Vol. 9 №. 1 **Trans. Nonferrous Met. Soc. China** Mar. 1999

ROBUST ESTIMATION OF VARIANCE COMPONENTS MODEL①

Wang Zhizhong and Zhu Jianjun
Institute of Surverying Engineering and Land Information,
Central South University of Technology, Changsha 410083, P. R. China

ABSTRACT Classical least squares estimation consists of minimizing the sum of the squared residuals of observation. Many authors have produced more robust versions of this estimation by replacing the square by something else, such as the absolute value. These approaches have been generalized, and their robust estimations and influence functions of variance components have been presented. The results may have wide practical and theoretical value.

Key words influence function robust estimation variance component

1 INTRODUCTION

The least squares method is a very popular estimation technique in the linear regression (adjustment) model. But in spite of its mathematical beauty and computational simplicity, this estimation suffers a dramatic lack of robustness. Indeed, one single outlier can have an arbitrarily large effect on the estimation. Therefore, many scholars have studied this problem and presented many approaches of the robust estimations. "Robust estimation" was presented by Box in 1953 and was developed by Huber in 1964, 1965 and Hample in 1968[1]. The robust estimate was introduced into surveying data processing by Krarup and Kubik in 1967. Zhou Jianwen (1989) and Yan Yuanxi (1991) also had conducted much work in this field.

There is an extensive literature on robust estimation in the case of the single error component. There is, however, only a small body of literature on robust estimation in the variance components model. Arvesen and Layard (1975) used the jackknife to estimate the variance ratio in the two-component (one way) unbalanced model. Shoemaker (1980) and Rock (1983) obtained estimations and tests by robust modifications of the mean squares of the classical analysis of balanced designs. Fellner (1986) obtained the robust estimation of variances by modifying the defining equations for the restricted maximum likelihood estimations under normality along the lines of Huber's proposal 2.

The goal of this paper is also to investigate the robust estimations in the variance components model. In Section 2, we give the approximative maximum liklihood estimations of variance components, which also are the estimations of variance components proposed by Förstner (1979) and Ou Ziqiang (1989). In Section 3, we give the influence functions, which are the foundations of studying robustness properties. Finally, in Section 4 we present the robust estimation of variance components, which generalize robustified least squares method.

2 APPROXIMATIVE MAXIMUM LIKELIHOOD ESTIMATION

Methods of estimating variance components have been intensively investigated in the statistics and geodetic literature, such as Refs. [2] and [3]. The paragraph discusses the approximative maximum likelihood estimation of vari-

① Project 49774209 supported by the National Natural Science Foundation of China
Received May 27, 1997; accepted Feb. 8, 1998

ance components.

We consider the following linear model

$$y_{ij} = x_{ij}^{\mathrm{T}}\beta + e_{ij} \qquad \begin{cases} i = 1, 2, \cdots, k \\ j = 1, 2, \cdots, n_i \end{cases} \qquad (1a)$$

or $\quad \boldsymbol{Y} = X\beta + \boldsymbol{e} \qquad (1b)$

$$\mathrm{Cov}(\boldsymbol{Y}) = \Sigma = \mathrm{diag}[\sigma_1^2 \boldsymbol{I}_{n_1}, \sigma_2^2 \boldsymbol{I}_{n_2}, \cdots, \sigma_k^2 \boldsymbol{I}_{n_k}] \qquad (1c)$$

where $y_{ij} \in \mathbf{R}$ is the jth observation of the ith group, its distribution is denoted by F_{ij}; $\boldsymbol{y}_i$ is the ith group observation; $\boldsymbol{x}_{ij}^{\mathrm{T}} \in \mathbf{R}^p$ the jth row of the ith group of design matrix $X_{n\times p}$ with rank $(X) = p$, $n_1 + n_2 + \cdots + n_k = n$; $\beta \in \mathbf{R}^p$, a p-vector of unknown parameter; $\sigma_i^2 > 0$, the ith unknown variance component.

The observation error equations are

$$\boldsymbol{V} = \boldsymbol{Y} - X\hat{\beta} \text{ or } v_{ij} = y_{ij} - \boldsymbol{x}_{ij}^{\mathrm{T}}\hat{\beta} \qquad (2a)$$

where $\boldsymbol{V}$ is an $n \times 1$ residual vector and $\hat{\beta}$ the estimations vector of parameter β.

If the observations $\boldsymbol{Y}$ are assumed to be normally distributed, the liklihood function $L(\boldsymbol{Y}, \beta, \sigma_i^2)$ of the observations $\boldsymbol{Y}$ with unknown parameters β and σ_i^2 is given by

$$L(\boldsymbol{Y}, \beta, \sigma_i^2) = \frac{1}{(2\pi)^{n/2}(\det \Sigma)^{1/2}} \times \exp\left[-\frac{1}{2}(\boldsymbol{Y} - X\beta)^{\mathrm{T}}\Sigma^{-1}(\boldsymbol{Y} - X\beta)\right] \qquad (2b)$$

es

The likelihood function may be written as the product of two likelihood functions L_1 and L_2[10].

$$L(\boldsymbol{Y}, \beta, \sigma_i^2) = L_1(\boldsymbol{Y}, \sigma_i^2)\, L_2(\boldsymbol{Y}, \beta, \sigma_i^2) \qquad (3a)$$

with

$$L_1(\boldsymbol{Y}) = L_1(\boldsymbol{Y}, \sigma_i^2) = \frac{1}{(2\pi)^{\frac{n-p}{2}}[\det \Sigma \det(X^{\mathrm{T}}\Sigma^{-1}X)]^{1/2}} \times \exp\left[-\frac{1}{2}(\boldsymbol{Y} - X\hat{\beta})^{\mathrm{T}}\Sigma^{-1}(\boldsymbol{Y} - X\hat{\beta})\right] \qquad (3b)$$

and

$$L_2(\boldsymbol{Y}) = L_2(\boldsymbol{Y}, \beta, \sigma_i^2) = \frac{1}{(2\pi)^{p/2}(\det(X^{\mathrm{T}}\Sigma^{-1}X))^{-\frac{1}{2}}} \times \exp\left[-\frac{1}{2}(X^{\mathrm{T}}\Sigma^{-1}X\beta - X^{\mathrm{T}}\Sigma^{-1}\boldsymbol{Y})^{\mathrm{T}} \times (X^{\mathrm{T}}\Sigma^{-1}X)^{-1}(X^{\mathrm{T}}\Sigma^{-1}X\beta - X^{\mathrm{T}}\Sigma^{-1}\boldsymbol{Y})\right] \qquad (3c)$$

where

$$\hat{\beta} = (X^{\mathrm{T}}\Sigma^{-1}X)^{-1}X^{\mathrm{T}}\Sigma^{-1}\boldsymbol{Y} \qquad (3d)$$

By using the formula of Taylor series, we easily prove that L_1 may be approximatively written as

$$L_1(\boldsymbol{Y}, \sigma_i^2) \propto \sum_{i=1}^{k}(\sigma_i^2)^{-r_i/2}\exp(-S_i/2\sigma_i^2) \qquad (4)$$

where

$$r = \sum_{i=1}^{k} r_i$$

$$r_i = \mathrm{tr}(W_0 Q_i)\,\sigma_{i0}^2$$

$$S_i = \sum_{j=1}^{n_i} v_{ij}^2 = \sum_{j=1}^{n_i}(y_{ij} - \boldsymbol{x}_{ij}^{\mathrm{T}}\hat{\beta})^2$$

$$W_0 = \Sigma_0^{-1} - \Sigma_0^{-1}X(X^{\mathrm{T}}\Sigma_0^{-1}X)^{-1}X^{\mathrm{T}}\Sigma_0^{-1}$$

$$Q_i = \mathrm{diag}[0, \cdots, \boldsymbol{I}_{n_i}, \cdots, 0)]$$

$$\Sigma_0 = \mathrm{diag}[\sigma_{10}^2 \boldsymbol{I}_{n_1}, \sigma_{20}^2 \boldsymbol{I}_{n_2}, \cdots, \sigma_{k0}^2 \boldsymbol{I}_{n_k}]$$

where σ_{i0}^2 is the approximative value of σ_i^2, r_i the superfluous observation number of the ith group, v_{ij} takes the approximate values.

In order to estimate β and σ_i^2, according to the principle of the maximum likelihood estimation, L_1 and L_2 are to be maximized, i.e.,

$$\sum_{j=1}^{n_i}\sum_{i=1}^{k}\rho\left(\frac{y_{ij} - \boldsymbol{x}_{ij}^{\mathrm{T}}\beta}{\sigma_i}\right) = \frac{1}{2}(\boldsymbol{Y} - X\beta)^{\mathrm{T}} \times \Sigma^{-1}(\boldsymbol{Y} - X\beta) = \min \qquad (5)$$

DP

and

$$\sum_{i=1}^{k}\left[\sum_{j=1}^{n_i}\rho\left(\frac{y_{ij} - \boldsymbol{x}_{ij}^{\mathrm{T}}\beta}{\sigma_i}\right) + \frac{r_i \ln \sigma_i^2}{2}\right] = \mathrm{ri}\sum_{i=1}^{k}\left[\frac{S_i}{2\sigma_i^2} + \frac{r_i \ln \sigma_i^2}{2}\right] = \min \qquad (6)$$

or

$$\sum_{j=1}^{n_i}\sum_{i=1}^{k}\psi\left(\frac{y_{ij} - \boldsymbol{x}_{ij}^{\mathrm{T}}\beta}{\sigma_i}\right)\frac{\boldsymbol{x}_{ij}}{\sigma_i} = X^{T}\Sigma^{-1}(\boldsymbol{Y} - X\beta) = 0 \qquad (7)$$

vo

and on

$$\sum_{j=1}^{n_i}\psi\left(\frac{y_{ij} - \boldsymbol{x}_{ij}^{\mathrm{T}}\beta}{\sigma_i}\right)\left(\frac{y_{ij} - \boldsymbol{x}_{ij}^{\mathrm{T}}\beta}{\sigma_i}\right) - r_i = \frac{S_i}{\sigma_i^2} - r_i = 0 \quad i = 1, 2, \cdots, k \qquad (8)$$

where

$$\psi(x) = \rho'(x)$$

From Eqns. (7) and (8), we obtain the approximative maixmum likelihood estimations of β, σ_i^2

$$\hat{\beta} = (X^{\mathrm{T}}\Sigma^{-1}X)^{-1}X^{\mathrm{T}}\Sigma^{-1}\boldsymbol{Y} \qquad (9a)$$

$$\hat{\sigma}_i^2 = S_i / r_i, \quad i = 1, 2, \cdots, k \tag{9b}$$

It is an approximate estimation of variance components by Föstner(1979)[4] and can be found in Ref.[3]. This estimation is iteratedly unbiased. The approximative maximum likelihood estimations of β, σ_i^2 can be concluded by the following steps: by using estimations to improve the approximative values, Eqns.(9) are applied successively, until the convergence of estimations are obtained. Because the estimations $\hat{\beta}$, $\hat{\sigma}_i^2$ are obtained by the extremal condition Eqns.(5) and (6), we can generalize them to obtain robust estimation, robust Bayes and robust empirical Bayes estimation for variance components.

3 INFLUENCE FUNCTIONS

We generalize Eqns.(7) and (8) and call simultaneous M-estimation of location and scales any statistics $(T_n, S_n^{(1)}, S_n^{(2)}, \cdots, S_n^{(k)})$ determined by $k+1$ equations of the form

$$\sum_{j=1}^{n_i} \sum_{i=1}^{k} \psi\left(\frac{y_{ij} - x_{ij}^{\mathrm{T}} T_n}{S_n^{(i)}}\right) \frac{x_{ij}}{S_n^{(i)}} = 0 \tag{10}$$

$$\sum_{j=1}^{n_i} \chi\left(\frac{y_{ij} - x_{ij}^{\mathrm{T}} T_n}{S_n^{(i)}}\right) - r_i = 0, \quad i = 1, 2, \cdots, k \tag{11}$$

where

$$\chi\left(\frac{y_{ij} - x_{ij}^{\mathrm{T}} T_n}{S_n^{(i)}}\right) = \psi\left(\frac{y_{ij} - x_{ij}^{\mathrm{T}} T_n}{S_n^{(i)}}\right) \frac{y_{ij} - x_{ij}^{\mathrm{T}} T_n}{S_n^{(i)}}$$

Defining

$$T(F) = T(F_{11}, \cdots, F_{1n_1}, \cdots, F_{kn_1}, \cdots, F_{kn_k})$$

$$S^{(i)}(F) = S^{(i)}(F_{i1}, F_{i2}, \cdots, F_{in_i})$$

we have

$$\sum_{j=1}^{n_i} \sum_{i=1}^{k} \int \psi\left(\frac{y_{ij} - x_{ij}^{\mathrm{T}} T(F)}{S^{(i)}(F)}\right) \times \frac{x_{ij}}{S^{(i)}(F)} \mathrm{d}F_{ij} = 0 \tag{12}$$

$$\sum_{j=1}^{n_i} \int \chi\left(\frac{y_{ij} - x_{ij}^{\mathrm{T}} T(F)}{S^{(i)}(F)}\right) \mathrm{d}F_{ij} - \int r_i \mathrm{d}F_{ij} = 0, \quad i = 1, 2, \cdots, k \tag{13}$$

Neither ψ nor χ needs to be determined by a probability density as in Eqns.(10) and (11). In most cases, however, ψ will be an odd and χ be an even function, the influence functions can be found straightforwardly by inserting $G_{ij;y} = (1 - \varepsilon) F_{ij} + \varepsilon \Delta y$ for F_{ij} into Eqns.(12) and (13), where Δy is the point mass 1 by y and $0 \leqslant \varepsilon \leqslant 1$, and then taking the derivative with respect to ε at $\varepsilon = 0$. We obtain that $k+1$ influence curves $IF(y, F, T)$ and $IF(y_i, F, S^{(i)})$ $(i = 1, 2, \cdots, k)$ satisfy the system of equations

$$\begin{aligned} &\sum_{j=1}^{n_i} \sum_{i=1}^{k} \int \psi(z_{ij}) \frac{x_{ij}}{S^{(i)}(F)} \mathrm{d}(\Delta y_{ij} - F_{ij}) - \\ &\sum_{j=1}^{n_i} \sum_{i=1}^{k} \int \left\{ \psi'(z_{ij}) \frac{x_{ij} x_{ij}^{\mathrm{T}}}{[S^{(i)}(F)]^2} IF(y, F, T) + \psi'(z_{ij}) \frac{x_{ij} z_{ij}}{[S^{(i)}(F)]^2} IF(y_i, F, S^{(i)}) + \psi(z_{ij}) \frac{x_{ij}}{[S^{(i)}(F)]^2} IF(y_i, F, S^{(i)}) \right\} F_{ij} \\ &= 0 \end{aligned} \tag{14}$$

$$\begin{aligned} &\sum_{j=1}^{n_i} \left\{ \int \left[\chi(z_{ij}) - \frac{r_i}{n_i} \right] \mathrm{d}(\Delta y_{ij} - F_{ij}) - \int \left[\chi'(z_{ij}) \frac{x_{ij}^{\mathrm{T}} IF(y_i, F, T)}{S^{(i)}(F)} + \chi'(z_{ij}) \frac{z_{ij}}{S^{(i)}(F)} IF(y, F, S^{(i)}) \right] \mathrm{d}F_{ij} \right\} \\ &= 0 \quad i = 1, 2, \cdots, k \end{aligned} \tag{15}$$

where z_{ij} is short for $z_{ij} = \dfrac{y_{ij} - x_{ij}^{\mathrm{T}} T(F)}{S^{(i)}(F)}$

Making use of Eqns.(12) and (13), we have

$$\begin{aligned} &\sum_{j=1}^{n_i} \sum_{i=1}^{k} \int \psi'(z_{ij}) \frac{x_{ij} x_{ij}^{\mathrm{T}}}{[S^{(i)}(F)]^2} \mathrm{d}F_{ij}(y)\, IF(y, F, T) \\ &\sum_{j=1}^{n_i} \sum_{i=1}^{k} \int \frac{z_{ij} \psi'(z_{ij}) + \psi(z_{ij})}{[S^{(i)}(F)]^2} x_{ij} \mathrm{d}F_{ij}(y) \\ &IF(y_i, F, S^{(i)}) = \sum_{j=1}^{n_i} \sum_{i=1}^{k} \frac{\psi(z_{ij})}{S^{(i)}(F)} x_{ij} \end{aligned} \tag{16}$$

$$\sum_{i=1}^{n_i} \int \chi'(z_{ij}) z_{ij} \mathrm{d}F_{ij} IF(y_i, F, S^{(i)}) = \left(\sum_{j=1}^{n_i} \chi(z_{ij}) - r_i\right) S^{(i)}(F) \tag{17}$$

Because F_{ij} is symmetric, ψ is odd and χ is even. Defing $T(F)$ and $S^{(i)}(F)$ Fisher Consistency, we have

$$T(F) = \beta,\ S^{(i)}(F) = \sigma_i,\quad i = 1, 2, \cdots, k \tag{18a}$$

$$\int [z_{ij}\psi'(z_{ij}) + \psi(z_{ij})]\,\mathrm{d}F_{ij} = 0 \tag{18b}$$

$$\int \chi'(z_{ij})\,\mathrm{d}F_{ij} = 0 \tag{18c}$$

So, some integrals in Eqns.(16) and (17) vanish for reasons of symmetry and there are considerable simplifications.

$$IF(y, F, \beta) = \left[\sum_{j=1}^{n_i}\sum_{i=1}^{k}\int \psi'\left(\frac{y_i - x_{ij}^{\mathrm{T}}\beta}{\sigma_i}s\right)\cdot \frac{x_{ij}x_{ij}^{\mathrm{T}}}{\sigma_i^2}\mathrm{d}F_{ij}\right]^{-1}\sum_{j=1}^{n_i}\sum_{i=1}^{k}\frac{\psi\left(\frac{y_i - x_{ij}^{\mathrm{T}}\beta}{\sigma_i}\right)}{\sigma_i}\,\mathrm{d}s\, x_{ij} \tag{19}$$

$$IF(y_i, F, \sigma_i) = \frac{\sigma_i\left[\sum_{j=1}^{n_i}\chi\left(\frac{y_i - x_{ij}^{\mathrm{T}}\beta}{\sigma_i}\right) - r_i\right]}{\sum_{j=1}^{n_i}\int \chi'\left(\frac{y_i - x_{ij}^{\mathrm{T}}\beta}{\sigma_i}\right)\frac{(y_i - x_{ij}^{\mathrm{T}}\beta)}{\sigma_i^2}\,\mathrm{d}F_{ij}}$$

$$i = 1, 2, \cdots, k \tag{20}$$

Therefore, we get the influence functions of approximative maximum likelihood estimations

$$IF(Y, F, \beta) = (X^{\mathrm{T}}\Sigma^{-1}X)^{-1}X^{\mathrm{T}}\Sigma^{-1}\times(Y - X\beta) \tag{21}$$

$$IF(y_i, F, \sigma_i) = \frac{S_i/\sigma_i^2 - r_i}{2n_i}\sigma_i,\quad i = 1, 2, \cdots, k \tag{22}$$

when $k = 1$, we obtain the IFs of mixmum likelihood estimations in the Gauss-Mark old model

$$IF(Y, F, \beta) = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}(Y - X\beta) \tag{23}$$

$$IF(Y, F, \sigma) = \frac{S/\sigma^2 - (n - p)}{2n}\sigma \tag{24}$$

when $k = 1$ and σ^2 is known, the IFs of LS estimations are Eqn.(23). This is the result of Huber(1983)[1].

Based on the influence functions Eqns.(19) and (20), we can study local robustness properties, deepen our understanding of certain estimations, and derive new estimations with prespecified characterstics. For example, based on the influence functions Eqns.(21) and (22), we can obtain that the approximative maximum likelihood estimations Eqn.(9) have an asymptotic efficiency $e = 1$, but are not B-robust, etc.

4 THE ROBUST ESTIMATORS

The extremal condition of Eqns.(5) and (6) can be generalized as

$$\Omega_1 = \sum_{j=1}^{n_i}\sum_{i=1}^{k}\rho(z_{ij}) = \min \tag{25}$$

$$\Omega_2 = \sum_{i=1}^{k}\left[\sum_{j=1}^{n_i}\rho(z_{ij})_{h} + \frac{1}{2}r_i\ln\sigma_i^2\right] = \min \tag{26}$$

where $\rho(\cdot)$ is the suitable function satisfying.

(1) ρ is symmetric, continuously different and $\rho(0) = 0$.

(2) there exists $c > 0$ such that ρ is strictly increasing on $[0, c]$ and constant on $[c, \infty]$.

Taking derivatives of Eqns.(25) and (26), we obtain the following equations:

$$\sum_{j=1}^{n_i}\sum_{i=1}^{k}\frac{\partial\rho(z_{ij})}{\partial\beta} = \sum_{j=1}^{n_i}\sum_{i=1}^{k}\left[\frac{\partial\rho(z_{ij})}{\partial z_{ij}}\frac{1}{z_{ij}}\right]\frac{\partial z_{ij}}{\partial\beta}z_{ij} = -\sum_{j=1}^{n_i}\sum_{i=1}^{k}\sigma_i^{-2}w_{ij}x_{ij}v_{ij} = 0 \tag{27}$$

$$\sum_{j=1}^{n_i}\frac{\partial\rho(z_{ij})}{\partial\sigma_i^2} + \frac{r_i}{2\sigma_i^2} = \sum_{j=1}^{n_i}\left[\frac{\partial\rho(z_{ij})}{\partial z_{ij}}\frac{1}{z_{ij}}\frac{\partial z_{ij}}{\partial\sigma_i^2}z_{ij}\right] + \frac{r_i}{2\sigma_i^2} = -\sum_{j=1}^{n_i}\frac{1}{2\sigma_i^4}w_{ij}v_{ij}^2 + \frac{r_i}{2\sigma_i^2} = 0$$

$$i = 1, 2, \cdots, k \tag{28}$$

where $w_{ij} = \frac{\psi(z_{ij})}{z_{ij}} = \frac{\partial\rho(z_{ij})}{\partial z_{ij}}\frac{1}{z_{ij}}$ is called weight factor[5].

Eqns.(27) and (28) can be rewritten as

$$\hat{\beta} = (X^{\mathrm{T}}\bar{P}X)^{-1}X^{\mathrm{T}}\bar{P}Y \tag{29a}$$

$$\hat{\sigma}_i^2 = \frac{1}{r_i}\sum_{j=1}^{n_i}w_{ij}v_{ij}^2\quad i = 1, 2, \cdots, k \tag{29b}$$

$$\bar{P} = \mathrm{diag}\left[\frac{w_{11}}{\sigma_1^2}, \cdots, \frac{w_{1n_1}}{\sigma_1^2}, \cdots, \frac{w_{k1}}{\sigma_k^2}, \cdots, \frac{w_{kn_k}}{\sigma_k^2}\right] \tag{30}$$

From Eqn.(29), we know that the robust estimations have the same forms as the approximative maximum likelihood estimators.

From Eqns.(19) and (20), we also obtain the influence functions of the robust estimations.

$$IF(Y, F, T) = (X^{T} MX)^{-1} X^{T} \bar{P} \times (Y - X\beta) \quad (31)$$

$$IF(y_i, F, \sigma_i) = \frac{pl \left[\sum_{j=1}^{n_i} \frac{w_{ij} v_{ij}^2}{\sigma_i^2} - r_i \right] \sigma_i}{\sum_{j=1}^{n_i} \int \frac{v_{ij}^2}{\sigma_i^2} \left[\psi'(z_{ij}) + w_{ij} \right] dF_{ij}}$$

$$i = 1, 2, \cdots, m \quad (32)$$

where

$$M = \mathrm{diag}\left[\frac{q_{11}}{\sigma_1^2}, \cdots, \frac{q_{1n}}{\sigma_1^2}, \cdots, \frac{q_{k1}}{\sigma_k^2}, \cdots, \frac{q_{kn_k}}{\sigma_k^2} \right]$$

$$q_{ij} = \int \psi'(z_{ij}) dF_{ij} \quad \begin{matrix} i = 1, 2, \cdots, k \\ j = 1, 2, \cdots, n_i \end{matrix}$$

The robust estimations can be concluded by the following steps:

(1) Because the approximative maximum likelihood estimations have been determined in section 2, we can make use of the approximative maximum likelihood estimation $\hat{\beta}$ to replace β, thus we obtain the residuals $v_{ij} = y_{ij} - x_{ij}^{T}\beta$. Also we can make use of the approximative maximum likelihood estimation $\hat{\sigma}_i^2$ to replace σ_i^2 in Eqn.(30).

(2) The weight factor w_{ij} can be determined according to Ref.[5]. For example:

when $\rho(z_{ij}) = z_{ij}^2$,

$w_{ij} = 1$;

when $\rho(z_{ij}) = |z_{ij}|$,

$$w_{ij} = \frac{1}{|z_{ij}|},$$

here $z_{ij} = \frac{v_{ij}}{\sigma_i} = \frac{y_{ij} - x_{ij}^{T}\beta}{\sigma_i}$

(3) Determine $\hat{\sigma}_i^2$ by Eqn.(29b) and $\bar{P}$ by Eqn.(30).

(4) Determine the $\hat{\beta}$ by Eqn.29(a)

(5) If the necessary convergence criterion is satisfied, quit. Otherwise return to step 2 to continue.

5 CONCLUSION

The work summarized here comprises: the approximative maximum likelihood estimations Eqn.(9), the influence functions Eqns.(19) and (20), the robust estimations Eqn.(29). The paper also gives the computational steps of solving the approximative maximum likelihood estimations and the robust estimations.

It must be emphasized that the developments of the robust estimations of variance components are very much preliminary. There are no doubt many ways in which the work could be improved, expanded and modified. The author thinks that the robust bays and robust empirical estimations for variance components are worthwhile studying.

REFERENCES

1 Huber P J. J Am Statist Assoc, 1983, 78: 68 - 80.

2 Koch K R. Bulletin Géodésique, 1986, 60: 329 - 338.

3 Ou Ziqiang. Bulletin Géodésique, 1989, 63: 139 - 148.

4 Förstner W. Allgemeine Vermessungs-Nachrichten, 1979, 96: 446 - 453.

5 Zhou Jiangwen. Acta Geodetica et Cartographica Sinica, 1989, 18: 115 - 120.

6 Huber P J. Robust Statistic. New York: Weley, 1981: 1.

7 Huber P J. Ann Math Statist, 1964, 35: 73 - 101.

8 Hampel F R. Robust Statistic. New York: Weley 1986: 1 - 20.

9 Yang Yuanxi. Bulletin Géodésique. 1991, 65: 145 - 150.

10 Ou Ziqiang. AVN-International edition. 1990, 7: 25 - 31.

11 Arvesen J N and Layard M W. J Annal of Statistics, 1975, 3: 1123 - 1134.

12 Fillner W H. Technometrics, 1986, 28(1): 51 - 60.

13 Wang Zhizhong. Trans Nonferrous Met Soc China, 1997, 7(4): 160 - 163.

(Edited by He Xuefeng)