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Design of decentralized robust stabilizing controller for interconnected time-varying uncertain systems①

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Abstract: For interconnected uncertain systems which are time-varying and assumed to satisfy the matching conditions, a sufficient condition for decentralized stabilization feedback control laws is derived. This condition is expressed as the solvability problem of linear matrix inequalities (LMIs). Based on that, a convex optimization problem with linear matrix inequality (LMI) constraints is formulated to design a decentralized state feedback control with smaller gain parameters which enables the closed-loop system asymptotically stable.

Key words: uncertainty; interconnected systems; LMI; decentralized control; robustness
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1 INTRODUCTION

In recent years, decentralized control for interconnected large systems has been paid much attention[1]. Since there exists uncertainties in the system models, performance can not be attained if the controller design is only based on nominal plant, which results in the widely study of robust stabilization for uncertain interconnected large systems[2~7]. However, in most papers, only sufficient conditions were given in Riccati equalities or inequalities, which needed preadjustment of parameters in advance, and was very complicated and inconvenient.

LMI has been paid much attention for its high solvability and become an effective method for robust analysis and synthesis[8,9]. In this paper, LMI is used to study the decentralized stabilization for time-varying uncertain interconnected large systems, which avoids Riccati method and has not been seen in other papers. For a class of time-varying uncertain interconnected large systems, which satisfy the matching condition, we obtain the sufficient condition for decentralized state-feedback stabilizability, meanwhile, give the convex optimization method for designing controller with smaller gain parameters.

2 PROBLEM DESCRIBING

Consider a class of time-varying uncertain interconnected large systems $\boldsymbol{L}$ with $\boldsymbol{N}$ subsystems $\boldsymbol{L}_i$, satisfying the matching condition, the subsystems can be described as follows:

$$\begin{aligned} \boldsymbol{L}_i: \dot{\boldsymbol{x}}_i = {} & [\boldsymbol{A}_i + \boldsymbol{B}_i\Delta\boldsymbol{A}_i(r_i(t))]\boldsymbol{x}_i(t) + \\ & [\boldsymbol{B}_i + \boldsymbol{B}_i\Delta\boldsymbol{B}_i(s_i(t))]\boldsymbol{u}_i(t) + \\ & \sum_{\substack{j=1 \\ j\neq i}}^{N}\boldsymbol{B}_i\boldsymbol{H}_{ij}(\boldsymbol{x}_j(t), t) \\ & i = 1, 2, \cdots, N \end{aligned} \tag{1}$$

where $\boldsymbol{x}_i(t) \in \boldsymbol{R}^{n_i}$ is the state vector, $\boldsymbol{u}_i(t) \in \boldsymbol{R}^{m_i}$ is the control vector; $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ are nominal matrixes with suitable dimensions; $\Delta\boldsymbol{A}_i(r_i(t))$, $\Delta\boldsymbol{B}_i(s_i(t))$ are continuous uncertainties on $r_i(t)$ and $s_i(t)$ with the compatible dimensions as $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$; $\boldsymbol{H}_{ij}$ is the interconnected matrix of the jth subsystem to the ith subsystem; $r_i(t) \in \boldsymbol{R}^{l_{ri}}$, $s_i(t) \in \boldsymbol{R}^{l_{si}}$ belong to Lebesgue measurable compact sets $\boldsymbol{R}_i$ and $\boldsymbol{S}_i$:

$$\left.\begin{aligned} \boldsymbol{R}_i = : \{r \,\|\, r_{ij} | \leqslant \bar{r}_i,\ j = 1, \cdots, l_{ri}\} \\ \boldsymbol{S}_i = : \{s \,\|\, s_{ij} | \leqslant \bar{s}_i,\ j = 1, \cdots, l_{si}\} \\ i = 1, 2, \cdots, N \end{aligned}\right\} \tag{2}$$

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Suppose $\Delta \boldsymbol{A}_i$ and $\Delta \boldsymbol{B}_i$ can be expressed as the sum of matrixes with rank 1, i.e.

$$\left.\begin{aligned} \Delta \boldsymbol{A}_i(r_i(t)) &= \sum_{j=1}^{l_{ri}} \boldsymbol{A}_{ij} r_{ij} \\ \Delta \boldsymbol{B}_i(s_i(t)) &= \sum_{j=1}^{l_{si}} \boldsymbol{B}_{ij} s_{ij} \\ i &= 1, 2, \cdots, N \end{aligned}\right\} \quad (3)$$

where $\boldsymbol{A}_{ij}$, $\boldsymbol{B}_{ij}$ satisfy

$$\boldsymbol{A}_{ij} = \boldsymbol{d}_{ij}\boldsymbol{e}_{ij}^{\mathrm{T}}, \quad \boldsymbol{B}_{ij} = \boldsymbol{f}_{ij}\boldsymbol{g}_{ij}^{\mathrm{T}} \quad (4)$$

$\boldsymbol{d}_{ij}$, $\boldsymbol{e}_{ij}$, $\boldsymbol{f}_{ij}$, $\boldsymbol{g}_{ij}$ are vectors with rank 1. For convenience, introducing the following symbols:

$$\left.\begin{aligned} \boldsymbol{T}_i &= : \bar{r}_i \sum_{j=1}^{l_{ri}} \boldsymbol{d}_{ij}\boldsymbol{d}_{ij}^{\mathrm{T}}, \quad \boldsymbol{U}_i = : \bar{r}_i \sum_{j=1}^{l_{ri}} \boldsymbol{e}_{ij}\boldsymbol{e}_{ij}^{\mathrm{T}} \\ \boldsymbol{V}_i &= : \bar{s}_i \sum_{j=1}^{l_{si}} \boldsymbol{f}_{ij}\boldsymbol{f}_{ij}^{\mathrm{T}}, \quad \boldsymbol{Q}_i = : \bar{s}_i \sum_{j=1}^{l_{si}} \boldsymbol{g}_{ij}\boldsymbol{g}_{ij}^{\mathrm{T}} \end{aligned}\right\} \quad (5)$$

suppose for all $i, j \in \{1, \cdots, N\}$, there exists a positive constants ξ_{ij} satisfying

$$\| \boldsymbol{H}_{ij}(\boldsymbol{x}_j, t) \| \leqslant \xi_{ij} \| \boldsymbol{x}_j \| \quad (6)$$

Suppose the states for all subsystems can be measured, the aim of this paper is to design a local memoryless state-feedback control law for each subsystem:

$$\boldsymbol{u}_i(t) = \boldsymbol{K}_i \boldsymbol{x}_i(t) \quad (7)$$

where $\boldsymbol{K}_i \in \boldsymbol{R}^{mj \times nj}$ are local feedback gain matrixes to stabilize the closed-loop system.

The following are several important lemmas.

Lemma 1 Schur complement suppose symmetric matrix $\boldsymbol{F} = \boldsymbol{F}^{\mathrm{T}} \in \boldsymbol{R}^{(N+M)\times(N+M)}$ can be divided into four blocks as following:

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B}^{\mathrm{T}} \\ \boldsymbol{B} & \boldsymbol{C} \end{bmatrix}$$

if $\boldsymbol{C} \in \boldsymbol{R}^{M\times M}$ is nonsingular, then $\boldsymbol{F} > 0$, if and only if $\boldsymbol{C} > 0$, and $\boldsymbol{A} - \boldsymbol{B}^{\mathrm{T}}\boldsymbol{C}^{-1}\boldsymbol{B} > 0$; if $\boldsymbol{A} \in \boldsymbol{R}^{N\times N}$ is nonsingular, then $\boldsymbol{F} > 0$, if and only if $\boldsymbol{A} > 0$, and $\boldsymbol{C} - \boldsymbol{B}\boldsymbol{A}^{-1}\boldsymbol{B}^{\mathrm{T}} > 0$.

Lemma 2 Suppose $\boldsymbol{X}$ and $\boldsymbol{Y}$ are vectors or matrixes with suitable dimensions, then for any positive $\alpha > 0$, we have

$$\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y} + \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{X} \leqslant \alpha \boldsymbol{X}^{\mathrm{T}}\boldsymbol{X} + \frac{1}{\alpha}\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}$$

Lemma 3 Suppose $\boldsymbol{Y}$ and $\boldsymbol{Q}$ are vectors or matrixes with suitable dimensions, where $\boldsymbol{Q} > 0$, α, β are given positive numbers, then

$\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y} < \alpha \boldsymbol{I}$, if and only if:

$$\begin{bmatrix} -\alpha \boldsymbol{I} & \boldsymbol{Y}^{\mathrm{T}} \\ \boldsymbol{Y} & -\boldsymbol{I} \end{bmatrix} < 0$$

$\boldsymbol{Q}^{-1} < \beta \boldsymbol{I}$, if and only if:

$$\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{I} \\ \boldsymbol{I} & \beta \boldsymbol{I} \end{bmatrix} > 0$$

Proof: It can be proven by **Lemma 1** and matrixes' transformations.

Remark 1: $\| \boldsymbol{M} \|$ is defined as the maximum singular value of $\boldsymbol{M}$, $\| \alpha \|$ is the 2-norm of α, $\boldsymbol{I}$ represents different unit matrixes at different places.

3 DESIGN OF DECENTRALIZED STABILIZING CONTROLLER

In this section, sufficient conditions for stabilizability of time-varying uncertain interconnected large system is given, meanswhile, the design of decentralized stabilizing control low with smaller feedback gain is given too.

With Eqn. (7), the closed-loop is

$$\begin{aligned} \dot{\boldsymbol{x}}_i(t) = & [\boldsymbol{A}_i + \boldsymbol{B}_i \Delta \boldsymbol{A}_i(r_i(t))]\boldsymbol{x}_i(t) + \\ & [\boldsymbol{B}_i + \boldsymbol{B}_i \Delta \boldsymbol{B}_i(s_i(t))]\boldsymbol{K}_i \boldsymbol{x}_i(t) + \\ & \sum_{\substack{j=1 \\ j\neq i}}^{N} \boldsymbol{B}_i \boldsymbol{H}_{ij}(\boldsymbol{x}_i(t), t) \end{aligned} \quad (8)$$

let Lyapunov function

$$\boldsymbol{V}(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x} = \sum_{i=1}^{N} \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i = \sum_{i=1}^{N} \boldsymbol{V}(\boldsymbol{x}_i)$$

where

$$\boldsymbol{x} = [\boldsymbol{x}_1^{\mathrm{T}}, \ \boldsymbol{x}_2^{\mathrm{T}}, \cdots \boldsymbol{x}_N^{\mathrm{T}}]^{\mathrm{T}}$$

$$\boldsymbol{P} = \mathrm{diag}\{\boldsymbol{P}_1, \ \boldsymbol{P}_2, \cdots, \boldsymbol{P}_N\}$$

we have

$$\begin{aligned} \dot{\boldsymbol{V}}(x) &= \sum_{i=1}^{N} \dot{\boldsymbol{V}}_i(\boldsymbol{X}_i) \\ &= \sum_{i=1}^{N} (\dot{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i + \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\dot{\boldsymbol{x}}_i) \\ &= \sum_{i=1}^{N} \{\boldsymbol{x}_i^{\mathrm{T}}(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{A}_i + \\ &\quad \Delta \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{B}_i\Delta \boldsymbol{A}_i + \boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}_i + \\ &\quad \boldsymbol{P}_i\boldsymbol{B}_i\boldsymbol{K}_i + \boldsymbol{K}_i^{\mathrm{T}}\Delta \boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}_i + \\ &\quad \boldsymbol{P}_i\boldsymbol{B}_i\Delta \boldsymbol{B}_i\boldsymbol{K}_i)\boldsymbol{x}_i + \\ &\quad 2\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{B}_i \sum_{\substack{j=1 \\ j\neq i}}^{N} \boldsymbol{H}_{ij}(\boldsymbol{x}_j(t), t)\} \end{aligned}$$

Put Eqns. (3) ~ (5) into the above equation, then we get

$$\begin{aligned} \dot{\boldsymbol{V}}(\boldsymbol{x}) &= \sum_{i=1}^{N} \dot{\boldsymbol{V}}_i(\boldsymbol{X}_i) \\ &= \sum_{i=1}^{N} (\dot{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i + \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\dot{\boldsymbol{x}}_i) \end{aligned}$$

$$= \sum_{i=1}^{N} \{ x_i^T (A_i^T P_i + P_i A_i + \sum_{j=1}^{l_{ri}} (B_i A_{ij} r_{ij})^T P_i + \sum_{j=1}^{l_{ri}} P_i B_i A_{ij} r_{ij} + K_i^T B_i^T P_i + P_i B_i K_i + \sum_{j=1}^{l_{si}} K_i^T (B_i B_{ij} s_{ij})^T P_i + \sum_{j=1}^{l_{si}} P_i B_i B_{ij} s_{ij} K_i) x_i + 2 x_i^T P_i B_i \sum_{\substack{j=1 \\ j \neq i}}^{N} H_{ij} (x_j (t), t) \} \tag{9}$$

From **Lemma 2**, we can get

$$\sum_{j=1}^{l_{ri}} (B_i A_{ij} r_{ij})^T P_i + \sum_{j=1}^{l_{ri}} P_i B_i A_{ij} r_{ij} \leqslant \alpha_i P_i B_i T_i B_i^T P_i + \frac{1}{\alpha_i} U_i \tag{10}$$

$$\sum_{j=1}^{l_{si}} K_i^T (B_i B_{ij} s_{ij})^T P_i + \sum_{j=1}^{l_{si}} P_i B_i B_{ij} s_{ij} K_{ij} \leqslant \beta_i P_i B_i V_i B_i^T P_i + \frac{1}{\beta_i} K_i^T Q_i^T K_i \tag{11}$$

Since $a^2 + b^2 \geqslant 2ab$, $\forall\, a, b \in \mathbf{R}$, we have

$$\sum_{i=1}^{N} 2 x_i^T P_i B_i \sum_{\substack{j=1 \\ j \neq i}}^{N} H_{ij} (x_j (t), t) \leqslant 2 \sum_{i=1}^{N} \| B_i^T P_i x_i \| \sum_{\substack{j=1 \\ j \neq i}}^{N} \xi_{ij} \| x_j \| \leqslant \sum_{i=1}^{N} (\sum_{\substack{j=1 \\ j \neq i}}^{N} \xi_{ij} \| B_i^T P_i x_i \|^2 + \sum_{\substack{j=1 \\ j \neq i}}^{N} \xi_{ji} \| x_i \|^2) = \sum_{i=1}^{N} (F_{1i} \| B_i^T P_i x_i \|^2 + F_{2i} \| x_i \|^2) \tag{12}$$

where $F_{1i} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \xi_{ij}$, $F_{2i} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \xi_{ji}$

Put Eqns. (10) ~ (12) into Eqn. (9), the following can be obtained

$$\dot{V}(x) \leqslant \sum_{i=1}^{N} \{ x_i^T (A_i^T P_i + P_i A_i + K_i^T B_i^T P_i + P_i B_i K_i + \alpha_i P_i B_i T_i B_i^T P_i + \frac{1}{\alpha_i} U_i + \beta_i P_i B_i V_i B_i^T P_i + \frac{1}{\beta_i} K_i^T Q_i K_i) x_i + F_{1i} \| B_i^T P_i x_i \|^2 + F_{2i} \| x_i \|^2 \}$$

$$= \sum_{i=1}^{N} \{ x_i^T (A_i^T P_i + P_i A_i + K_i^T B_i^T P_i + P_i B_i K_i + \alpha_i P_i B_i T_i B_i^T P_i + \frac{1}{\alpha_i} U_i + \beta_i P_i B_i V_i B_i^T P_i + \frac{1}{\beta_i} K_i^T Q_i K_i) x_i + F_{1i} x_i^T P_i B_i B_i^T P x_i + F_{2i} x_i^T x_i \}$$

$$= \sum_{i=1}^{N} x_i^T (A_i^T P_i + P_i A_i + K_i^T B_i^T P_i + P_i B_i K_i + \alpha_i P_i B_i T_i B_i^T P_i + \frac{1}{\alpha_i} U_i + \beta_i P_i B_i V_i B_i^T P_i + \frac{1}{\beta_i} K_i^T Q_i K_i + F_{1i} P_i B_i B_i^T P + F_{2i} I_i) x_i$$

From Lyapunov stability theorem, if the inequality

$$A_i^T P_i + P_i A_i + K_i^T B_i^T P_i + P_i B_i K_i + \beta_i P_i B_i V_i B_i^T P_i + \frac{1}{\beta_i} K_i^T Q_i K_i + F_{1i} P_i B_i B_i^T P + F_{2i} I + \alpha_i P_i B_i T_i B_i^T P_i + \frac{1}{\alpha_i} U_i < 0 \tag{13}$$

has a positive defined symmetric solution P_i, then large system Eqn. (1) can be state-feedback stabilized.

From Eqn. (5), U_i, Q_i, $(i = 1, 2, \cdots, N)$ are positive defined or semi-positive defined matrixes, which can be divided into $U_i = U_i^{1/2} U_i^{1/2}$, $Q_i = Q_i^{1/2} Q_i^{1/2}$, then left-multiply and right-multiply Eqn. (13) with P_i^{-1}, let $X_i = P_i^{-1}$, $Y_i = K_i X_i$, from **Lemma 1**, matrix inequality Eqn. (13) is equivalent to the following LMI

$$\begin{bmatrix} \bar{A}_i & X_i U_i^{\frac{1}{2}} & Y_i^T Q_i^{\frac{1}{2}} & X_i \\ U_i^{\frac{1}{2}} X_i & -\alpha_i I & 0 & 0 \\ Q_i^{\frac{1}{2}} Y_i & 0 & -\beta_i I & 0 \\ X_i & 0 & 0 & -\frac{1}{F_{2i}} I \end{bmatrix} < 0 \tag{14}$$

where

$$\bar{A}_i = A_i^T X_i + X_i A_i + Y_i^T B_i^T + B_i Y_i + \alpha_i B_i T_i B_i^T + \beta_i B_i V_i B_i^T + F_{1i} B_i B_i^T$$

Therefore, if Eqn. (14) is solvable, the closed-loop is asymptotically stable. we have the following theorem.

Theorem 1 For time-varying uncertain interconnected large system Eqn. (1), if there exist positive defined matrix $\boldsymbol{X}_i$, matrix $\boldsymbol{Y}_i$, positive number α_i, β_i and the LMI Eqn. (14), then the large system (1) can be decentralized state-feedback stabilizable, with the decentralized state-feedback gain $\boldsymbol{K}_i = \boldsymbol{Y}_i\boldsymbol{X}_i^{-1}$.

From **Theorem 1**, the sufficient condition is given, which, however , can not guarantee the feedback gain as smaller as possible. In engineering applications, control laws with smaller feedback gain are always adopted to guarantee performance and better disturbance-rejection.

To get smaller feedback gain matrix, we consider

$$\boldsymbol{Y}_i^{\mathrm{T}}\boldsymbol{Y}_i < \theta_i\boldsymbol{I}_i, \quad \boldsymbol{X}_i^{-1} < \gamma_i\boldsymbol{I} \tag{15}$$

where $\theta_i > 0$, $\gamma_i > 0$, then $\boldsymbol{K}_i^T\boldsymbol{K}_i = \theta_i\boldsymbol{X}_i^{-1}\boldsymbol{Y}_i^T\boldsymbol{Y}_i\boldsymbol{X}_i^{-1} < \theta_i\gamma_i^2\boldsymbol{I}_i$.

Therefore, smaller feedback matrix can be obtained by minimizing θ_i, γ_i.

From **Lemma 3**, Eqn. (15) is equivalent to

$$\begin{bmatrix} -\theta_i\boldsymbol{I}_i & \boldsymbol{Y}_i^{\mathrm{T}} \\ \boldsymbol{Y}_i & -\boldsymbol{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \boldsymbol{X}_i & \boldsymbol{I} \\ \boldsymbol{I} & \gamma_i\boldsymbol{I} \end{bmatrix} > 0$$

then, decentralized control law with smaller feedback gain for system Eqn. (1) can be solved by the following optimization problem

$$\min\left(\sum_{i=1}^{N}\theta_i + \sum_{i=1}^{N}\gamma_i\right)$$

with constraints

$$\begin{bmatrix} \overline{\boldsymbol{A}}_i & \boldsymbol{X}_i\boldsymbol{U}_i^{\frac{1}{2}} & \boldsymbol{Y}_i^{\mathrm{T}}\boldsymbol{Q}_i^{\frac{1}{2}} & \boldsymbol{X}_i \\ \boldsymbol{U}_i^{\frac{1}{2}}\boldsymbol{X}_i & -\alpha_i\boldsymbol{I}_i & 0 & 0 \\ \boldsymbol{Q}_i^{\frac{1}{2}}\boldsymbol{Y}_i & 0 & -\beta_i\boldsymbol{I}_i & 0 \\ \boldsymbol{X}_i & 0 & 0 & -\dfrac{1}{F_{2i}}\boldsymbol{I} \end{bmatrix} < 0$$

$$\begin{bmatrix} -\theta_i\boldsymbol{I} & \boldsymbol{Y}_i^{\mathrm{T}} \\ \boldsymbol{Y}_i & -\boldsymbol{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \boldsymbol{X}_i & \boldsymbol{I} \\ \boldsymbol{I} & \gamma_i\boldsymbol{I} \end{bmatrix} > 0$$

This is a convex optimization problem with LMI constraints, which can be solved by LMI toolbox.

4 EXAMPLE

Consider the following time-varying uncertain interconnected system which contains two subsystems where

$$\boldsymbol{L}_1: \dot{\boldsymbol{x}}_1(t) = \begin{bmatrix} -1 & 2 \\ -1 + r_1(t) & r_1(t) \end{bmatrix}\boldsymbol{x}_1 + \begin{bmatrix} 0 \\ 1 + s_1(t) \end{bmatrix}\boldsymbol{u}_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix}[\sin x_{21} \quad \cos x_{21}][\boldsymbol{x}_2]$$

$$\boldsymbol{L}_2: \dot{\boldsymbol{x}}_2(t) = \begin{bmatrix} -3 & 0 \\ 2r_2(t) & -1 + r_2(t) \end{bmatrix}\boldsymbol{x}_2 + \begin{bmatrix} 0 \\ 1 + s_2(t) \end{bmatrix}\boldsymbol{u}_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix}[\sin x_{12} \quad \cos x_{12}][2\boldsymbol{x}_1]$$

where $|r_1(t)| \leqslant 0.5$, $|r_2(t)| \leqslant 1$, $|s_1(t)| \leqslant 0.4$, $|s_2(t)| \leqslant 0.3$.

Using the approach introduced, the convex optimization problem is solved in LMI tool, the decentralized stabilizing control law is

$$\boldsymbol{u}_1(t) = [-0.6829 \quad -5.3526]\boldsymbol{x}_1(t)$$

$$\boldsymbol{u}_2(t) = [-2.0779 \quad -3.2677]\boldsymbol{x}_2(t)$$

Remark 2 The LMIs have many parameters, however, they can be solved at one time in LMI tool. It is very convenient and need not to adjust of parameters, which overcomes the shortcoming of Riccati equalities approach.

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