NUMERICAL SIMULATION FOR SHAPE OF COLD ROLLING ALUMINUM STRIP ON 4-HIGH MILL WITH ROLL SHIFTED①

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ABSTRACT The distribution of pressure between rolls was simulated by the polynomial expression, the coupled equations were established according to the roll deformation compatibility equations, and then solved by the least square and collocation method to get the distribution of pressure between rolls, and the roll system elastic deformation was obtained. A mathematical model for the shape of cold rolling strip on 4-high mill with roll shifted was established, which was on the base of combination of the least square and collocation method to solve roll system elastic deformation and the variation method to solve the lateral metal flow. Using the model, the distributions of tension stresses of aluminum strip were simulated, and the influence regulation of work roll shifted distance and bending force on the shape were given.

Key words mill with roll shifted least square and collocation method variation method shape

1 INTRODUCTION

As the mill with roll shifted has good shape-control characteristic, it has been applied to production of cold rolled and hot rolled strip. It is very important to establish accurate mathematical model for shape[1, 2]. The least square and collocation method were not only applied to solve linear problems, but also to solve nonlinear problems, whose adaptability is rather wide. As the order of the demanded coupled equations is very few and has nothing with the number of collocation points, the demanded computing resources and time are less than those of other methods[3]. In this paper, the distribution of pressure between rolls was simulated by the polynomial expression, the coupled equations were established according to the roll deformation compatibility equations, and then solved to get the distribution of pressure between rolls by the least square and collocation method, which was combined with the variation method to calculate the elastic deformation of roll system and the distributions of tension stresses, and to give the influence regulation of shifted distance and bending force on the shape.

2 MATHEMATICAL MODEL OF SHAPE ON 4-HIGH MILL WITH ROLL SHIFTED

2.1 Model of roll system elastic deformation on 4-high mill with roll shifted

The calculated model of roll system elastic deformation is shown as Fig. 1.

The distribution of the pressure between rolls is expressed as follow

$$f(y) = \sum_{i=0}^{M} q_i \left(\frac{2y}{L_c}\right)^i \tag{1}$$

where L_c —contact length between rolls, $L_c = L_L + L_R$; q_i —simulation coefficients of $f(y)$.

2.1.1 Backup roll equilibrium equations

The force equilibrium equations in vertical direction is

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$$P_Z = P_L + P_R = \int_{-L_L}^{L_R} f(y)\,dy \qquad (2)$$

where P_Z —total push-up force; P_L, P_R —push-up forces of the left and right hydraulic cylinders.

The torque equilibrium equation of back-up roll is

$$\Sigma M_A = 0 \qquad (3)$$

so $$P_R = 0.5P_Z + \frac{1}{L}\int_{-L_L}^{L_R} yf(y)\,dy \qquad (4)$$

and $$P_L = 0.5P_Z - \frac{1}{L}\int_{-L_L}^{L_R} yf(y)\,dy \qquad (5)$$

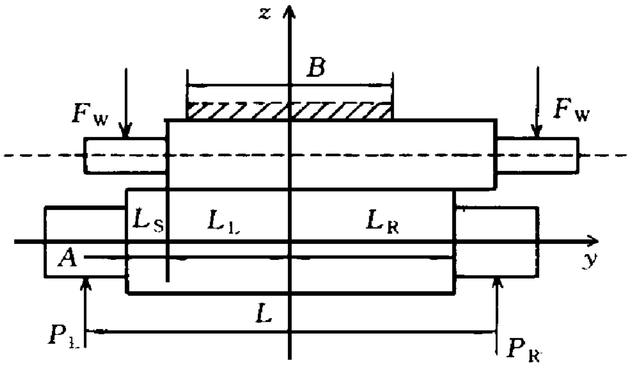

Fig. 1 Calculation model of roll system elastic deformation

2.1.2 Calculation of deflections of backup roll

The calculation model of back-up roll deflections is shown as Fig. 2, it can be computed by the energy method.

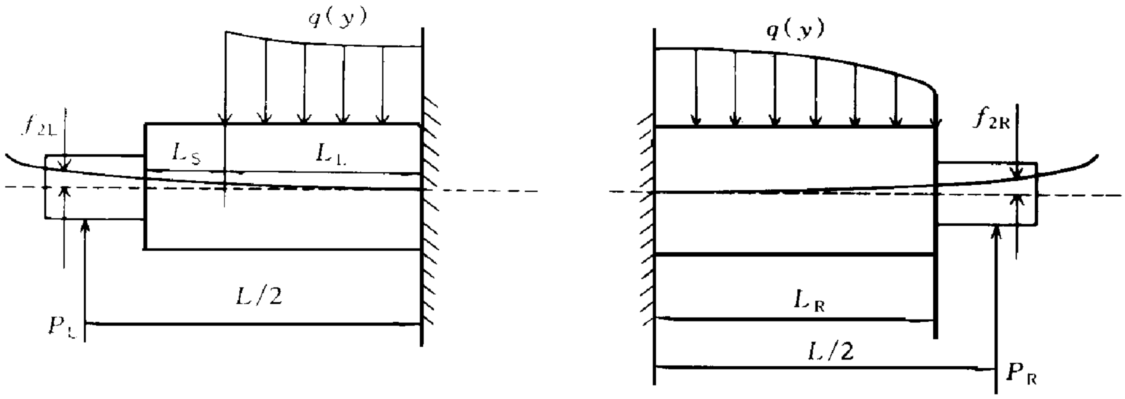

Fig. 2 Calculation model of backup roll deflections

$$f_{2R}(y) = \sum_{i=0}^{M} q_i[A_{2RP}(i, y) + A_{2RQ}(i, y)] + B_{2RP}(y) + B_{2RQ}(y) \qquad (6)$$

$$f_{2L}(y) = \sum_{i=0}^{M} q_i[A_{2LP}(i, y) + A_{2LQ}(i, y)] + B_{2LP}(y) + B_{2LQ}(y) \qquad (7)$$

where $$A_{2RP}(i, y) = [y^2(\frac{L}{2} - \frac{y}{3})\frac{L_R^2(\frac{2L_R}{L_c})^i - L_L^2(-\frac{2L_L}{L_c})^i}{2L(i+2)} - \frac{\frac{y^2}{2}L_R^2(\frac{2L_R}{L_c})^i - \frac{y^4}{(i+3)(i+4)}(\frac{2y}{L_c})^i}{i+2} - \frac{-y^3\frac{L_c}{6}(\frac{2L_R}{L_c})^{i+1} + \frac{y^3L_c}{(i+3)(i+4)}(\frac{2y}{L_c})^{i+1}}{2(i+1)}]/(E_BI_B);$$

$$A_{2RQ}(i, y) = \frac{10}{9G_BA_B}\{y\frac{L_R^2(\frac{2L_R}{L_c})^i - L_L^2(-\frac{2L_L}{L_c})^i}{L(i+2)} - \frac{L_c}{2(i+1)}[y(\frac{2L_R}{L_c})^{i+1} - \frac{L_c}{2(i+2)}(\frac{2y}{L_c})^{i+2}]\}$$

$$B_{2RP}(y) = \frac{y^2 P_Z}{4E_B I_B}\left(\frac{L}{2} - \frac{y}{3}\right); \quad B_{2RQ}(y) = \frac{5yP_Z}{9G_B A_B}$$

where E_B, G_B —Elastic modulus and shear modulus of back-up; I_B, A_B —Moment of inertia and section of back-up roll.

The forms of expressions of $A_{2LP}(i, y)$, $A_{2LQ}(i, y)$, $B_{2LP}(y)$ and $B_{2LQ}(y)$ are similar to those of $A_{2RP}(i, y)$, $A_{2RQ}(i, y)$, $B_{2RP}(y)$ and $B_{2RQ}(y)$.

2.1.3 Calculation of work roll deflections

The calculation model of work roll deflections is shown as Fig. 3, it also can be computed by the energy method.

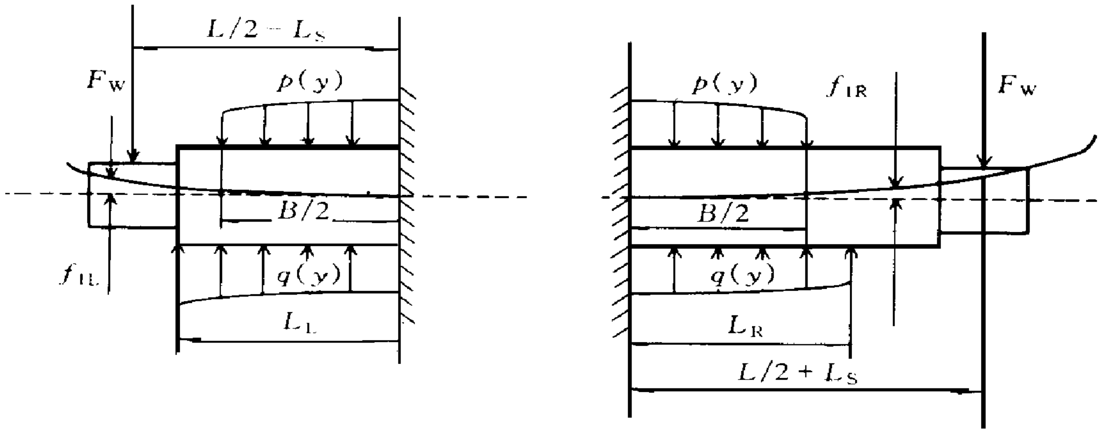

Fig. 3 Calculation model of work roll deflections

When $0 \leqslant y < 0.5B$,

$$f_{1R}(y) = \sum_{i=0}^{M} q_i[A_{1RP}(i, y) + A_{1RQ}(i, y)] + B_{1RP}(y) + B_{1RQ}(y) + \gamma_1 y \tag{8}$$

When $0.5B \leqslant y \leqslant L_R$,

$$f_{1R}(y) = \sum_{i=0}^{M} q_i[A'_{1RP}(i, y) + A'_{1RQ}(i, y)] + B'_{1RP}(y) + B'_{1RQ}(y) + \gamma_1 y \tag{9}$$

When $-0.5B \leqslant y \leqslant 0$,

$$f_{1L}(y) = \sum_{i=0}^{M} q_i[A_{1LP}(i, y) + A_{1LQ}(i, y)] + B_{1LP}(y) + B_{1LQ}(y) + \gamma_1 y \tag{10}$$

When $-L_L \leqslant y < -0.5B$,

$$f_{1L}(y) = \sum_{i=0}^{M} q_i[A'_{1LP}(i, y) + A'_{1LQ}(i, y)] + B'_{1LP}(y) + B'_{1LQ}(y) + \gamma_1 y \tag{11}$$

where B —Strip width;

$$A_{1RP}(i, y) = \Bigg[\frac{\frac{y^2}{2}L_R^2\left(\frac{2L_R}{L_c}\right)^i - \frac{y^4}{(i+3)(i+4)}\left(\frac{2y}{L_c}\right)^i}{i+2} + \frac{-\frac{L_c y^3}{6}\left(\frac{2L_R}{L_c}\right)^{i+1} + \frac{y^3 L_c}{(i+3)(i+4)}\left(\frac{2y}{L_c}\right)^{i+1}}{2(i+1)}\Bigg]/(E_w I_w)$$

$$A_{1RQ}(i, y) = \frac{5L_c}{9G_w A_w(i+1)}\left[y\left(\frac{2L_R}{L_c}\right)^{i+1} - \frac{L_c}{2(i+2)}\left(\frac{2y}{L_c}\right)^{i+2}\right]$$

$$B_{1RP}(y) = \Bigg\{-\frac{F_w}{2}y^2\left(\frac{L}{2} + L_S - \frac{y}{3}\right) - \sum_{i=0}^{M} p_i\Bigg[\frac{-\frac{y^4}{(i+3)(i+4)}\left(\frac{2y}{B}\right)^i + \frac{y^2 B}{8}}{i+2} - \frac{B}{2(i+1)}\left(\frac{y^3}{6} - \frac{y^3}{(i+3)(i+4)}\right)\left(\frac{2y}{B}\right)^{i+1}\Bigg]\Bigg\}/(E_w I_w)$$

$$B_{1RQ}(y) = \frac{10}{9G_wA_w}\{-F_wy - \sum_{i=0}^{M}p_i\frac{B}{2(i+1)}[y - \frac{B}{2(i+2)}(\frac{2y}{B})^{i+2}]\};$$

$$A'_{1RP}(i, y) = A_{1RP}(i, y); \quad A'_{1RQ}(i, y) = A_{1RQ}(i, y);$$

$$B'_{1RP}(y) = \{-\frac{F_w}{2}y^2(\frac{L}{2} + L_S - \frac{y}{3}) - \sum_{i=0}^{M}p_i(\frac{B}{2})^3[\frac{y}{2(i+3)} - \frac{B}{12(i+4)}]/(E_wI_w);$$

$$B'_{1RQ}(y) = \frac{10}{9G_wA_w}[-F_wy - \sum_{i=0}^{M}p_i\frac{(\frac{B}{2})^2}{(i+2)}]$$

and where E_w, G_w —elastic modulus and shear modulus of work roll; I_w, A_w —moment of inertia and section of work roll; p_i —simulation coefficients of rolling pressure $p(y_J)$; γ_1 —relative rotating angle between work and back-up rolls at the piont $y_J = 0$.

The structures of expressions of $A_{1LP}(i, y)$, $A'_{1LP}(i, y)$, $A_{1LQ}(i, y)$, $A'_{1LQ}(i, y)$, $B_{1LP}(y)$, $B'_{1LP}(y)$, $B_{1LQ}(y)$ and $B'_{1LQ}(y)$ are similar to those of $A_{1RP}(i, y)$, $A'_{1RP}(i, y)$, $A_{1RQ}(i, y)$, $A'_{1RQ}(i, y)$, $B_{1RP}(y)$, $B'_{1RP}(y)$, $B_{1RQ}(y)$ and $B'_{1RQ}(y)$.

2.1.4 Equations of deformation compatibility between work and backup rolls

Equations of deformation compatibility between work and backup rolls are

$$Z_{wB}(y_J) = f_{2R}(y_J) - f_{1R}(y_J) - M_w(y_J) - M_B(y_J) + Z_{wB}(0) \tag{12}$$

$$Z_{wB}(y_J) = f_{2L}(y_J) - f_{1L}(y_J) - M_w(y_J) - M_B(y_J) + Z_{wB}(0) \tag{13}$$

where $M_w(y_J)$, $M_B(y_J)$ —distributions of work and back-up roll crowns; $Z_{wR}(y_J)$ —elastic flattening between work and back-up rolls at the point y_J, it is calculated by the following expression:

$$Z_{wB}(y_J) = C_{wB}(y_J)f(y_J)$$

where

$$C_{wB}(y_J) = 2\{\frac{1-\gamma_w^2}{\pi E_w}[\ln\frac{D_w}{B_c(y_J)} + 0.407] + \frac{1-\gamma_B^2}{\pi E_B}[\ln\frac{D_B}{B_c(y_J)} + 0.407]\};$$

$$B_c(y_J) = \sqrt{\frac{2D_wD_B}{D_w + D_B}(\frac{1-\gamma_w^2}{\pi E_w} + \frac{1-\gamma_B^2}{\pi E_B})f(y_J)};$$

and where γ_w, γ_B —poisson ratios of work and back-up rolls; D_w, D_B —barrel diameters of work and back-up rolls; $B_c(y_J)$ —flattening width of work and back-up rolls at the point y_J.

Substituting Eqns. (6), (7), (8), (9), (10) and (11) into (12) and (13), we have

when $0 \leqslant y_J < 0.5B$,

$$\sum_{i=0}^{M}A_{RP}(i, y_J)q_i - \gamma_1y_J - C_{wB}(y_J)\sum_{i=0}^{M}q_i(\frac{2y_J}{L_c})^i + Z_{wB}(0) + BM_{RP}(y_J) = 0 \tag{14}$$

when $0.5B \leqslant y_J \leqslant L_R$,

$$\sum_{i=0}^{M}A'_{RP}(i, y_J)q_i - \gamma_1y_J - C_{wB}(y_J)\sum_{i=0}^{M}q_i(\frac{2y_J}{L_c})^i + Z_{wB}(0) + BM'_{RP}(y_J) = 0 \tag{15}$$

when $-0.5B \leqslant y_J \leqslant 0$,

$$\sum_{i=0}^{M}A_{LP}(i, y_J)q_i - \gamma_1y_J - C_{wB}(y_J)\sum_{i=0}^{M}q_i(\frac{2y_J}{L_c})^i + Z_{wB}(0) + BM_{LP}(y_J) = 0 \tag{16}$$

when $-L_L \leqslant y_J < -0.5B$,

$$\sum_{i=0}^{M}A'_{LP}(i, y_J)q_i - \gamma_1y_J - C_{wB}(y_J)\sum_{i=0}^{M}q_i(\frac{2y_J}{L_c})^i + Z_{wB}(0) + BM'_{LP}(y_J) = 0 \tag{17}$$

where

$$A_{RP}(i, y_J) = A_{2RP}(i, y_J) + A_{2RQ}(i, y_J) - A_{1RP}(i, y_J) - A_{1RQ}(i, y_J);$$

$$BM_{RP}(y_J) = B_{2RP}(y_J) + B_{2RQ}(y_J) - B_{1RP}(y_J) - B_{1RQ}(y_J) - M_w(y_J) - M_B(y_J);$$

$$A'_{RP}(i, y_J) = A_{2RP}(i, y_J) + A_{2RQ}(i, y_J) - A'_{1RP}(i, y_J) - A'_{1RQ}(i, y_J);$$

$$BM'_{RP}(y_J) = B_{2RP}(y_J) + B_{2RQ}(y_J) - B'_{1RP}(y_J) - B'_{1RQ}(y_J) - M_w(y_j) - M_B(y_J)$$

The forms of expressions of $A_{\mathrm{LP}}(i, y_J)$, $A'_{\mathrm{LP}}(i, y_J)$, $BM_{\mathrm{LP}}(y_J)$ and $BM'_{\mathrm{LP}}(y_J)$ are similar to those of $A_{\mathrm{RP}}(i, y_J)$, $A'_{\mathrm{RP}}(i, y_J)$, $BM_{\mathrm{RP}}(y_J)$ and $BM'_{\mathrm{RP}}(y_J)$. Pressure distribution between rolls is calculated by collocation and iteration methods in the case of known $Z_{wB}(0)$.

2.1.5 Calculation of pressure between work and back-up rolls

N_1 points are collocated uniformly in the range of $-L_{\mathrm{L}} \leqslant y < -0.5B$, N_2 points in the range of $-0.5B \leqslant y \leqslant 0$, N_3 points in the range of $0 \leqslant y < 0.5B$ and N_4 points in the range of $0.5B \leqslant y \leqslant L_{\mathrm{R}}$, thus the residual error equations of Eqns. (14), (15), (16) and (17) can be written as

$$\left.\begin{aligned}
R_J &= \sum_{i=0}^{M}\left[A'_{\mathrm{LP}}(i, y_J) - C_{wB}(y_J)\left(\frac{2y_J}{L_{\mathrm{c}}}\right)^i\right]q_i - \gamma_1 y_J + BM'_{\mathrm{LP}}(y_J) + Z_{wB}(0) \\
&\qquad (1 \leqslant J \leqslant N_1) \\
R_J &= \sum_{i=0}^{M}\left[A_{\mathrm{LP}}(i, y_J) - C_{wB}(y_J)\left(\frac{2y_J}{L_{\mathrm{c}}}\right)^i\right]q_i - \gamma_1 y_J + BM_{\mathrm{LP}}(y_J) + Z_{wB}(0) \\
&\qquad (N_1 + 1 \leqslant J \leqslant N_1 + N_2) \\
R_J &= \sum_{i=0}^{M}\left[A_{\mathrm{RP}}(i, y_J) - C_{wB}(y_J)\left(\frac{2y_J}{L_{\mathrm{c}}}\right)^i\right]q_i - \gamma_1 y_J + BM_{\mathrm{RP}}(y_J) + Z_{wB}(0) \\
&\qquad (N_1 + N_2 + 1 \leqslant J \leqslant N_1 + N_2 + N_3) \\
R_J &= \sum_{i=0}^{M}\left[A'_{\mathrm{RP}}(i, y_J) - C_{wB}(y_J)\left(\frac{2y_J}{L_{\mathrm{c}}}\right)^i\right]q_i - \gamma_1 y_J + BM'_{\mathrm{RP}}(y_J) + Z_{wB}(0) \\
&\qquad (N_1 + N_2 + N_3 + 1 \leqslant J \leqslant N_1 + N_2 + N_3 + N_4) \quad (
\end{aligned}\right| \tag{18}$$

Let $N = N_1 + N_2 + N_3 + N_4$, the coupled Eqns. (18) has N equations and $M + 2$ unkown parameters, and it is a nonlinear coupled equations, as N is greater than $M + 2$, it is solved by the least square method and iteration method. The calculation precision of engineering can be met when M is equal to 12.

The solved process is shown as follows:

(1) Assume $q_i(i = 0, 1, 2, \cdots, M)$;

(2) Calculate $f(y_J)$, $B_{\mathrm{c}}(y_J)$ and $C_{wB}(y_J)(1 \leqslant J \leqslant N)$;

(3) Compute q_i and γ_1 with the least square method.

The coupled Eqns. (18) can be written in the following form

$$\boldsymbol{R}_{VS} = \boldsymbol{AQ} - \boldsymbol{F}_{VS} \tag{19}$$

where $\boldsymbol{R}_{VS} = [\boldsymbol{R}_1, \boldsymbol{R}_2, \boldsymbol{R}_3, \cdots, \boldsymbol{R}_N]^{\mathrm{T}}$; $\boldsymbol{Q} = [q_0, q_1, q_2, \cdots, q_M, \gamma_1]^{\mathrm{T}}$;

$\boldsymbol{F}_{VS}$ is $N \times 1$ order coefficients matrices made up of $-BM'_{\mathrm{LP}}(y_J) - Z_{wB}(0)$, $-BM_{\mathrm{LP}}(y_J) - Z_{wB}(0)$, $-BM_{\mathrm{RP}}(y_J) - Z_{wB}(0)$ and $-BM'_{\mathrm{RP}}(y_J) - Z_{wB}(0)$;

$\boldsymbol{A}$ is $N \times (M + 2)$ order coefficients matrices made up of $A'_{\mathrm{LP}}(i, y_J) - C_{wB}(y_J)\left(\frac{2y_J}{L_{\mathrm{c}}}\right)^i$, $A_{\mathrm{LP}}(i, y_J) - C_{wB}(y_J)\left(\frac{2y_J}{L_{\mathrm{c}}}\right)^i$, $A_{\mathrm{RP}}(i, y_J) - C_{wB}(y_J)\left(\frac{2y_J}{L_{\mathrm{c}}}\right)^i$, $A'_{\mathrm{RP}}(i, y_J) - C_{wB}(y_J) \times \left(\frac{2y_J}{L_{\mathrm{c}}}\right)^i$ and γ_1.

Let Π express a sum of squareness of residual error equations, that is

$$\Pi = \boldsymbol{R}_{VS}^{\mathrm{T}}\boldsymbol{R}_{VS} \tag{20}$$

The following system of algebraic equations is obtained by minimizing the total error Π with respect to the free parameters

$$\frac{\partial \Pi}{\partial q_i} = 0, \quad \frac{\partial \Pi}{\partial \gamma_1} = 0 \quad (i = 0, 1, 2, \cdots, M) \tag{21}$$

The coupled Eqns. (21) can be written as

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{AQ} - \boldsymbol{A}^{\mathrm{T}}\boldsymbol{F}_{VS} = 0 \tag{22}$$

In the above coupled equations, $M + 2$ equations are established to solve $M + 2$ optimal approximate results of free parameters q_i and γ_1.

(4) Calculate the distribution of pressure $f(y_J)$ between rolls according to q_i in the step(3), and the points of $f(y_J) \leqslant 0$ are removed, that is, the uncontacted ones at the edge of work and back-up rolls, then simulate the distribution of presure $f(y_J)$ with the least square method to get q'_i, which is compared with q_i in the step(1), if $\max|q'_i - q_i| \leqslant \varepsilon$, optimal approximate results of q_i and γ_1 are obtained, otherwise, exponential smoothing method should be used to modify q_i, towards the step(2).

The modification of $Z_{wB}(0)$ is that if the $B_c(y_J)$ is the constant, let $Z_{wB}(0) = C_{wB}(0)q_0$, the coupled Eqns. (18) is a linear coupled equations, then the following expression is established as long as that the collocation number N is greater than a certain value, and iterative method is not used to modify $Z_{wB}(0)$,

$$|\Sigma p(y_J)\Delta y_J + 2F_w - \Sigma f(y_J)\Delta y_J| / [\Sigma f(y_J)\Delta y_J] \leqslant 10^{-3} \tag{23}$$

If $B_c(y_J)$ is not a constant, the secant method is used to modify $Z_{wB}(0)$ to meet Eqn. (23)[4]. In this paper the secant method is applied to modify $Z_{wB}(0)$.

2.2 Elastic flattening between the work roll and strip

The elastic flattening between the work roll and strip is calculated by Tozawa formula[5].

3 METAL MODEL

Metal lateral flow is solved with the variation method.

The transverse distributions of the front and back tension stresses can be expressed by the following equations[6, 7]

$$\sigma_1(y) = \frac{T_1}{B\bar{h}} + \frac{E}{1-\gamma^2}\left[\frac{h(y)}{\bar{h}} - \frac{H(y)}{\bar{H}} + u'(y) - \frac{\Delta B}{B}\right] \tag{24}$$

$$\sigma_0(y) = \frac{T_0}{B\bar{H}} + \frac{E}{1-\gamma^2}\left\{\frac{\bar{H}h(y)[1+u'(y)]}{\bar{h}H(y)[1+2u(\frac{B}{2})/B]}\right\} \tag{25}$$

where T_1, T_0—total front and back tensions; $\bar{H}$, $\bar{h}$—integral averages of $H(y)$ and $h(y)$ across the whole width of strip; $u(y)$, ΔB—lateral displacement function of strip at the exit and width spread; E, γ—elastic modulus and poisson ratio of strip.

The lateral displacement function of strip at the exit can be calculated by the variation method[8], the distribution of rolling pressure can be computed by Stone formula. The model of elastic deformation of roll system is combined with metal model to simulate the shape of cold rolling strip.

4 SIMULATED RESULTS OF SHAPE ON 4-HIGH MILL WITH ROLL SHIFTED

In order to verify the established mathematical model, the shape and profile of cold rolling aluminum strip at the exit on 4-high mill are simulated. A special position (shifted distance $L_S = 0.$) is selected to simulate on 4-high mill with roll shifted, the simulated results are shown as Fig. 4. According to calculation conditions in the Ref. [6, 7], the calculated results have shown good agreement with experimental results[6, 7], the theory was proved to be correct by the author.

Fig. 5 shows the influence regulation of the shifted distance and bending force on the shape, from it, we can see that the shifted distance improves the ability and the bending force controls the shape.

5 CONCLUSIONS

The model of the roll system elastic deformation on 4-high mill with roll shifted is established by the least square and collocation method for the first time in this paper. The least square and collocation

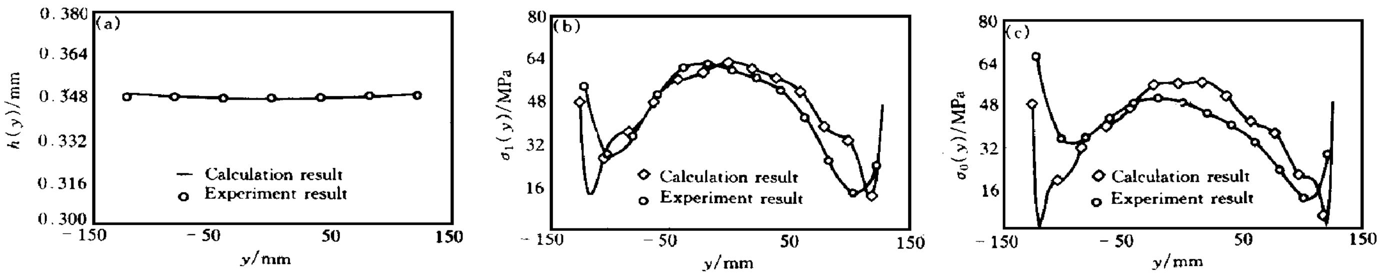

Fig. 4 Comparisons of calculated and experimental results

(a) —Distribution of thickness at the exit; (b) —Distribution of front tension stress; (c) —Distribution of back tension stress

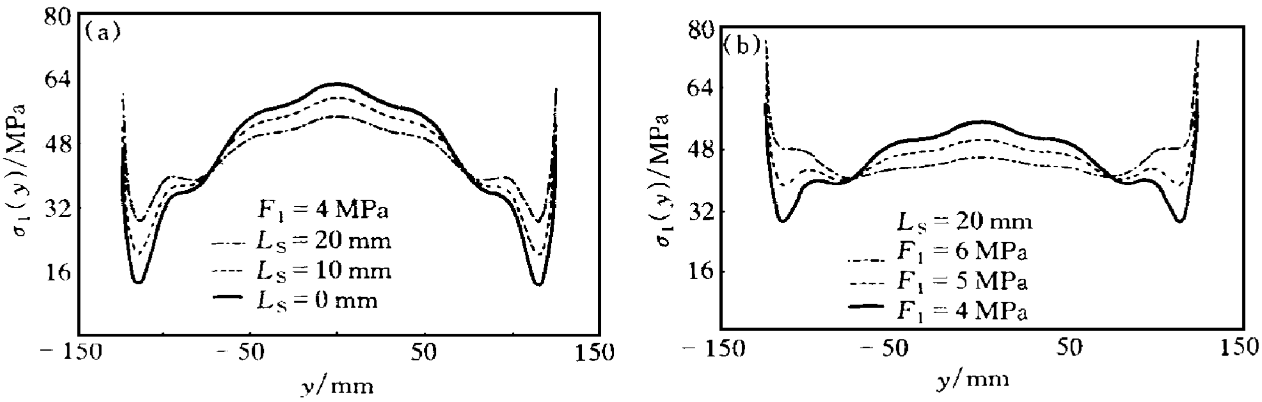

Fig. 5 Influences of the shifted distance and bending force on shape

(a) —Influence of the shifted distance on shape; (b) —Influence of the bending force on shape

methods are combined with the variation method to establish the mathematical model for the shape of cold rolling strip on 4-high mill with roll shifted. The distributions of tension stresses of aluminum strip are simulated by using the model, and the influence regulation of roll shifted distance and bending force on the shape are given. When the distribution of pressure between rolls are solved by the least square and collocation method, because of the few unknown parameters, the calculated speed is very fast and the efficience is rather high. The method is suitable for the study on the shape of wide strip mill and optimal design of technology parameters.

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