IDENTIFICATION OF STIFFNESS COEFFICIENTS OF LAMINATED COMPOSITE PLATES①

Wang Xiaochun, Shen Xinpu†, Masuda Yuichiro††, Xu Bingye†
North China University of Technology, Beijing 100041
† *Department of Mechanical Engineering, Tsinghua University, Beijing 100084*
†† *Department of Engineering, Ibaraki University, Hitachi City 316, Japan*

ABSTRACT An inverse method was proposed for the identification of the stiffness coefficients of laminated composite plates. The weighted least square iterative computational equations were given and differential instead of partial derivative was used to compute sensitivity coefficients. The convergences expressed by the iterative computations were discussed, and inverse computation was carried out for 3 laminated composite plates.
Key words inverse analysis composite laminated plate stiffness coefficient

1 INTRODUCTION

In the analyses of mechanical behavior of laminated composite plates, three kinds of macromechanical parameters, A_{ij}, B_{ij} and D_{ij} are used, and they represent the extensional, coupling and bending stiffness coefficients, respectively. When the pattern of layers, the thickness of each layer and the mechanical properties of the materials are known, the three coefficients can be calculated. However, owing to the influence produced by the various factors in the process, the stiffness coefficients calculated in terms of the properties of materials of each layer may not be in accordance with practical conditions. So, it is necessary to develop a new method to measure the stiffness coefficients directly. Inverse problem is one of the main tendencies of computational mechanical study in recent years[1-7]. The study of objects, which cannot be directly observed or actually measured, like the properties of materials, interior defects of structures and deep-seated geological structures, should be carried out by means of the inverse method. In this paper, the inverse method was applied to the identification of the stiffness coefficients of laminated composite plates, and an inverse computation was obtained by measuring displacements. In this method, the basic equation used was the weighted least square iterative equation. For the sake of adaptability, no other constraint conditions were added except differential in stead of partial derivative. Therefore, it can also be used in the identification of other mechanical property parameters.

2 NUMERICAL ITERATIVE EQUATION OF THE INVERSE ANALYSIS

Supposing a given problem can be directly calculated by the finite element method, if the parameters of the material are given, at the same time, by experiment, the displacement values of some points of the model can be measured. Let $\boldsymbol{S}^k$ be the unknown parameter vector of the material when making the kth iterative computations, $\boldsymbol{u}^k$, corresponding to $\boldsymbol{S}^k$, be the displacement vector calculated by direct analysis of the finite element method, $\overline{\boldsymbol{u}}$ be the measured values of displacement.

In order to establish the recursion relation between material parameters $\boldsymbol{S}^k$ and $\boldsymbol{S}^{k-1}$, in the neighborhood of $\boldsymbol{u}^k$, $u(S)$ is expanded according to Taylor Series. For the linear elasticity of

① Received Oct. 29, 1996; accepted Feb. 18, 1997

minor deformation, minor quantities of the second order are neglected.

$$\boldsymbol{u}^k = \boldsymbol{u}^{k-1} + \frac{\partial \boldsymbol{u}^k}{\partial \boldsymbol{S}^k}(\boldsymbol{S}^k - \boldsymbol{S}^{k-1}) \tag{1}$$

Let $$\frac{\partial \boldsymbol{u}^k}{\partial \boldsymbol{S}^k} = [C]_{m \times n} \tag{2}$$

it can be deduced

$$\boldsymbol{u}^k = \boldsymbol{u}^{k-1} + [C]\boldsymbol{S}^k - \boldsymbol{S}^{k-1} \tag{3}$$

where m is the number of the measured points of displacement, n, the number of material parameters. In the light of principle of the least square method, the objective function J can be set up

$$J = \{\overline{\boldsymbol{u}} - \boldsymbol{u}^k\}^{\mathrm{T}}\{\overline{\boldsymbol{u}} - \boldsymbol{u}^k\} \tag{4}$$

From the minimizing condition of the objective function,

$$\frac{\partial J}{\partial \boldsymbol{S}^k} = 0 \tag{5}$$

it can be deduced

$$\boldsymbol{S}^k = \boldsymbol{S}^{k-1} + [[C]^{\mathrm{T}}[C]]^{-1}[C]^{\mathrm{T}}\{\overline{\boldsymbol{u}} - \boldsymbol{u}^{k-1}\} \tag{6}$$

This is the least square iterative computational equation.

In order to take the convergence caused by the errors of each measured point of displacement into account, an error weighted function $[W]$ is introduced to establish a new objective function J

$$J = \{\overline{\boldsymbol{u}} - \boldsymbol{u}^k\}^{\mathrm{T}}[W]\{\overline{\boldsymbol{u}} - \boldsymbol{u}^k\} \tag{7}$$

From the minimizing condition of the objective function J, it can be deduced

$$\boldsymbol{S}^k = \boldsymbol{S}^{k-1} + [[C]^{\mathrm{T}}[W] \cdot [C]]^{-1}[C]^{\mathrm{T}}[W]\{\overline{\boldsymbol{u}} - \boldsymbol{u}^{k-1}\} \tag{8}$$

where $[W]$ is the matrix of the weighted coefficients, and

$$[W] = \mathrm{diag}[1/\sigma_j^2] \tag{9}$$

where σ_j^2 is the standard deviation of the error of displacements at the jth measured point in each iterative computation,

$$\sigma_j^2 = [e_j - \sum_{i=1}^{m} e_i/m]^2 \tag{10}$$

$$e_j = \overline{u}_j - u_j^k \tag{11}$$

From the above, it is known that when computing the matrix of weighted coefficients $[W]$, it is not necessary to introduce a new variable. And, if after being weighted, convergence cannot be achieved, let $\sigma_j^2 = 1$. In this case $[W]$ will degenerate to a unit matrix, and eq. (6) and eq. (8) are equivalent.

Eq. (8) is iterative computation equation obtained from weighted least square method. When the initial value of material parameters $\boldsymbol{S}^0$ and the measured value of displacement $\overline{u}$ are given, and repeated iterative computations with eq. (8) are carried out, the approximate value of material parameters can be obtained.

$[C]$ in eq. (8) is an important coefficient matrix. From eq. (2), it is known that the physical sense of the component C_{ij}^k is the change of the computational value of the ith measured point of displacement $\boldsymbol{u}_i^k$ caused by the change of the jth unknown material parameter S_j. This coefficient is defined as the sensitivity coefficient. As it is impossible to establish an analytic function relation between $\boldsymbol{u}$ and $\boldsymbol{S}$, it has to be turned to the numerical computation for the solution of C_{ij}. The commonest approach to compute sensitivity coefficients is to use differential in place of partial derivative i e

$$C_{ij} = \frac{\partial u_i}{\partial S_j} = \frac{\triangle u_i(\triangle S_j)}{\triangle S_j} \tag{12}$$

where $\triangle S_j$ represents a minute change of unknown S_j. If § is used to denote a small positive number, then

$$\triangle S_j = \S S_j \tag{13}$$

The displacement increment $\triangle u_i(\triangle S_j)$ of the ith measured point can be obtained by the following formula:

$$\triangle u_i(\triangle S_j) = u_i(S_j + \triangle S_j) - u_i(S_j) \tag{14}$$

3 COMPUTATIONAL EXAMPLES

To realize the above computation, in this paper with FORTRAN language, the computational programs of direct analysis and inverse analysis of laminated composite plate were compiled. Also the identification of stiffness coeffieient of some typical laminated plates was carried out by simulation computation. In order to examine the universality of method and remove the effect of experimental errors on precision of computation, the precision value of finite element method was used instead of the measured values of displacement in computation.

3.1 Example of symmetrically laminated composite plate

For the symmetrically laminated plate, the coupling stiffness coefficient $B_{ij} = 0$. There is no coupling in plane deformation and bending deformation, so that extensional and bending stiffness coefficients can be computed, respectively. The vector of extensional stiffness coefficients to be computed is

$$[S]_E = [A_{11}, A_{12}, A_{16}, A_{22}, A_{26}, A_{66}]^T \tag{15}$$

bending stiffness coefficient to be computed is

$$[S]_B = [D_{11}, D_{12}, D_{16}, D_{22}, D_{26}, D_{66}]^T \tag{16}$$

The computation model for identifying extensional stiffness coefficients is shown in Fig. 1. The number of measured points $m = 18$. Only the horizontal displacements on X-direction are taken as the measured values of displacement for each measured point. The differential step lengths are $\delta_1 = 0.1$, $\delta_2 \sim \delta_6 = 0.015$, respectively. The initial value, real value of the extensional stiffness coefficient and the approximate value for 10th iteration are listed in Table 1. The convergence by iterative computation is shown in Fig. 3. From those, it can be known that the convergence rate is larger with the inverse computation, the change of differential parameters is stable and the precision of convergence is higher.

Fig. 2 shows the computational model for the computation of bending stiffness coefficients. Only the component perpendicular to the plane is taken as the measured values of displacement at each measured point. The number of measured points $m = 20$, differential step lengths are: $\delta_1 = 0.05$, $\delta_2 \sim \delta_6 = 0.01$, respectively. The related figures and the states of convergence are shown in Table 1 and Fig. 4, respectively. From the results, it can be shown that the convergence is quite good.

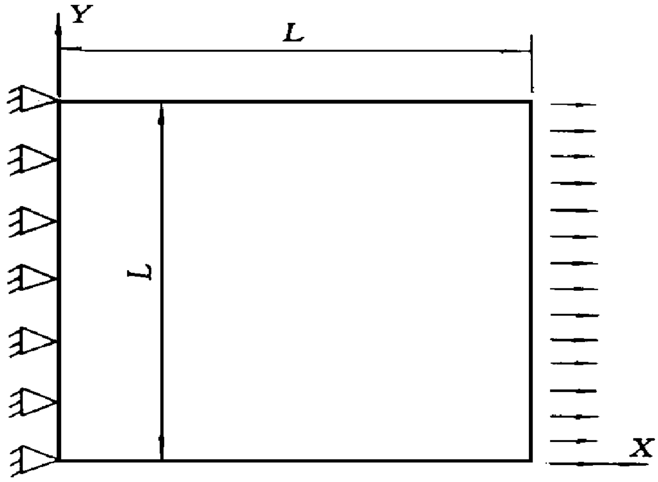

Fig. 1 A model for computation of extensional stiffness coefficients

number of mode: 121;
number of element: 200; $L = 300$ mm

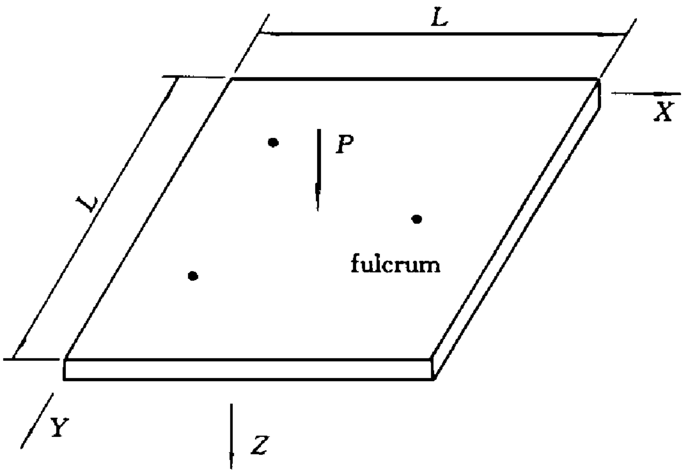

Fig. 2 A model for computation of coupling and bending stiffness coefficients

Table 1 Extensional and bending stiffness coefficients of symmetrically laminated plate

	A_{11}	A_{12}	A_{16}	A_{22}	A_{26}	A_{66}	D_{11}	D_{12}	D_{16}	D_{22}	D_{26}	D_{66}
Initial value S^0	392	98	147	116	49	98	980	294	490	196	196	392
Computational results S^{10}	507	142	240	119	93	169	1 534	434	728	362	284	512
Real value	509	144	243	120	95	171	1 529	434	728	362	284	513

A_{ij} / MN·m^{-1}; D_{ij} / Nm

3.2 Regular nonsymmetrical orthogonally laminated plates

For regular nonsymmetrical orthogonally laminated plates, with 0°/90°/0°/90° being the layer-pattern, if the thickness of each layer is the same, there will be only four independent stiffness coefficients. Here, the material parameter vector chosen is

$$[S]=[B_{11}, D_{11}, D_{12}, D_{66}]^T \quad (17)$$

The related data are shown in Table 2.

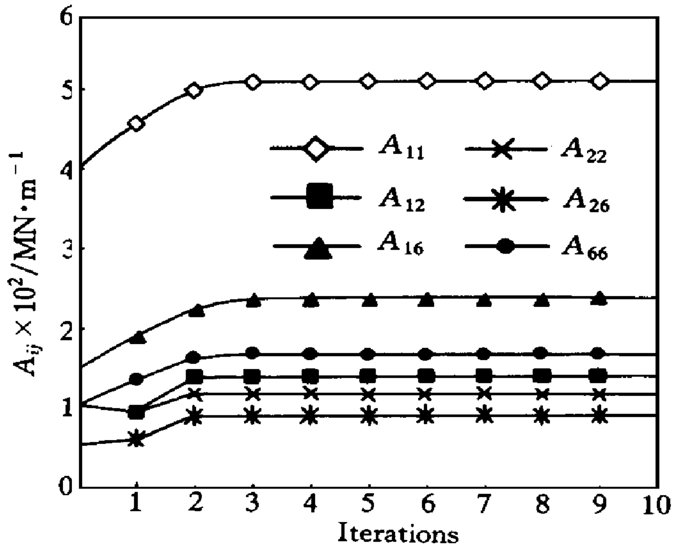

Fig. 3 The convergence situation for computation of extensional stiffness coefficients of symmetrically laminated plate

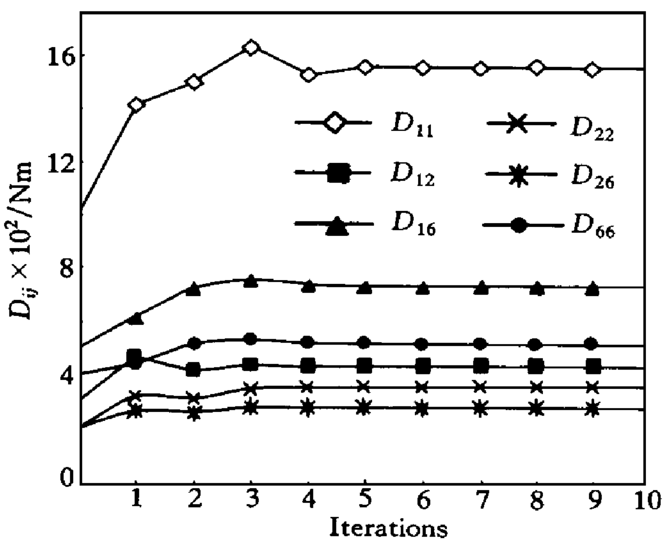

Fig. 4 The convergence situation for computation of bending stiffness coefficients of symmetrically laminated plate

Table 2 Coupling and bending stiffness coefficients of nonsymmetrical orthogonally laminated plates

	B_{11}	D_{11}	D_{12}	D_{66}
Initial value S^0	98	490	20	98
Computational results S^{10}	196	1 334	48	128
Real value	195	1 331	49	128

B_{ij} /KN; D_{ij} /Nm

The differential step length is $\delta_i = 0.1$, and the number of measured points $m = 20$. Computational model in Fig. 2 is used. Fig. 5 and Fig. 6 are the states of convergence by iterative computation. The numerical results show that the convergence is also quite good.

3.3 Anti-symmetrically laminated plates

For anti-symmetrically laminated plates, with $+\alpha/-\alpha/+\alpha/-\alpha$ being the layer-pattern, if the thickness of each layer is the same, there will be six independent stiffness coefficients. Here, the material parameter vector chosen is

$$[S]=[B_{16}, B_{26}, D_{11}, D_{12}, D_{22}, D_{66}]^T \quad (18)$$

The related data are listed in Table 3. The differential step lengths are $\delta_1 = \delta_2 = 0.01$, $\delta_3 = 0.05$, and $\delta_4 = \delta_5 = 0.03$. The number of measured points $m = 20$. Computational model in Fig. 2 is used. Fig. 7 and Fig. 8 show the states of convergence, from the above, it can be

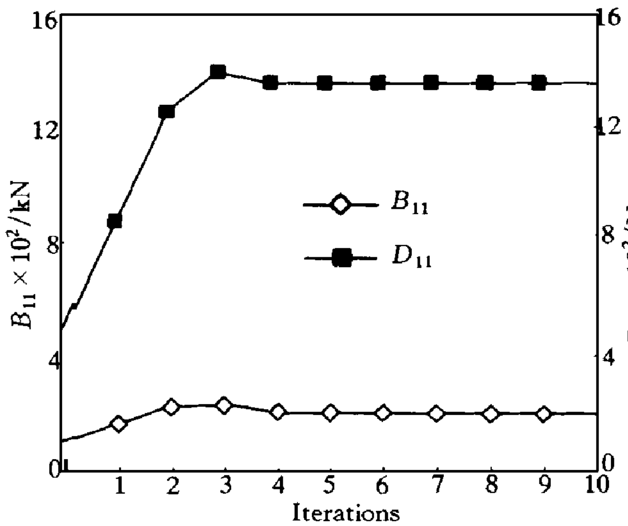

Fig. 5 The convergence situation for computation of coupling and bending stiffness coefficients of nonsymmetrically orthogonally laminated plates

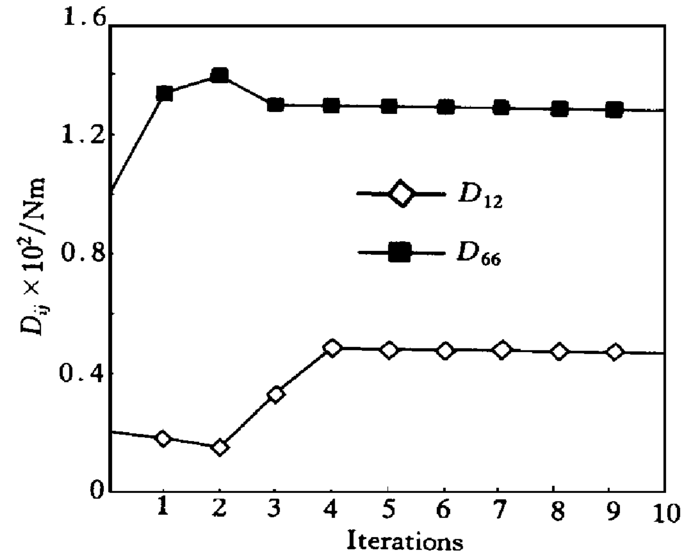

Fig. 6 The convergence situation for computation of bending stiffnes coefficients of nonsymmetrically orthogonally laminated plates

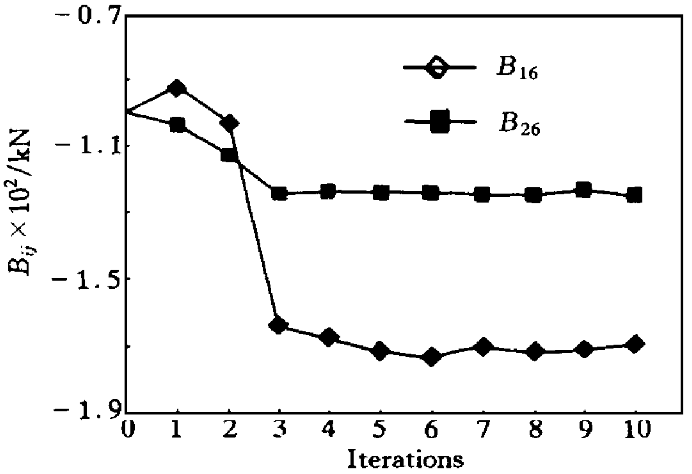

Fig. 7 The convergence situation for computation of coupling stiffness coefficients of anti-symmetrical laminated plates

known that the convergence is also quite good.

4 CONCLUSIONS

A complete inverse computational algorithm for solving the stiffness coefficients of laminated composite plates is proposed. In order to reduce the complexity due to the functional relation of displacements with material parameters, differential is used in stead of partial derivative in the

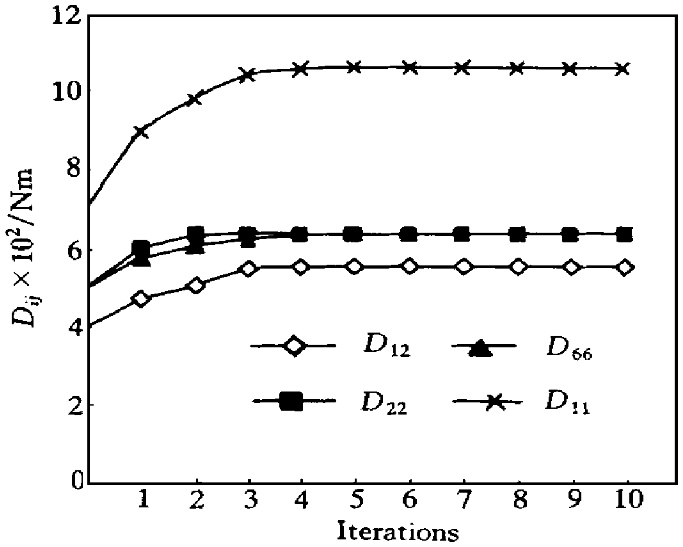

Fig. 8 The convergence situation for computation of bending stiffness coefficients of anti-symmetrical laminated plates

Table 3 Coupling and bending stiffness coefficients of anti-symmetrical laminated plate

	B_{16}	B_{26}	D_{11}	D_{12}	D_{22}	D_{26}
initial value S^0	-98	- 98	686	392	491	491
computational results S^{10}	- 166	- 123	1 035	546	630	626
real value	- 166	- 122	1 035	546	630	626

B_{ij} / KN; D_{ij} / Nm

computation of sensitivity coefficients.

REFERENCES

1 Kubo Shiro, J Soc Mat Sci, (in Japanese), Japan 1992, 41 (470), 1595- 1604.
2 Courage W M G, Schreurs P J G, Janssen J D. Computers and Structures, 1990, 34 (2), 231~ 237.
3 Stephen R, Kennon, George S. Inter Jour Numer Methods in Eng. 1986, 22: 363.
4 Cshnur D S, Zabaras Nicholas. Inter Jour Numer Methods in Eng. 1990, 30: 57.
5 Busby H R, Trujiuo D M. Inter Jour Numer Methods in Eng. 1985, 21: 349.
6 Mota Soares C M, Moreira de Freitas M, Araujo A L. Composite Structures, 1993, 25: 277- 285.
7 Kim W C, Dharan C K H. Composite Structures, 1995, 30: 241- 251.

(Edited by Huang Jinsong)