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NUMERICAL PREDICTION OF PULSATING FLOW IN A LIQUID LINE WITH BRANCH OF HYDRAULIC SERVO SYSTEM①

He Yongsen and Liu Shaoying
Changsha Institute of Mining Research, Changsha 410012, P. R. China
Kobayashi Toshio
Institute of Industrial Science, University of Tokyo, Japan

ABSTRACT Viscous flow equations for two-dimension were transformed into frequency characteristics formulae. Numerical prediction method, simulation model and application software have been used for calculating the dynamic characteristics (gain, phase, resonance angular frequency and maximal amplitude ratio, etc.) of pulsating flow in branch in the hydraulic servo systems, which can make the numerical prediction with digital computer highly effective and accurate. A new inspection method for hydraulic system was also provided.

Key words viscous flow numerical prediction pulsating flow hydraulic servo system

1 INTRODUCTION

The dynamic characteristics of fluid transmisson lines of the hydraulic control system have received the attention of many researchers in a number of fields such as mechanical engineering and so on [1, 2]. The accurate knowledge of the dynamic characteristics of pulsating flow in complex liguid lines of the hydraulic servo system is impotant.

The present study proposed the numerical method to be suited for predicting the frequency characteristics of pulsating flow in a liquid line with branch of the hydraulic servo system, so that the dynamic characteristics of the liquid line may be easily predicted with reasonable accuracy in the actual applications.

2 GOVERNING EQUATIONS

The computational fluid is the pulsating flow in a liquid line with branch of the hydraulic servo system as shown in Fig. 1.

Viscous flow equations for two-dimension

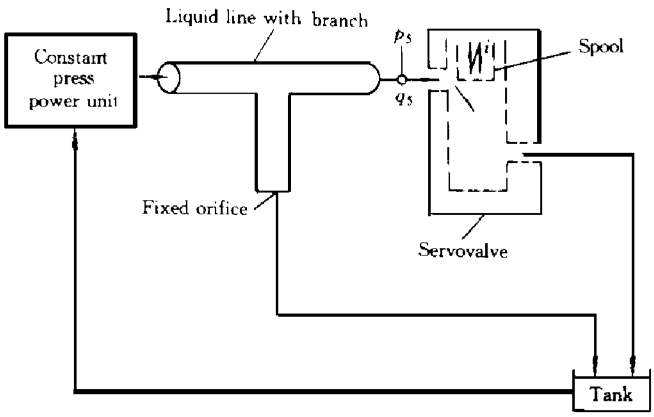

Fig. 1 Hydraulic servo system

are expressed as follows[3]:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{1}$$

$$\frac{1}{K_e}\frac{\partial p}{\partial t} + \frac{\partial v}{\partial r} + \frac{v}{r} + \frac{\partial u}{\partial x} = 0 \tag{2}$$

where u is an axial velocity component, v a radial velocity component, p the pressure, υ the kinematic viscosity and K_e the equivalent bulk modulus of liquid.

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3 NUMERICAL METHOD

The liquid line with the branch is shown in Fig. 2.

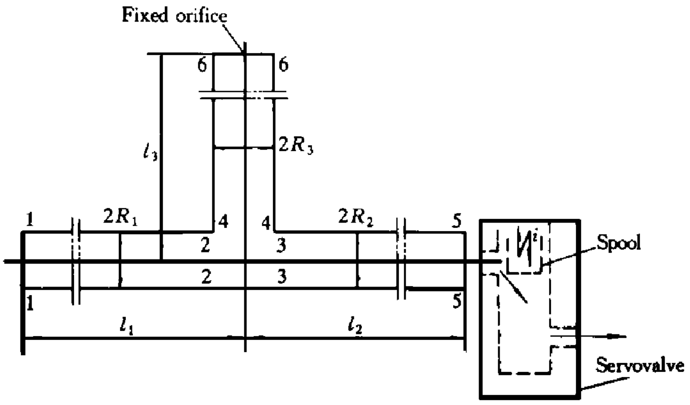

Fig. 2 Liquid line with branch

Concerning the flowrate at the branch, the continuity law is used and concerning the pressure at the branch, the common value is used for three pipes and moreover the pressure change due to the branch is neglected in the analysis. Assume $p_2 = p_3 = p_4$ and $q_2 = q_3 + q_4$.

Let $P(s)$ and $Q(s)$ be Laplace transfom of p and q respectively, taking the Laplace transform of Eqns. (1) and (2), the following transfer matrixes are obtained.

$$\begin{bmatrix} P_1(s) \\ Q_1(s) \end{bmatrix} = \begin{bmatrix} \cosh \gamma_1(s) l_1 & Z_{01}\sinh \gamma_1(s) l_1 \\ (1/Z_{01})\sinh \gamma_1(s) l_1 & \cosh \gamma_1(s) l_1 \end{bmatrix} \cdot \begin{bmatrix} P_2(s) \\ Q_2(s) \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} P_3(s) \\ Q_3(s) \end{bmatrix} = \begin{bmatrix} \cosh \gamma_2(s) l_2 & Z_{02}\sinh \gamma_2(s) l_2 \\ (1/Z_{02})\sinh \gamma_2(s) l_2 & \cosh \gamma_2(s) l_2 \end{bmatrix} \cdot \begin{bmatrix} P_5(s) \\ Q_5(s) \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} P_4(s) \\ Q_4(s) \end{bmatrix} = \begin{bmatrix} \cosh \gamma_3(s) l_3 & Z_{03}\sinh \gamma_3(s) l_3 \\ (1/Z_{03})\sinh \gamma_3(s) l_3 & \cosh \gamma_3(s) l_3 \end{bmatrix} \cdot \begin{bmatrix} P_6(s) \\ Q_6(s) \end{bmatrix} \quad (5)$$

where

$$\gamma_k(s) l_k = T e_k S / \left(\frac{I_0(Z_k)}{I_2(Z_k)}\right)^{\frac{1}{2}}$$

$$Z_{0k} = \frac{\rho a_k}{\pi R_k^2} \times \left(\frac{I_0(Z_k)}{I_2(Z_k)}\right)^{\frac{1}{2}}$$

here $k = 1, 2, 3$

The Bessel function terms are expressed as follows:

$$\left[\frac{I_0(Z_k)}{I_2(Z_k)}\right]^{1/2} = \partial \left[1 - \frac{2J_1(jR_k\sqrt{s/\upsilon})}{jR_k\sqrt{s/\upsilon}\, J_0(jR_k\sqrt{s/\upsilon})}\right]^{-1/2}$$

$k = 1, 2, 3$

$$a = \frac{\sqrt{K/\rho}}{\sqrt{1 + \frac{K}{E}\frac{2R}{\delta}(1 - \mu^2)}}$$

$$T_e = l/a$$

where $\gamma(s) l$ is a propagated operator, Z_0 is an impedance, J_0 and J_1 are the Bessel function of the first kind of the zero order and first order respectively; I_0 and I_2 are the modified Bessel fuction of the first kind of the zero order and the second order respectively; S is the operator of Laplace ($S = j\omega$, $j = \sqrt{-1}$, ω the angular frequency), K the bulk modulus of liquid, l the length of pipe, δ the thickness of the pipe wall; e is the yong ratio, μ the poisson ratio, Z the complex ($Z = R\sqrt{s/\upsilon}$), R the Radius of pipe, ρ the density, a the sound speed.

The mean opening of the control orifice of the servovalve is I_m as shown Fig. 3.

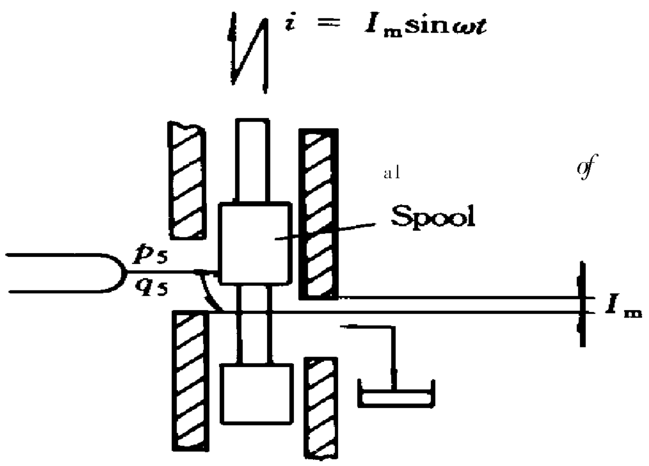

Fig. 3 Servovalve

The frequency response P_5 to input $i = I_m \sin\omega t$ is given as

$$P_5 = P_{5a}\sin(\omega t + \theta)$$

The pressure-flowrate characteristics of the control orifice is linearized at the neighborhood of the operating point, then have

$$Q_5(s) = C_1 I(s) + C_2 P_5(s) \quad (6)$$

where $I(s)$ is Laplace transform of excess opening $i(t)$ of the control orifice from the mean opening, and C_1 and C_2 are constant coefficients.

The pressure-flowrate characteristics of the fixed orifice which is set at the downstream end of the branched pipe is linearized at the neighborhood of the operating point, then have

$$Q_6(s) = K_2 P_6(s) \tag{7}$$

where K_2 is the constant coefficient of fixed orifice.

The end condition at the upstream end is as following:

$$P_1(s) = 0 \tag{8}$$

because the variation of the pressure is zero.

The Laplace transform of the both side of formulae p_2 and q_2 are given as

$$P_2(s) = P_3(s) = P_4(s) \tag{9}$$

$$Q_2(s) = Q_3(s) + Q_4(s) \tag{10}$$

Substitute Eqns. (6) ~ (10) into (3) ~ (5), the following equations are given as

$$\begin{vmatrix} 0 \\ Q_1(s) \end{vmatrix} = \begin{vmatrix} \cosh\gamma_1(s)l_1 & Z_{01}\sinh\gamma_1(s)l_1 \\ (1/Z_{01})\sinh\gamma_1(s)l_1 & \cosh\gamma_1(s)l_1 \end{vmatrix} \cdot \begin{vmatrix} P_2(s) \\ Q_3(s) + Q_4(s) \end{vmatrix} \tag{11}$$

$$\begin{vmatrix} P_3(s) \\ Q_3(s) \end{vmatrix} = \begin{vmatrix} \cosh\gamma_2(s)l_2 & Z_{02}\sinh\gamma_2(s)l_2 \\ (1/Z_{02})\sinh\gamma_2(s)l_2 & \cosh\gamma_2(s)l_2 \end{vmatrix} \cdot \begin{vmatrix} P_5(s) \\ C_1I(s) + C_2P_5 \end{vmatrix} \tag{12}$$

$$\begin{vmatrix} P_4(s) \\ Q_4(s) \end{vmatrix} = \begin{vmatrix} \cosh\gamma_3(s)l_3 & Z_{03}\sinh\gamma_3(s)l_3 \\ (1/Z_{03})\sinh\gamma_3(s)l_3 & \cosh\gamma_3(s)l_3 \end{vmatrix} \cdot \begin{vmatrix} P_6(s) \\ K_2P_6(s) \end{vmatrix} \tag{13}$$

Substitute $S = j\omega$ into Eqns. (11) ~ (13), the following equations are given as

$$\cosh\gamma_1(j\omega)\, l_1P_2(j\omega) + Z_{01}\sinh\gamma_1(j\omega) \cdot l_1Q_2(j\omega) = 0 \tag{14}$$

$$Q_1(j\omega) = (1/Z_{01})\sinh\gamma_1(j\omega)\, l_1P_2(j\omega) + \cosh\gamma_1(j\omega)\, l_1Q_2(j\omega) \tag{15}$$

$$P_3(j\omega) = \cosh\gamma_2(j\omega)\, l_2P_5(j\omega) + Z_{02}\sinh\gamma_2(j\omega)\, l_2[C_1I(j\omega) + C_2P_5(j\omega)] \tag{16}$$

$$Q_3(j\omega) = (1/Z_{02})\sinh\gamma_2(j\omega)\, l_2P_5(j\omega) + \cosh\gamma_2(j\omega)\, l_2[C_1I(s) + C_2P_5(j\omega)] \tag{17}$$

$$P_4(j\omega) = \cosh\gamma_3(j\omega)\, l_3P_6(j\omega) + Z_{03}\sinh\gamma_3(j\omega)\, l_3K_2P_6(s) \tag{18}$$

$$Q_4(j\omega) = (1/Z_{03})\sinh\gamma_3(j\omega)\, l_3P_6(j\omega) + \cosh\gamma_3(j\omega)\, l_3K_2P_6(j\omega) \tag{19}$$

The frequency transfer function is given as

$$G_5(j\omega) = \frac{P_5(j\omega)}{I(j\omega)} = -\frac{E - AC}{AB - D} \tag{20}$$

where

$$A = \frac{\dfrac{1}{Z_{03}}\sinh\gamma_3(s)l_3 - \dfrac{(\cosh\gamma_1(s)l_1)^2}{Z_{01}\sinh\gamma_1(s)l_1}}{\cosh\gamma_1(s)l_1 - \dfrac{(\sinh\gamma_1(s)l_1)^2}{\cosh\gamma_1(s)l_1}} - \frac{K_2\cosh\gamma_3(s)l_3 + (1/Z_{03})\sinh\gamma_3(s)l_3}{K_2Z_{03}\sinh\gamma_3(s)l_3 + \cosh\gamma_3(s)l_3}$$

$$B = \cosh\gamma_2(s)l_2 + C_2Z_{02}\sinh\gamma_2(s)l_2$$

$$C = C_1Z_{02}\sinh\gamma_2(s)l_2$$

$$D = C_2\cosh\gamma_2(s)l_2 + (1/Z_{02})\sinh\gamma_2(s)l_2$$

$$E = C_1\cosh\gamma_2(s)l_2$$

The gain and phase are given as

$$|G_5(j\omega)| = \sqrt{\{RE[G_5(j\omega)]\}^2 + \{IM[G_5(j\omega)]\}^2} \tag{21}$$

$$\angle G_5(j\omega) = -\operatorname{arctg}\{IM[G_5(j\omega)]/RE[G_5(j\omega)]\} \tag{22}$$

4 RESULTS AND DISCUSSION

The computational example is pulsating flow in a liquid line with branch of the hydraulic servo system, as shown in Fig. 1.

The constants of servovalve and liquid are shown in Table 1. The computational results of the frequency characteristics $|G_5(j\omega)|$ (Gain) and $\angle G_5(j\omega)$ (phase) are shown in Fig. 4. The computational results are compared with the experimental data by Nakano *et al*[4].

Fig. 4 suggests that the calculated results of the gain and phase agreed relatively well with the experiments except minimum amplitude ratio. The calculated results of resonance angular frequency and maximal amplitude ratio agreed relatively well with the experiments.

Table 1 Computational conditions(Experimental data[4])

Parameters	Expression and unit
Port area of the control orifice	$S = \pi B(I_m + i)$, cm^2
Diameter of Spool	$B = 0.8$ cm
Mean opening of servovalve	$I_m = 70$ μm
Constant coefficients of servovalve	$C_1 = \frac{\partial q_5}{\partial i}\Big\|_{p_{5m}=6.0\,\mathrm{MPa}} = 1.91\ \frac{\mathrm{cm^3/s}}{\mu\mathrm{m}}$
	$C_2 = \frac{\partial q_5}{\partial p_5}\Big\|_{I_m=70\,\mu\mathrm{m}} = 16.2\ \frac{\mathrm{cm^3/s}}{\mathrm{MPa}}$
Constant coefficient of fixed orifice	$K_2 = \frac{\partial q_6}{\partial p_6}\Big\|_{I_m=70\,\mu\mathrm{m}} = 16.2\ \frac{\mathrm{cm^3/s}}{\mathrm{MPa}}$
Mean flowrate	$q_{5m} = 10.6$ L/min
Mean pressure	$p_{5m} = 6.0$ MPa
Specific weight of oil	$\gamma = 0.85 \times 10^{-3}$ kg/cm^3
Kinematic viscosity of oil	$\upsilon = 0.3$ cm^2/s(40 ℃)
Bulk modulus of oil	$K = 1.7 \times 10^3$ MPa

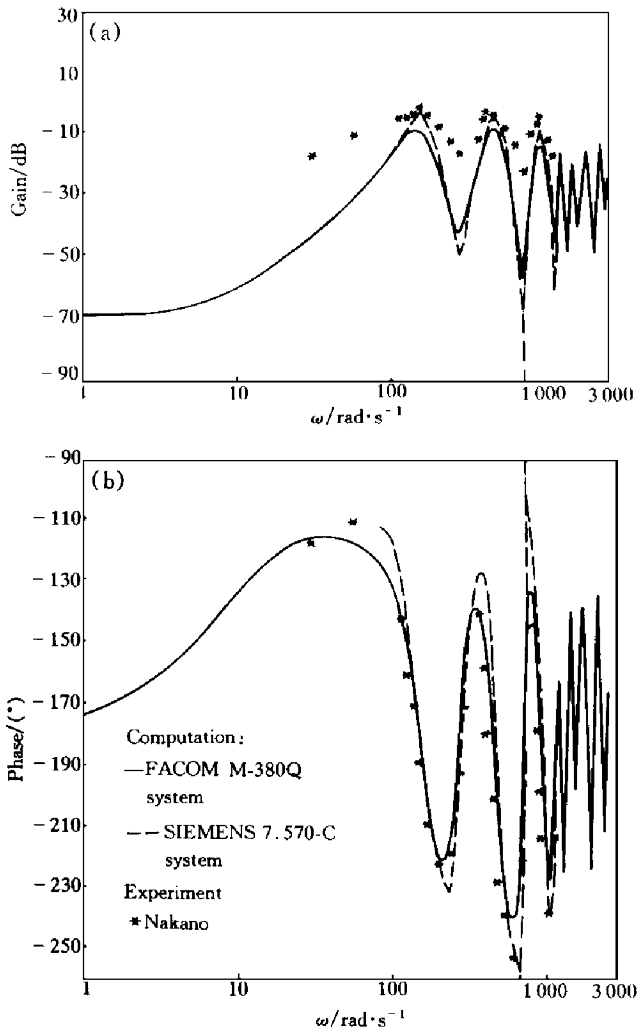

Fig. 4 Comparison of frequency characteristics (Gain and phase)

(a) —$|G_s(j\omega)|$; (b) —$\angle G_s(j\omega)$

5 CONCLUSIONS

It is confirmed from the computational results that the viscous flow equations for two-dimension can simulate a frequency characteristics of pulsating flow in a liquid line with branch of the hydraulic servo system, but not suitable for the low-frequency range. The numerical method and simulation model can make the numerical prediction of the dynamic characteristics highly efficient, cost effective and accurate from the practical viewpoint.

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