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Research on rheologic characteristics of soil based on endo-chronic theory①

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Abstract: Rheologic characteristics of soil are very important in soil mechanics. Almost every aspect of research on soil mechanics deals with rheologic characteristics of soil. Although more than half century has passed, study on rheologic characteristics of soil only made a very small progress, which was due to the two facts: (1) the complexity of rheologic characteristics of soil; (2) the research methods based on experimental formulas. Based on thermodynamic theory and endo-chronic theory, a new intrinsic constitutive equation for rheologic characteristic of soil was developed, which combined with two groups of internal state variables and their correlative intrinsic time. The new intrinsic constitutive equation did very well in predict essential features of the rheologic behavior of soil, with which the dilatancy of soil during the rheologic process could be described.

Key words: soil mechanics; rheology; endo-chronic theory; viscoplasticity **Document code**: A

1 INTRODUCTION

Research on intrinsic constitutive equation of rheologic characteristics of soil is very important in solving problems in engineering. During past seventy years, progress had been made in this field, while it was difficult to develop an intrinsic constitutive equation, which could predict essential features of rheologic characteristics of soil objectively. This may owe mainly to complexity of the rheologic characteristics of soil and extension of the problem. There is no intrinsic constitutive equation, which can describe the rheologic characteristics of soil under complex load condition, or constitutive equation, which can describe the dilatancy of soil during the rheologic process at present. On the other hand, some new and more difficult problems were met in engineering during the recent years, with developing on civil engineering and mine engineering. Therefore, there is an urgent need to develop realistic sophisticated constitutive equations for rheologic characteristics of soil.

To meet this requirement, the authors developed a new intrinsic constitutive equation for rheologic characteristics of soil based on the theory of intrinsic time, and verified the new constitutive equation with laboratory triaxial tests.

2 THEORETICAL FORMULATION

Endo-chronic theory was introduced by Valanis[1,2] in order to describe irreversible thermodynamic process in 1971. Based on endo-chronic theory, the state of dissipative material in irreversible thermodynamic process can be described with one point in intrinsic space instead of point in stress space or strain space. Intrinsic time is a function of Euclid module of the increment of the arc of the stress or strain. It never decreases and can be used to determine the state of material in irreversible thermodynamic process alone. Endo-chronic theory does not base on the concept of yield surface, while, it works well with the existence of yield surface. The analytical form of the yield surface can even be solved with endo-chronic theory.

Soil can be treated as dissipative material, its rheologic process under external load is irre-

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versible thermodynamic process and can be treated with endo-chronic theory. Considering the rheologic characteristics of soil, we can describe soil rheologic process with n number of internal variable q^a. Each internal variable q^a can be separated into two parts: one is $q_{kk}{}^a$ for the representation of hydrostatic response caused by hydrostatic stress σ_{kk}, and its related intrinsic time Z_H; the other is $q_{ij}{}^a$ for the representation of deviatoric response caused by deviatoric stress S_{ij}, and its related intrinsic time Z_S. According to the definition introduced by Valanis, the increment of intrinsic time can be written as following:

$$dZ = \chi(\zeta)dt \tag{1}$$

where Z is intrinsic time, t is Newton time, ζ is scale of intrinsic time. Considering the dilatancy of soil during the rheologic process, dZ_H and dZ_S can be defined as:

$$\left.\begin{aligned} dZ_H &= \frac{\beta_{21}dt}{t} + \frac{\beta_{22}d\zeta_H}{\zeta_H} + \frac{\beta_{23}d\zeta_S}{\zeta_S} \\ dZ_S &= \frac{\beta_{11}dt}{t} + \frac{\beta_{12}d\zeta_S}{\zeta_S} \end{aligned}\right\} \tag{2}$$

in which[2,3]:

$$\left.\begin{aligned} d\zeta_S &= \left\| de_{ij} - \frac{k_1 dS_{ij}}{2\mu} \right\| \\ d\zeta_H &= \left| d\varepsilon_{kk} - \frac{k_0 d\sigma_{kk}}{2K} \right| \end{aligned}\right\} \tag{3}$$

where S_{ij}, σ_{kk} are deviatoric and hydrostatic stress, respectively; e_{ij} and ε_{kk} are deviatoric and hydrostatic strain respectively; μ, K are tangent and volumetric elastic modulus, respectively. β_{11}, β_{12}, β_{21}, β_{22}, β_{23}, K, k_1 and k_0 are material constants. For k_1 and k_0, the value is between $0\sim1$, and

$$\left.\begin{aligned} &k_1 = 0\ ,\ k_0 = 0 \\ &\text{with viscoplasticity sumption} \\ &\beta_{23} = 0 \\ &\text{without shear stress} \end{aligned}\right\} \tag{4}$$

According to the method presented by Wu[4] and Valanis[2], we can obtain the follow equation under viscoplastic condition:

$$S_{ij} = S_{ij}^0 \frac{\partial \varepsilon_{ij}^p}{\partial Z_S} + 2\mu_{11}\varepsilon_{ij}^p + 2\mu_{12}\int_0^{Z_S} e^{\alpha_S(Z_S - Z'_S)} \frac{\partial \varepsilon_{ij}^p}{\partial Z'_S} dZ'_S \tag{5}$$

where S_{ij}^0, μ_{11}, μ_{12} and α_S are constants, ε_{ij}^p is plastic deviatoric strain, and

$$\sigma_{kk} = \sigma_{kk}^0 \frac{\partial \varepsilon_{kk}^p}{\partial Z_H} + 2\mu_{21}\varepsilon_{kk}^p + 2\mu_{22}\int_0^{Z_H} e^{\alpha_H(Z_H - Z'_H)} \frac{\partial \varepsilon_{kk}^p}{\partial Z'_H} dZ'_H \tag{6}$$

where σ_{kk}^0, μ_{21}, μ_{22} and α_H are constants, ε_{kk}^p is plastic volumetric strain.

It is very difficult to obtain the common solution of Eqn. (5) or (6). There is no requirement to do that in fact. Our goal is to find the solution of the questions we will meet in engineering with Eqns. (5) and (6). The basic and most important questions in engineering are settlement of construction and stability of slope. They are all problems of no more than two dimensions, and can be studied with triaxial test. So, only the solutions of Eqns. (5) and (6) under condition of triaxial test are presented in this paper.

According to the method introduced by Li[5], the solution of Eqn. (5) can be given by:

$$q = \left[E_1 + E_c\left(\frac{t}{t_0}\right)^{-m}\gamma^{-n}\right]\gamma \tag{7}$$

where q is deviatoric stress; γ is deviatoric strain; E_1, E_c, m and n are constants; t is Newton time; t_0 is unit time(such as 1 s, 1 min or 1 a etc.).

We can also obtain the solution of Eqn. (6) in the same way:

$$p = \left[E_{vl} + E_{vc}\left(\frac{t}{t_0}\right)^{-f}\gamma^{-w}\left|\varepsilon_{kk}\right|^{-S}\right]\varepsilon_{kk} \tag{8}$$

where p is normal stress; ε_{kk} is normal strain; E_{vl}, E_{vc}, f, w and s are constants. Eqns. (7) and (8) are the constitutive equations for the rheologic characteristics of soil under the condition of triaxial test.

3 VERIFICATION OF INTRINSIC CONSTITUTIVE EQUATIONS

To verify Eqns. (7) and (8), the constants must be calculated at first. There are two methods to solve the equations, those are geometric method and numeric method[6].

The verification of Eqns. (7) and (8) was performed with the results of triaxial and torsional tests, the constants were calculated at first with geometric method or numeric method, then theoretical value of stress or strain were compared with the results of the tests.

Figs. 1 and 2 are the theoretical creep curve comparing with the results of torsional tests[7,8]. The compatibility of the theoretical value with the result of tests is quite satisfactory.

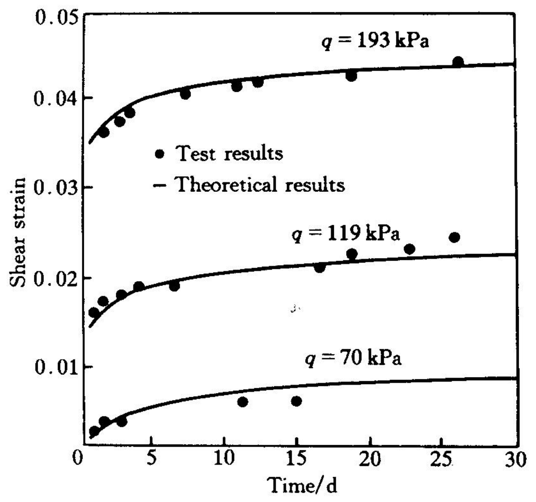

Fig.1 Theoretical curves versus results of torsional tests

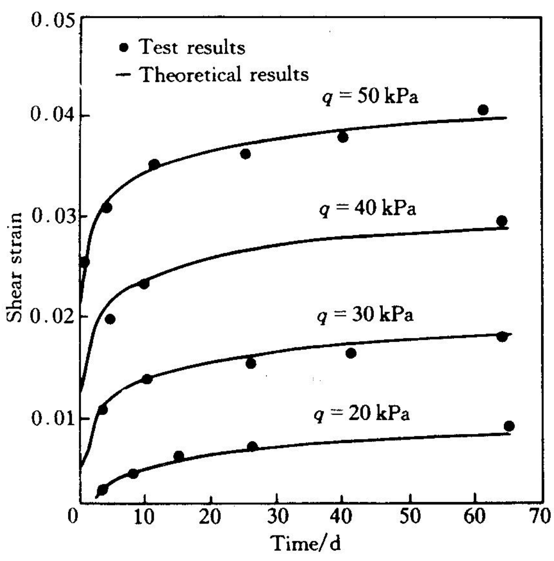

Fig.2 Theoretical curves versus results of torsional tests

Fig. 3 shows the theoretical creep curve, without deviatoric stress, comparing with the results of triaxial tests without deviatoric stress[9]. The compatibility of the theoretical result with the result of tests is also quite satisfactory.

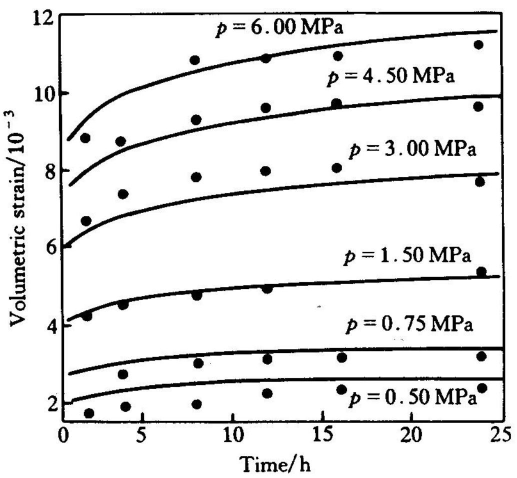

Fig.3 Theoretical curves of volumetric creep versus test results

Verification of Eqn. (8) was based on the famous experiments presented by Bishop[10]. Fig. 4 shows the distribution of the relative error between theoretical value and test value of the

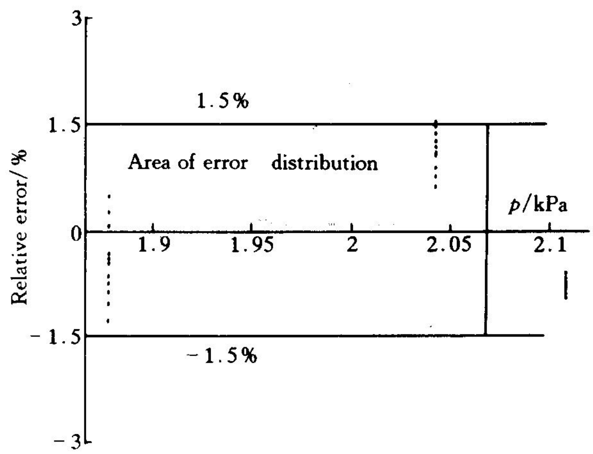

Fig.4 Distribution of relative error with shear stress

effective pressure, and the error scatters within a small area. This means Eqn. (8) performed well in predicting the relation among volumetric stress, strain and time.

Figs. 5 and 6 are distributions of the relative error between theoretical value and test value of the effective pressure under the condition without shear stress (i.e. $w=0$ in Eqn. (8)). The figures show that shear stress affects the relation between volumetric stress and strain of the normal consolidated soil slightly. But this effect on the pre-consolidated soil is obvious. This coincided perfectly with our knowledge on soil's dilatancy.

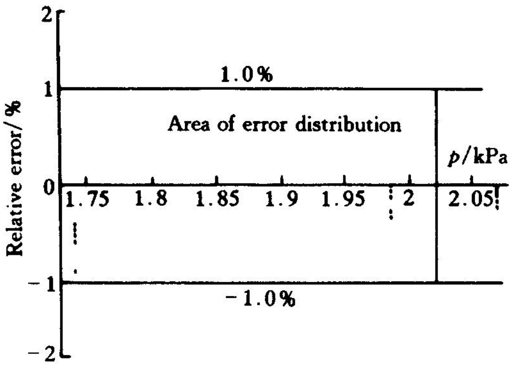

Fig. 5 Distribution of relative error of normal consolidated soil without shear stress

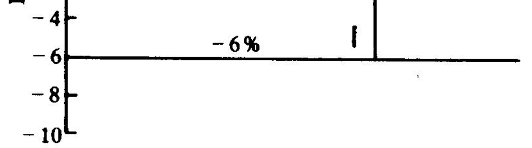

Fig. 6 Distribution of relative error of pre-consolidated soil without shear stress

Comparing theoretical results from Eqns. (7) and (8) with the results from tests, it is clear that the new constitutive equation based on endo-chronic theory performs very well in predicting the rheologic characteristics of soil.

4 CONCLUSION

Based on endo-chronic theory, the authors presented a new constitutive equation to predict the rheologic characteristics of soil under the condition of triaxial test. The verification with comparing theoretical results from Eqs. (7) and (8) with the results from tests showed that the new constitutive equation performs very well in predicting the rheologic characteristics of soil. As most problems met in civil engineering are problems under the condition of triaxial test, the new constitutive equation can be used widely in this field.

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