# DEPENDENCE OF PREDICTION MODEL OF FORMING LIMIT STRAINS ON FORMING METHOD AND MECHANICAL PROPERTIES OF SHEET METALS①

Zhou Weixian
*Department of Aeronautical Manufacturing Engineering, Northwestern Polytechnical University, Xi'an 710072*

**ABSTRACT** Taking into account the deformation characteristics of sheet metal forming under punch or fluid pressure by means of the states of tension-compression strains or biaxial tension strains, the possibility of development of the tensile instable deformation to localized necking deformation was analysed using the principle of minimum strain energy. It was found that this is impossible when the sheet metals are deformed by means of fluid pressure in the biaxial tension mode, and possible in other deformation conditions, but the possibility of breaking before the localized necking occurs due to the insufficient deformation ability of materials should be noted. Therefore, the choice of either localized necking-type model or breaking-type model for predicting the forming limit strains must be determined in the light of actual conditions.

**Key words** forming limit strains prediction model forming method mechanical properties

## 1 INTRODUCTION

Known for an effective guide for production, the forming limit curve (FLC) has drawn much attention for its prediction. However, no satisfactory theories have been advanced. The present leading M-K Theory[1] was based on the assumed initial inhomogeneity of mechanical properties or thickness in the sheet metals, and the limit strain concept in this theory is ambiguous. There are a few other theories[2, 3] which take rupture strains as limit strains. The concept of limit strains in those theories is explicit, but they attribute the rupture to the development of inclusions[2] or damages[3], therefore they seem to attend to trifles and tend to neglect the essentials.

The author believes that there are two chief reasons which hamper the deep-going study of the FLC prediction: (1) The mechanisms of localized necking deformation have not been made clear yet; (2) No analyses considering actual forming characteristics can be found, and all kinds of sheet metal forming are taken as the same[4], therefore various phenomena are mixed and can not be distinguished.

Practical observations show that different kinds of fractures exist, consequently there may be different mechanisms controlling the forming limits of sheet metals. As a result, it is reasonable to predict the limit strains using different models so as to be geared to actual circumstances. This paper aims to analyze and understand this problem so as to promote the development of forming limit strain prediction.

## 2 DEFINITION OF FORMING LIMIT STRAINS OF SHEET METALS

All the existing prediction theories for sheet metal forming in the tension-compression strain fields take the strains approaching localized necking as limit strains. In order to guarantee the usability of the formed parts, this definition is gen-

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erally accepted. However, it is meaningless for those forming parts without localized necking deforming, the rupture strains need to be definited as limit strains as there are no alternatives.

## 3 POSSIBILITY OF OCCURRENCE OF LOCALIZED NECKING DEFORMATION

As stated in Ref. [1], localized necking deformation occurs under the condition of plane strain increment. Therefore, in a certain forming process, if this condition may be met by taking into account the effect of tensile instable deformation on the actual stress field, then localized necking deformation may occur. Otherwise, there would be no occurrence of this kind of deformation.

### 3.1 Deformation characteristics of sheet metal forming in different forming methods

Generally speaking, there are two sheet metal forming methods, i.e. forming by punch pressure or forming by fluid pressure.

In the forming process by a punch, the normal stress of the sheet's part which does not contact with the punch equals zero. Although the normal stress of the sheet's part which is in contact with the punch does not equal zero, it is clear that only tangential displacement would occur and no normal displacement would occur here. Noting the patterns of instable deformation in the conditions of $dp = 0$ and $dP_1 = 0$[5], it can be seen that the instable deformation can only occur in the condition of $dP_1 = 0$ in this forming method.

In the forming process by fluid pressure, materials may make tangential and normal displacements. This means that the tensile deformation can be realized by tangential displacement, or achieved through changing the curvatures of the workpieces by means of normal displacement of materials, or fulfiled by both. The specific deformation paths are determined by the principle of minimum strain energy. The author's analyses below indicate that if the tensile deformation is realized by tangential displacement, the strain energy is not minimum. Therefore, in this forming mode, the instable deformation will not begin in the condition of $dP_1 = 0$.

For the cases under the combined actions of fluid pressure and other loadings, for example, a tube with loading on both ends, the deformation characteristics should be decided in the light of actual conditions.

### 3.2 Deformation characteristics in different strain fields

It has been proved by the author[5] that the instable deformation is one following the path leading to minimum strain energy, and emerges by germinating new inhomogeneous deformation. This means that the curvatures of the flow lines of principal stresses are under change, as shown in Fig. 1. According to the equilibrium equations, it can be known that the bulging of the curves will induce inward compression stress, and the indenting of the curves will induce outward tensile stress, thus leading to the change of the ratio, $\chi(=\sigma_2/\sigma_1)$, of the two principal stresses. It should be noted that in different stress fields, the change of the $\chi$ value is not the same because of the different correlativities between $d\varepsilon_1$ and $d\varepsilon_2$.

Because $d\varepsilon_t < 0$, the volume constancy condition in the tension-compression strain fields can be written as $|d\varepsilon_1| = |d\varepsilon_t| + |d\varepsilon_2|$, and it is clear that the variation of $d\varepsilon_2$ will inevitably cause the variation of $d\varepsilon_1$, i.e. the indenting of $\sigma_1$ flow line will undoubtedly bring about the bulging of $\sigma_2$ flow line, then only changes meeting the condition of $d\chi > 0$ can be induced. In the biaxial tension strain fields, $|d\varepsilon_1| = |d\varepsilon_t| - |d\varepsilon_2|$, where $d\varepsilon_t$ can act as an adjuster to make $d\varepsilon_1$ and $d\varepsilon_2$ not directly dependent. This means that the curvatures of the flow lines of the two principal stresses can vary freely, i.e. the changes meeting the conditions of $d\chi \geqslant 0$ or $d\chi \leqslant 0$ can be induced. The specific changes are determined by the need to meet the deformation path leading to minimum deformation energy.

It is also shown from the above analyses that the possible value of $\chi$-change in the biaxial tensile strain fields is independent of the actual stress fields and in the tensile-compression strain

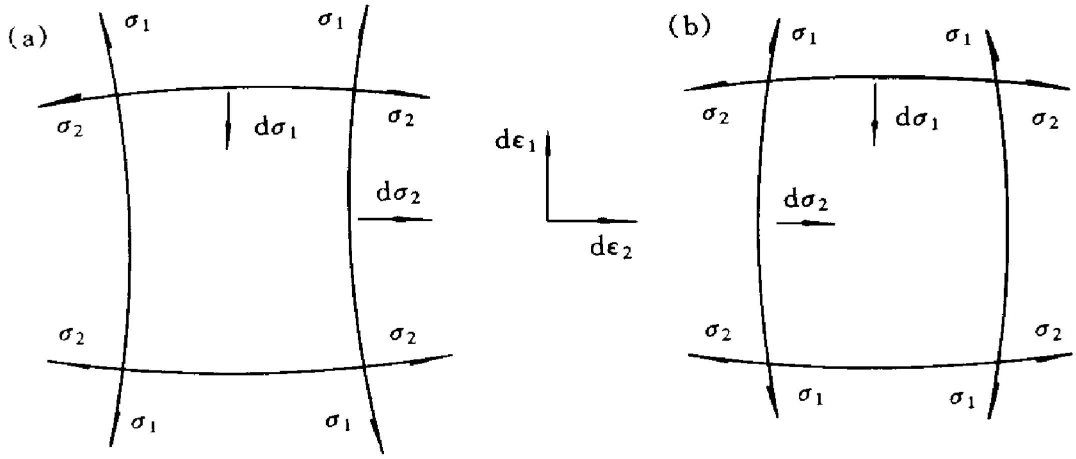


**Fig. 1 Variations of curvaturves of flow lines of principal stresses**

(a) $-d\varepsilon_1 > 0,\ d\varepsilon_2 < 0$; (b) $-d\varepsilon_1 > 0,\ d\varepsilon_2 > 0$

fields is not so.

### 3.3 Minimum strain energy path

#### 3.3.1 Forming using a punch

Neglecting the effect of contact pressure applied by the punch, the increment of strain energy in unit volume material can be expressed by[5]

$$\frac{dW}{d\varepsilon_i} = \sigma_i = \sqrt{1-\chi+\chi^2}\,\sigma_1 \tag{1}$$

whose further increment is

$$\frac{d^2W}{d\varepsilon_i^2} = \left(\frac{\chi-1/2}{1-\chi+\chi^2}\frac{d\chi}{d\varepsilon_i} + \frac{1}{\sigma_1}\frac{d\sigma_1}{d\varepsilon_i}\right)\sigma_i \tag{2}$$

It can be learned that $\frac{d\chi}{d\varepsilon_i} > 0$ in the tension-compression strain fields ($\chi < 1/2$) and $d\chi/d\varepsilon_i < 0$ in the biaxial tension strain fields ($\chi > 1/2$) are the minimum strain energy paths followed by the instable deformation.

#### 3.3.2 Forming by fluid pressure

The equilibrium equation for sheet metals under fluid pressure $p$ is as follows:

$$\left.\begin{aligned} p &= \left(\frac{\sigma_1}{R_1} + \frac{\sigma_2}{R_2}\right)t \\ &= (1+\rho\chi)\frac{\sigma_1 t}{R_1} \\ \chi &= \frac{\sigma_2}{\sigma_1},\quad \rho = \frac{R_1}{R_2} \end{aligned}\right| \tag{3}$$

From eq. (3) we can obtain $\sigma_1 = \frac{pR_1}{(1+\rho\chi)t}$. Substituting it into eq. (2) gives

$$\frac{d^2W}{d\varepsilon_i^2} = \left[\left(\frac{\chi-1/2}{1-\chi+\chi^2} - \frac{\rho}{1+\rho\chi}\right)\times \frac{d\chi}{d\varepsilon_i} - \frac{\chi}{1+\rho\chi}\frac{d\rho}{d\varepsilon_i} + \frac{1}{p}\frac{dp}{d\varepsilon_i} + \frac{1}{R_1}\frac{dR_1}{d\varepsilon_i} - \frac{d\varepsilon}{d\varepsilon_i}\right]\sigma_i \tag{4}$$

Under the actions of fluid pressure, the changes of geometric shapes of the workpieces are unobstructed. Before and after instability, there are freedoms of the variations of $\rho$ with $\chi$ or $\chi$ with $\rho$ in order to follow the minimum strain energy path. From eq. (4), we have

$$\frac{\partial}{\partial\rho}\left(\frac{d^2W}{d\varepsilon_i^2}\right) = -\frac{\chi}{1+\rho\chi} < 0 \tag{5a}$$

$$\frac{\partial}{\partial\chi}\left(\frac{d^2W}{d\varepsilon_i^2}\right) = \frac{\chi-1/2}{1-\chi+\chi^2} - \frac{\rho}{1+\rho\chi} \tag{5b}$$

Thus the strain energy always reduces with increasing $\rho$ value, i. e. the changes satisfying $d\rho/d\varepsilon_i > 0$ are realized. As for the changes of $\chi$ value in the tension-compression strain fields of $\chi < 1/2$, it is known from eq. (5b) that the changes satisfying $d\chi/d\varepsilon_i > 0$ are realized like the forming cases using a punch. When $\chi > 1/2$, no definite sign is found from eq. (5b). Whether of both $d\chi/d\varepsilon_i > 0$ and $d\chi/d\varepsilon_i < 0$ is possible? If it is $d\chi/d\varepsilon_i < 0$, then, from eq.

(5b), there are $\left(\frac{\chi-1/2}{1-\chi+\chi^2}-\frac{\rho}{1+\rho\chi}\right)\geqslant 0$, i.e. $(2\chi-1)/(2-\chi)\geqslant\rho$ in accordance with the strain path of minimum strain energy. But, it is found from the differential of the latter that $\frac{3}{(2-\chi)^2}\frac{d\chi}{d\varepsilon_i}\geqslant\frac{d\rho}{d\varepsilon_i}$, i.e. $d\chi/d\varepsilon_i>0$. This is to reject the presupposition. Therefore, in the biaxial tensile strain fields the changes that satisfy $d\chi/d\varepsilon_i>0$ and $(2\chi-1)/(2-\chi)\leqslant\rho$ are realized. The deformation condition $d\chi/d\varepsilon_i>0$ excludes the possibility of localized necking deformation in this case. There may be exceptions when external forces are exerted.

## 4 MOTION LOCUS OF MATERIAL PARTICLES UNDER ACTIONS OF FLUID PRESSURE

It has been proved above that, in the fluid pressure forming process, whether there occurs instability or not, the deformation of sheet metals must follow the relation $(2\chi-1)/(2-\chi)\leqslant\rho$. According to the plastic flow rule

$$\frac{d\varepsilon_i}{2\sqrt{1-\chi+\chi^2}}=\frac{d\varepsilon_1}{2-\chi}=\frac{d\varepsilon_2}{2\chi-1}=-\frac{d\varepsilon_t}{1+\chi}\qquad(6)$$

it is recognized that this relation is equivalent to $d\varepsilon_2/d\varepsilon_1\leqslant\rho$. Because $\rho\geqslant 0$, in the tension-compression strain fields, only $d\varepsilon_2/d\varepsilon_1<\rho$ can hold true. In the biaxial tension strain fields, either $d\varepsilon_2/d\varepsilon_1<\rho$ or $d\varepsilon_2/d\varepsilon_1=\rho$ conforming to the minimum strain energy path can be decided by analyzing the relation between the second-order differential of $\rho$ and the deformation energy. Re-differentiation of eq. (5a) yields

$$\frac{\partial^2}{\partial\rho^2}\left(\frac{d^2W}{d\varepsilon_i^2}\right)=\left(\frac{\chi}{1+\rho\chi}\right)^2>0$$

Therefore, the deformation follows the mode of minimum $d^2\rho/d\varepsilon_i^2$ value. Noting $dW/d\varepsilon_i=\sigma_i$ and introducing the hardening law $\sigma_i=K\varepsilon_i^n$, the left part of eq. (4) can be re-written as $d^2W/d\varepsilon_i^2=\sigma_i(n/\varepsilon_i)$, then

$$\frac{\chi}{1+\rho\chi}\frac{d\rho}{d\varepsilon_i}=\left(\frac{\chi-1/2}{1-\chi+\chi^2}-\frac{\rho}{1+\rho\chi}\right)\times\frac{d\chi}{d\varepsilon_i}-\left(\frac{n}{\varepsilon_i}-\frac{1}{p}\frac{dp}{d\varepsilon_i}-\frac{1}{R_1}\frac{dR_1}{d\varepsilon_i}+\frac{d\varepsilon_t}{d\varepsilon_i}\right)\qquad(7)$$

Further differentiation and arrangement gives

$$\begin{aligned}\frac{\chi}{1+\rho\chi}\frac{d^2\rho}{d\varepsilon_i^2}=&\Big[\Big(\frac{\chi-1/2}{1-\chi+\chi^2}-\frac{\rho}{1+\rho\chi}\Big)^2-\frac{2}{\chi(1+\rho\chi)}\Big(\frac{\chi-1/2}{1-\chi+\chi^2}-\frac{\rho}{1+\rho\chi}\Big)+\frac{1/2+\chi-\chi^2}{(1-\chi+\chi^2)^2}+\Big(\frac{\rho}{1+\rho\chi}\Big)^2\Big]\Big(\frac{d\chi}{d\varepsilon_i}\Big)^2+\\&\Big(\frac{\chi-1/2}{1-\chi+\chi^2}-\frac{\rho}{1+\rho\chi}\Big)\frac{d^2\chi}{d\varepsilon_i^2}+2A\frac{1-\chi/2}{\chi(1-\chi+\chi^2)}\frac{d\chi}{d\varepsilon_i}-\frac{dA}{d\varepsilon_i}+A^2\end{aligned}\qquad(8)$$

$$A=\left(\frac{n}{\varepsilon_i}-\frac{1}{p}\frac{dp}{d\varepsilon_i}-\frac{1}{R_1}\frac{dR_1}{d\varepsilon_i}+\frac{d\varepsilon_t}{d\varepsilon_i}\right)\leqslant 0\qquad(9)$$

Considering the relation $\left(\frac{\chi-1/2}{1-\chi+\chi^2}-\frac{\rho}{1+\rho\chi}\right)\leqslant 0$ and the signs of its multipliers, it can be concluded that the smaller the value of $\left|\frac{\chi-1/2}{1-\chi+\chi^2}-\frac{\rho}{1+\rho\chi}\right|$ is, i.e. the more the $\rho$ approaches $\left|\frac{2\chi-1}{2-\chi}\right|$, the smaller the $\frac{d^2\rho}{d\varepsilon_i^2}$ is. Therefore, in the biaxial tension strain fields, the $\rho=\frac{2\chi-1}{2-\chi}$ relation is fulfiled.

What is the physical meaning of $\rho=\frac{2\chi-1}{2-\chi}=\frac{d\varepsilon_2}{d\varepsilon_1}$? It is learned from the geometrical relations that, if a material particle has a displacement $dh$ along the normal line of the workpiece, then $d\varepsilon_1=\frac{dh}{R_1}$, $d\varepsilon_2=\frac{dh}{R_2}$, i.e. $\rho=\frac{R_1}{R_2}=\frac{d\varepsilon_2}{d\varepsilon_1}$. This means that no tangential displacement takes place. The above particle displacement model was proposed by Hill for analyzing the hydraulic bulging of circular sheet metals[6] and has not been proved yet.

In addition, if the instability occurs in the condition of $\mathrm{d}P_1 = 0$, i.e. there occurs abnormal increment in $\mathrm{d}\varepsilon_1$, then $\mathrm{d}\rho < 0$, or the difference between $\rho$ and $|(\mathrm{d}\varepsilon_2)/(\mathrm{d}\varepsilon_1)|$ increases, which is contradictory to the minimum strain energy path, consequently this deformation will not take place.

## 5 FACTORS AFFECTING DEFORMATION ABILITY OF MATERIALS

The above analyses are concerned with the deformation mechanisms. Obviously, the occurrence of localized necking deformation also depends upon the deformation ability of materials. Insufficiency of deformation ability will bring about rupture before localized necking.

Take 1Cr18Ni9Ti stainless steel ( $n = 0.45 \sim 0.50$ ) as an example. Assuming that it is a monotonic loading process before instability, i.e. $\frac{\mathrm{d}\chi}{\mathrm{d}\varepsilon_i} = 0$, then the instability occurs in the condition of $\frac{\mathrm{d}P_1}{\mathrm{d}\varepsilon_i} = 0$, i.e. $\frac{\mathrm{d}\sigma_1}{\sigma_1} = \mathrm{d}\varepsilon_1$. By means of eqs. (1) and (6) and the hardening law $\sigma_i = k\varepsilon_i^n$, then $\varepsilon_{i1}$ at the critical state of instability can be calculated and $\varepsilon_{i1} = \frac{2\sqrt{1-\chi+\chi^2}}{2-\chi}n$. When $\chi = 1$, $\varepsilon_{i1} = 2n = 0.9 \sim 1.0$. The deformation ability of a material can be approximately expressed by the reduction of area, $\psi$. For the stainless steels, $\psi = 0.585 \sim 0.635$, the corresponding effective strain $\varepsilon_i = \ln\frac{1}{1-\psi} = 0.88 \sim 1.008$. So, under this stress state, there is only a little potential left for the deformation after the instable deformation starts.

The situations are similar for 70∶30 brass with high $n$ value( $n = 0.47$). Therefore it is no wonder that no satisfactory results can be predicted using the existing theories.

## 6 CONCLUSIONS

(1) When a sheet metal is formed using a punch, the instable deformation starts in the condition of $\mathrm{d}P_1 = 0$. No matter what the strain states are, if only the deformation ability of materials is sufficient, it is reasonable to predict the limit strains using the localized necking-type model.

(2) When a sheet metal is formed by means of fluid pressure, instable deformation starts in the condition of $\mathrm{d}p = 0$, and it is advisable to use a localized necking-type model in the tension-compression strain fields and to use a rupture-type model in the biaxial tension strain fields to predict the limit strains.

(3) For materials of high $n$-values or relatively poor plasticity, they can not always be deformed to the level of localized necking deformation. Therefore it is practical to predict the limit strains using both models spontaneously and take the relatively smaller values as the limit strains.

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