3D SLOPE STABILITY ANALYSIS WITH KINEMATICAL ELEMENT TECHNIQUE[①]

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ABSTRACT Kinematical element technique is a numerical method to analyze ultimate bound problems of Mohr-Coulomb materials. Its important applications are the analysis of bearing capacity of foundations, the determination of active or passive earth pressure as well as the evaluation of slope stability in geo-engineering. The principle of kinematical element method in analyzing stability of slope was discussed, on the created program, sensitivity of safety factor of slope for different parameters has also been analyzed using three dimensional kinematical element method.

Key words kinematical element method stability analysis three dimensional slope

1 INTRODUCTION

The analytical object of slope stability is to determine the factor of safety (F_s) against sliding, the limit equilibrium analysis based on the Mohr-Coulomb law is well used for different slopes. The real geometry of the plastic zone of a slope is unknown before the stability analysis, so theoretical solution of the factor of safety is not easy to obtain even for two dimensional problems due to difficulty in mathematically analyzing. The key to obtain a more accurate result of factor of safety is to search a more accurate sliding surface of slope. Kinematical element method had been created by adopting an optimization process in finding sliding surface which is much closer to the real condition than that found with conventional methods, it brought about a more reliable and more accurate solution for slope stability analysis. This paper will discuss the kinematical element method in analyzing the stability of three dimension slope problems.

2 PRINCIPLE OF KINEMATICAL ELEMENT METHOD

It is a process to analyze ultimate bound problems using kinematical element method, at first an initial approximate failure mechanism is assumed on experience or testing analysis, the failure mechanism consists of kinematical elements by separating plastic zone with selected slip lines[1]. Kinematical element method suggests: (1) Every element is rigid; (2) There is no dilatancy between any two adjacent elements; (3) There is only tangential movement along the common boundary of two adjacent elements; (4) there is no rotation for any element.

Based on above assumptions, kinematical element method uses kinematics and static analysis to determine the displacements of elements and acted forces at boundaries respectively, then the real plastic zone and critical sliding surface are searched using an optimization process.

2.1 Kinematics analysis

The aim of kinematics analysis is to determine the relative movement between any two adjacent elements. In kinematics analysis process, flexible boundaries have to be selected on experience. If an external boundary of slope is free from loading and its moving direction can be pre-known, this kind of free boundary can be select-

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ed as flexible boundary in kinematics analysis. Giving displacement for every one of the flexible boundaries, it will induce displacements for all elements considering the assumed harmonic of displacements between elements, the harmonic equations form kinematics equation system[2-4]:

$$[K]\{v\} + \{\hat{v}\} = 0 \tag{1}$$

where $[K]$ —kinematics matrix, the elements of the matrix are function of direction cosine of boundaries, $\{v\}$ —unknown vector consists of displacements of elements, $\{\hat{v}\}$ —known vector including the given displacements for flexible boundaries.

After solving algebraic Eqn. (1), the unknown displacements of elements will be obtained, then the relative tangential displacement between any two adjacent elements can be evaluated.

2.2 Static analysis

On the base of kinematics analysis, the acting direction of shear forces acted at internal boundaries of elements can be determined by considering that the acting direction of a shear force is opposite to the tangential movement. Because any internal boundary of a kinematics element is a segment of assumed slip lines in the plastic zone of Mohr-Coulomb material, the normal and shear force acted at the internal boundary yield Mohr-Coulomb law, considering the limit equilibrium equation, the shear force S^i at the ith internal boundary can be expressed with the normal force N^i at the boundary:

$$S^i = N^i f + cR^i \tag{2}$$

where f is factor of friction, c is cohesion, R^i is the area of the ith internal boundary.

From Eqn.(2) it is known that only one of these two forces is independent. Supposing that the normal force is variable, so equilibrium equations of any element can be expressed with normal forces, body forces, factor of friction and cohesion. Assembling equilibrium equations over all elements, we can obtain the following static equation system, in which the unknowns are the normal forces acted at internal and flexible boundaries[2-4]:

$$[K_s]\{N\} + \{F\} = 0 \tag{3}$$

where $[K_s]$ —static matrix determined with coordinates of the nodes and factor of friction, $\{N\}$ —unknown vector consists of normal forces acted at internal and flexible boundaries, $\{F\}$ —known vector related to body forces, neutral pore pressure and cohesion.

The solution of above static algebraic equations gives normal forces at all internal and flexible boundaries. In the static analysis only equilibrium of resultant is considered. If the acting points of the forces at boundaries had been determined, the equilibrium of resultant moment of every element can give additional equations to check the static condition of elements.

2.3 Optimization

Kinematics and static analysis of kinematical element technique are both based on an assumed failure mechanism. For two dimensional case, the decision failure mechanism of a slope is determined with an optimization process to find the minimum factor of safety[2-4]. For three dimensional slope the dissipated work done by the shear forces and related tangential displacements of internal boundaries is employed as the objective function of optimization analysis in the paper. Because that any element of matrix $[K_s]$ is a function of the coordinates of nodes of kinematical elements, it indicates that the total dissipated work E can be expressed with an implicit function of node coordinates of kinematics elements:

$$E = F(x_1, y_1, z_1, \cdots, x_m, y_m, z_m) \tag{4}$$

where m is the number of nodes; $x_1, y_1, \cdots, x_m, y_m$ and z_m are coordinates of the nodes respectively. The decision geometry of the plastic zone of a three dimension slope depends on an optimization process to find the minimum dissipated work E with variation of coordinates of the nodes. Suggesting that the real plastic zone is determined by the coordinates of nodes $x_1^*, y_1^*, z_1^*, \cdots, x_m^*, y_m^*$ and z_m^* so the minimum dissipated work will be

$$\begin{aligned} E_{\min} &= F_{\min}(x_1, y_1, z_1, \cdots, \\ &\quad x_m, y_m, z_m) \\ &= F(x_1^*, y_1^*, z_1^*, \cdots, \end{aligned}$$

$x^*_m, y^*_m, z^*_m)$ (5)

On the determined plastic zone by x_1^*, y_1^*, z_1^*, …, x^*_m, y^*_m, and z^*_m the factor of safety can be calculated at last.

3 COMPLEMENTED FORCES AT FLEXIBLE BOUNDARIES

3.1 Kinematical characteristic of slope

The choosing of flexible boundaries is important in kinematics analysis. If some free boundaries near the upper part of sliding body of a slope had been chosen as flexible boundaries, the flexible boundaries will move along the anti-normal direction of the boundaries respectively (Fig.1). Suggesting that there is unity virtual displacement in the anti-normal direction, so the kinematics equations similar to Eqn. (1) can be formed for three dimensional slope. The solution of the kinematical equation gives the displacements of all elements.

3.2 Complemented forces at flexible boundaries

Rewriting Eqn. (3) by separating the normal force at internal boundaries $\{F^i\}$ and the normal forces at flexible boundaries $\{F^{fl}\}$, the static equation system becomes

$$[K_s^{fl}\ K_s^i]\begin{Bmatrix} N^{fl} \\ N^i \end{Bmatrix} + \begin{Bmatrix} F^{fl} \\ F^i \end{Bmatrix} = 0 \qquad (6)$$

where $\{N^{fl}\}$ —sub-vector that consists of the normal forces at flexible boundaries, $\{N^i\}$ —sub-vector that consists of the normal forces at internal boundaries; $[K_s^{fl}]$, $[K_s^i]$ —two sub-matrices of static matrix $[K_s]$ induced by re-arranging vector $\{N\}$.

It is obvious that the flexible boundaries of slope are free from loading, the calculated normal forces at flexible boundaries $\{N^{fl}\}$ are fictitious and can be considered as complemented forces in the process to find the real plastic zone and critical sliding surface.

For an initially chosen failure mechanism or searched failure mechanism at a step of the optimization process, the complemented forces at flexible boundaries can be always determined by static analysis. On the obtained critical sliding surface with an optimization approach, the factor of safety can be determined while the resultant of the complemented forces is moved away from the flexible boundaries with an iteration calculations.

4 CALCULATION OF FACTOR OF SAFETY

From Eqn. (6) it is known that the normal force at any flexible boundary may not be just zero, the resultant of complemented forces at flexible boundaries indicates the stability condition of the slope. If the resultant is larger, equal to or less than zero, it means that the factor of safety of the slope is greater, equal to or less than 1 respectively. The determination of the factor of safety requires that the resultant of the complemented forces at flexible boundaries be vanished with an iterative process. For a sliding block, the factor of safety can be simply defined as

$$F_s = \tau_c / \tau_{cal} \qquad (7)$$

Fig.1 Flexible boundary of slope

where τ_c —critical shear stress, τ_{cal} —calculating shear stress.

This definition can be extended to evaluate the factor of safety of three dimensional slope with sliding blocks as kinematical elements. If an iterative process is employed to move the complemented forces away from flexible boundaries, both normal force and shear forces acted at internal boundaries have to be changed to keep the equilibrium of any element. From static analysis, Eqn. (6) can be expressed with factor of friction and cohesion:

$$[\ K_s^{fl}(f),\ K_s^i(f)] \begin{Bmatrix} N^{fl} \\ N^i \end{Bmatrix} + \begin{Bmatrix} F^{fl}(c) \\ F^i(c) \end{Bmatrix}^* = 0 \quad (8)$$

When the resultant of the complemented forces at flexible boundaries has been moved away, Eqn. (6) will become

$$\{N^{fl}\} = 0 \quad (9)$$

$$[K_s^i]^* \{N^i\}^* + \{F^i\}^* = 0 \quad (10)$$

where * indicates the matrix or vector formed during the process to move the complemented forces away from the flexible boundaries. The change of shear forces acted at the internal boundaries can be realized by substituting the factor of friction f and cohesion c with calculating value f_{cal} and c_{cal} respectively during the process, so Eqn. (10) becomes

$$[K_s^i(f_{cal})]^* \{N^i\}^* + \{F^i(c_{cal})\}^* = 0 \quad (11)$$

From the equation, $\{N^i\}^*$, f_{cal} and c_{cal}, which satisfy Eqn. (9) and equilibrium of any element, can be solved. From Eqn. (7), the factor of safety of a slope can also be defined with the following two formulas together:

$$F_s = c/c_{cal} \quad (12)$$

$$F_s = f/f_{cal} \quad (13)$$

Substituting Eqns. (12) and (13) into (11), there is

$$[K_s^i(f/F_s)]^* \{N^i\}^* + \{F^i(c/F_s)\}^* = 0 \quad (14)$$

From limit equilibrium Eqn. (2), it is known that elements in $[K_s^i(f/F_s)]^*$ are a linear function of $1/F_s$, so that Eqn. (14) can be simplified as

$$[K_s^i]^0 \begin{Bmatrix} N^i \\ 1/F_s \end{Bmatrix} + \{F^i(1/F_s)\}^* = 0 \quad (15)$$

Rewriting Eqn. (15) in an iterative form, there is

$$[K_s^i]^0 \begin{Bmatrix} N^i \\ 1/F_s \end{Bmatrix}_j^* + \{F(1/F_s)\}_{j-1}^* = 0 \quad (16)$$

where $(F_s)_j$ and $(F_s)_{j-1}$ are the factor of safety of the jth and $(j-1)$th iterative step respectively. At the start of analysis process, that is $j = 1$, an initial value of factor of safety has to be given, the initial value can be 1 in convenience. Assuming that there exists a given small positive value, if the factor of safety calculated at the jth and the $(j-1)$th iterative step satisfies:

$$\left| \frac{(F_s)_j - (F_s)_{j-1}}{(F_s)_j} \right| \leqslant \varepsilon \quad (17)$$

The $(F_s)_j$ will be just the factor of safety of the slope being searched.

5 NUMERICAL EXAMPLES

5.1 Simple case

On above mentioned principle, a related computer program of kinematical element method solving 3D slope problems had been created. For the purpose of comparison, a symmetry three dimensional slope was firstly analyzed with the program. Due to geometry symmetry, only half of the slope is separated into elements, the dip angle of the slope is 60° (Fig.2). The factors of safety calculated with kinematical element method and variation method are shown in Table 1.

From Table 1, it can be found that the factor of safety calculated with kinematical element method and variation method is quite close if only 8 kinematics elements are used.

5.2 Sensitivity analysis for different parameters

The 3D kinematical element program was employed to analyze the sensitivity of factor of

Table 1 Results of 3D slope stability analysis

Author	Element	c/ MPa	γ/ (MPa·m^{-1})	f	$c/(\gamma H)$	F_s
This paper	4	0.0208	0.01	0.268	0.116	1.40
	6	0.0208	0.01	0.268	0.116	1.32
	8	0.0208	0.01	0.268	0.116	1.26
Leshchinsky[5]					0.116	1.25

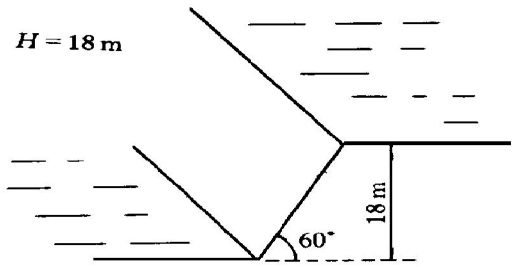

Fig.2 3D slope stability analysis model

safety of a symmetry slope, only half of the slope is considered which consists of 6 elements (Fig. 3). The factors of safety had been calculated for different cohesion while factor of friction and unit weight keep constant, the obtained factors of safety (F_s) are shown in Table 2. Table 3 shows the factors of safety for different unit weights in case factor of friction and cohesion keep constant.

Table 2 Factors of safety for different cohesions ($f = 0.5$, $\gamma = 0.027$ MPa/m)

c/MPa	0.010	0.015	0.020	0.025	0.030
F_s	0.964	1.120	1.312	1.356	1.567

Table 3 Factors of safety for different unit weights ($c = 0.002$ MPa, $f = 0.15$)

γ/(MPa·m^{-1})	0.60	0.50	0.40	0.30	0.20
F_s	0.552	0.754	0.951	1.269	1.611

From Table 2 and Table 3, it is shown that the factor of safety is in direct proportion to the cohesion and is in inverse proportion to the unit weight of the slope.

6 CONCLUSION

Kinematical element technique is an advanced method in analyzing ultimate bound problems in geo-engineering. If the dissipated work

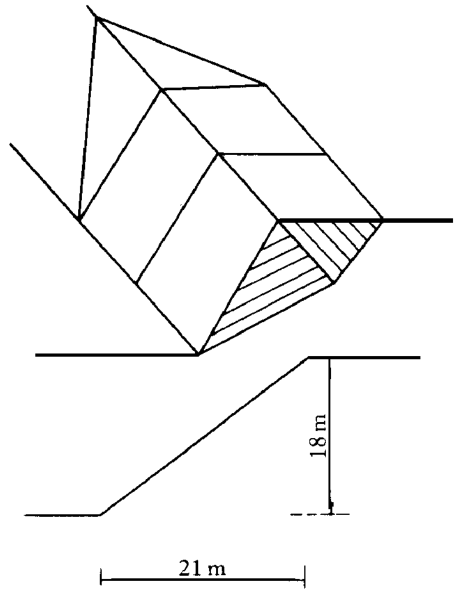

Fig.3 3-D slope stability analysis model

done by the failure mechanism is employed as an object function, kinematical element method can be extended to efficiently analyze the stability of three dimensional slopes, and the decision of the failure mechanism depends on how to find the minimum dissipated work with the variation of the coordinates of nodes of elements.

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