_1$$

When the null hypothesis is $E(\boldsymbol{d}) = \boldsymbol{B}_1\boldsymbol{c}_1$, the critical value of the Bayesian hypothesis test is $T_{k_{12}}$. When the null hypothesis is $E(\boldsymbol{d}) = \boldsymbol{B}_2\boldsymbol{c}_2$, the corresponding critical value is $T_{k_{12}}$. If $\pi \neq 0.5$, the value of $T_{k_{21}}$ is different from that of $T_{k_{12}}$. This means that the value of a_{B21} is different from that of $a_{B_{12}}$. Usually, if $\pi > 0.5$, the value of $a_{B_{21}}$ is larger than that of a_{B12}, that is, the separability of $\boldsymbol{B}_1\boldsymbol{c}_1$ from $\boldsymbol{B}_2\boldsymbol{c}_2$ is larger than that of $\boldsymbol{B}_2\boldsymbol{c}_2$ from $\boldsymbol{B}_1\boldsymbol{c}_1$.

4 EXAMPLE

Fig. 1 shows the monitoring network of a dam taken from Zhu in 1995[6]. The network consists of eight points. Points 2, 3 and 4 lie on deformation bodies. On the basis of other knowledge from geology, mechanics (see Ref. [6]), the value of the prior parameter for these points can be taken to be $\pi_0 = 0.3$. For other points, the value of the prior parameter can be taken to be $\pi_0 = 0.5$. The network is an angular network. The variance of the direction is $\sigma^2 = 0.7^2$. In this example, the separability of single point movement of the points 2, 3 and 4 *versus* every other points is computed. For comparison , the separability of the points 1, 5, 6, 7 and 8 is also

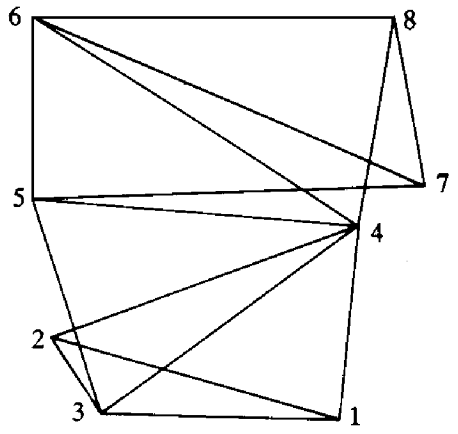

Fig. 1 A monitoring network

computed.

Selecting $\alpha = 0.05$, $\beta = 0.20$, one can obtain the boundary $\delta_0 = 9.451$. The results are shown in Table 1.

From Table 1 one can know:

Table 1 Sensitivity and separability of single point movement (mm)

Point	a_z	a_B	a_{12}	a_{B12}
1	5.15	5.15	5.15	5.15
2	7.63	6.51	7.63	6.51
3	6.82	5.82	6.82	5.82
4	6.28	5.36	6.28	5.36
5	4.63	4.63	4.63	4.63
6	3.19	3.19	3.19	3.19
7	6.12	6.12	6.12	6.12
8	4.01	4.01	4.01	4.01

Note: the values of a_z are determined by Eqn. (9), and those of a_{12} are determined by Eqn. (24)[1].

(1) If reasonable prior information is used, the sensibility will be improved

(2) With prior information, the separability of one model from another also can be improved.

(3) For the single point movement model, the separability supposed by Chen and Chrzawoski in 1994[1] is equal to the sensitivity proposed by Zheng and Zhu in 1986[2]. If only single point movement model is considered, the value of Eqn. (9) can be taken as the measure of the sensitivity and the separability in the design of a monitoring network. With regard of prior information, Eqn. (20) should be taken as a measure of the sensitivity and the separability. For other deformation model, Eqns. (22) and (24) can be taken as a measure of the sensitivity and separability. If prior information is used, Eqn. (26) should be taken as the measure of the sensitivity and the separability.

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